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TECHNICAL NOTE

OPTIMAL DIGITAL FILTERING REQUIRES A DIFFERENT CUT-OFF FREQUENCY STRATEGY FOR THE DETERMINATION OF THE HIGHER DERIVATIVES

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Abstract—The present study investigated four different filtering and differentiation sequences for the calculation of the higher derivatives from noisy displacement data when using a second-order Butterworth filter and first-order finite differences. These were: (1) the conventional sequence (i.e. filtering the displacement data and then differentiating); (2) filtering the displacement with a different cut-off frequency depending upon optimal 0th, 1st and 2nd derivatives; (3) double filtering and differentiation (only for acceleration); and (4) differentiation and then filtering separately in each derivative domain, i.e. treating the noisy higher derivatives as individual signals. Thirty levels of time domain and 30 levels of frequency domain computer-generated pure noise signals, were superimposed on 24 reference signals which simulated the medial–lateral, anterior–posterior and vertical displacement patterns of eight markers attached to the lower extremity segments during walking. The optimum cut-off frequency for the displacement, velocity and acceleration data was calculated as the one that produced the minimum root mean square error between the reference and noisy data in each derivative domain. The results indicated that the conventional strategy has to be reconsidered and modified, as the best results were obtained by the second strategy. The optimum cut-off frequency for acceleration was lower than that required for the velocity which in turn was lower than the optimum cut-off frequency for displacement. The findings of the present study will contribute to the development of existing and future automatic filtering techniques based on digital filtering. © 1997 Elsevier Science Ltd

Keywords: Digital filtering; Smoothing; Differentiation; Recursive.

INTRODUCTION

Most automatic filtering techniques used in biomechanics which are based on digital filtering (DAmico and Ferrigno, 1990; Jackson, 1979; Winter *et al.*, 1974; Yu, 1989) calculate the optimal cut-off frequency by considering only the displacement data. The higher derivatives are calculated by filtering the displacement data and then differentiating in the time or frequency domain. However, the frequency characteristics of a signal is different for derivatives of different order. Therefore, a different cut-off frequency might be required for each derivative.

In the study of Vaughan (1982) in which the criterion was gravitational acceleration, the best results with a digital filter were obtained when data were filtered twice. However it was not justified why the data were filtered twice, or why the lowest cut-off was not initially applied for filtering the displacement data. This double filtering-differentiation strategy applied for the calculation of acceleration has not yet been investigated.

Hatze (1981) introduced an automatic filtering method (ORFOS) which calculates the optimal filtering factor separately in every derivative domain. Hence instead of first filtering and then differentiating the displacement data, as with the conventional approach, the data were first differentiated and then filtered. This procedure guarantees a truly optimal regularisation for each of the derivatives (Hatze, 1981). This differentiation and filtering sequence is used only by ORFOS and has not yet been evaluated.

It is evident that there are at least three more filtering strategies, along with the one conventionally used, that have not yet been investigated. In summary the other three strategies are: (a) filtering the displacement data with a cut-off frequency different from the optimum cut-off frequency for displacement; (b) double filtering – differentiation; and (c) separate filtering in each derivative domain.

An understanding of which strategy is more effective will enable investigators, who use digital filters for smoothing, to obtain more accurate results. It will also offer a means to improve the performance of existing and future automatic smoothing techniques based on digital filters. The purpose of this study was therefore to compare all four strategies. Because filtering becomes more important when the first and second derivatives are examined, the present study will focus on the optimal derivation of velocities and accelerations.

METHODS

*Signal development**

Twenty-four signals (3 × 8) simulating the medial–lateral, anterior–posterior and vertical displacements of eight markers attached to the right lower segments, during approximately one walking cycle were used as reference signals. The source of these signals was the file 'woman' from GAITLAB (Vaughan *et al.*, 1992) which was sampled at 50 Hz. The signals were approximated by Fourier series and then reconstructed again in the time domain by using only the first six harmonics (cut-off frequency = 6.25 Hz). This procedure ensured that the new signals, along with their higher derivatives, are practically band-limited and that the associated accelerations were smooth (Fig. 1).

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* All signals as well as additional information on signal development can be downloaded from the International Society of Biomechanics WWW (http://www.kin.ucalgary.ca/isb/giakas).

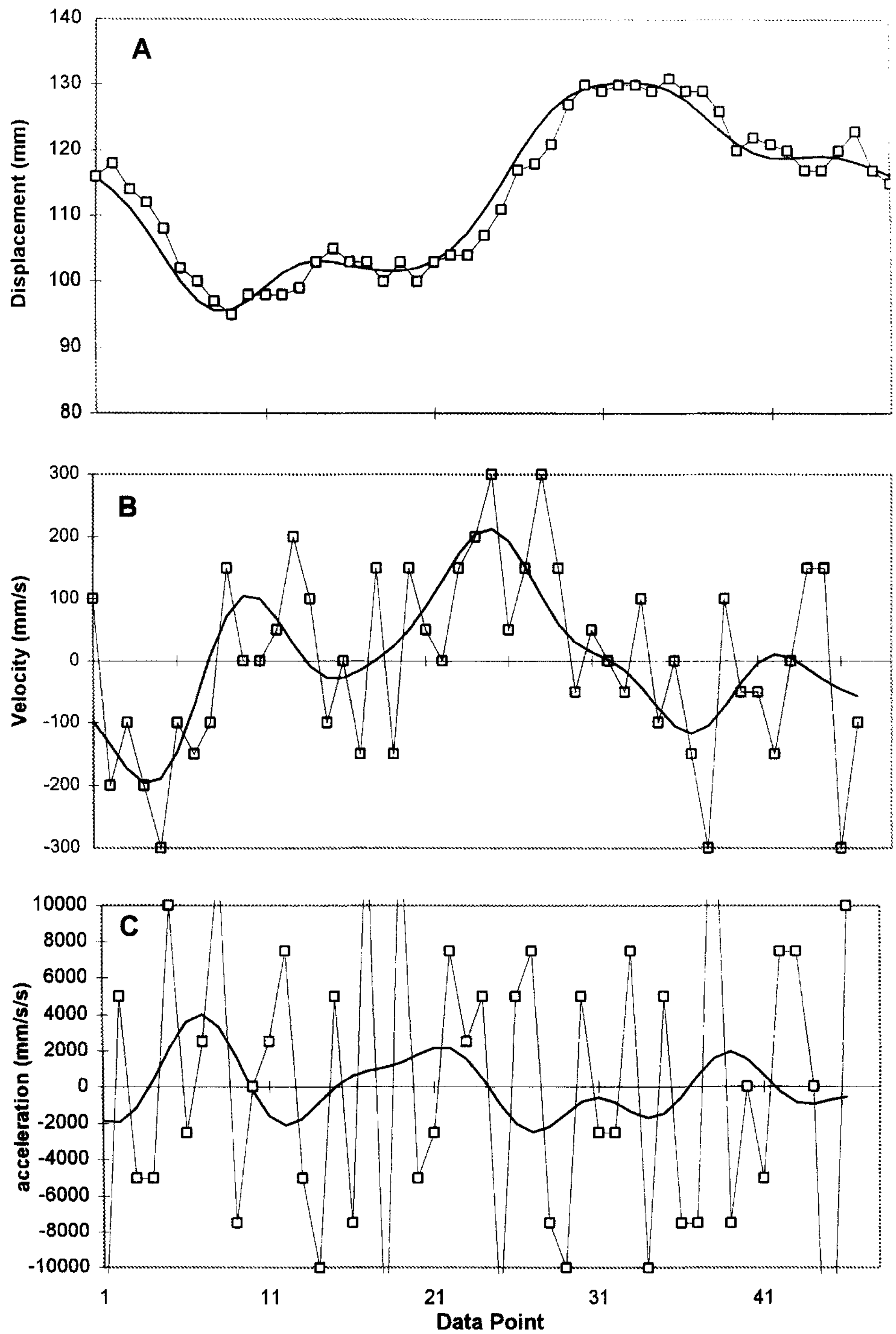


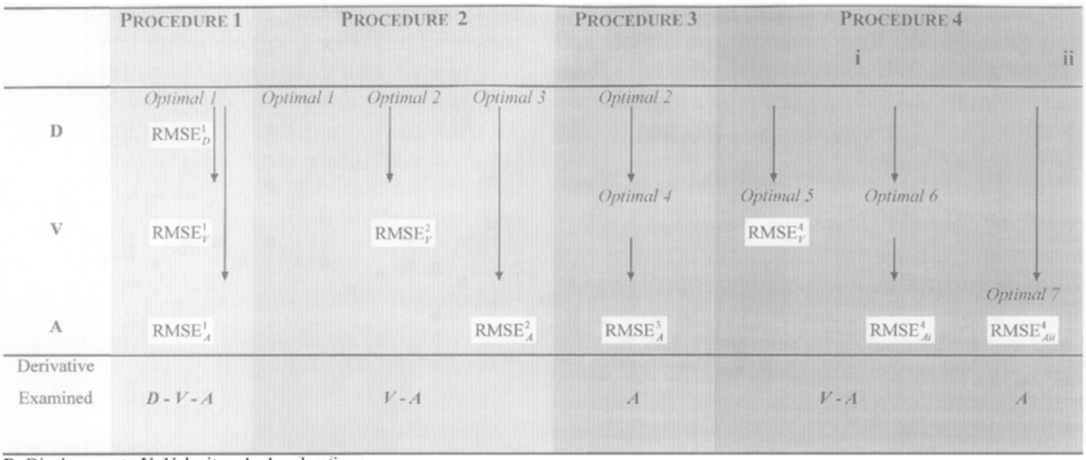
Fig. 1. Before (line with squares) and after filtering (plain line) the displacement data (A) of the medial–lateral movement of the marker attached upon the right greater trochanter using truncated Fourier series, and the calculated velocity (B) and acceleration (C) patterns.

A power spectrum analysis suggested that the new reference signals had more than 99.7% of the power of the original signal (Winter *et al.*, 1974).

Two types of noise, with 30 levels each, were generated: one type in the time domain ('random' noise) and the second in the frequency domain ('white' noise). Because of the small number of data points in each signal ($N = 48$), the two types of noise had slightly different characteristics. The 60 different combinations of noise type and level were superimposed onto the 24 reference walking signals giving a total number of 1440 noisy signals. The range of the mean added noise, 0.15–4.57 mm, was considered typical for video cameras and other optoelectronic systems (Dapena, 1978; Ehara *et al.*, 1995; Leroux *et al.*, 1990; Whittle, 1982). The mean signal-to-noise ratio ranged from 10^2 (level 30) to 10^9 (level 1) approximately.

Data analysis

A recursive second-order low-pass Butterworth filter was used to filter the data (Vaughan, 1982; Winter, 1979). A reversed mirror extrapolation technique was applied at both ends of the data as suggested by Smith (1989) before filtering. The length of the extrapolation at each end was half the data set length. First-order finite differences (Miller and Nelson, 1973; Vaughan, 1982), which have no smoothing effect



D: Displacement; **V**: Velocity; **A**: Acceleration

Fig. 2. The procedures (Procedure 1–4) followed. *Optimal* indicates filtering; lines with arrows indicate differentiation. *Optimal 1*: Optimal filtering of displacement; *Optimal 2*: Filtering of displacement for optimal velocity; *Optimal 3*: Filtering of displacement for optimal acceleration; *Optimal 4*: Filtering of smoothed velocity for optimal acceleration; *Optimal 5*: Filtering of unsmoothed velocity for optimal velocity; *Optimal 6*: Filtering of unsmoothed velocity for optimal acceleration; *Optimal 7*: Optimal filtering of unsmoothed acceleration. RMSE$^i_{D,V,A}$ (i = 1–4, indicates the Procedure number): Associated root mean square error for each data process.

Table 1. Cut-off frequencies required for optimal displacement (*Optimal* 1–Procedure 1). Key: #1 = Right metatarsal head V; #2 = R. Heel; #3 = R. Malleolus; #4 = R.Tibial tubercle; #5 = R. Femoral epicondyle; #6 = R. Greater trochanter; #7 = R. ASIS; #15 = Sacrum

	Anterior–Posterior						Medial–lateral						Vertical					
	Random			White			Random			White			Random			White		
#	Range	$\bar{M}$	± S.D.	Range	$\bar{M}$	± S.D.	Range	$\bar{M}$	± S.D.	Range	$\bar{M}$	± S.D.	Range	$\bar{M}$	± S.D.	Range	$\bar{M}$	± S.D.
1	5.8–10	8.0	± 1.2	6.4–10	7.7	± 1.0	3.8–10	5.7	± 1.4	4.4–7.0	5.1	± 0.9	4.6–10	7.4	± 1.6	5.6–9.6	6.7	± 0.9
2	5.8–10	8.0	± 1.3	6.4–10	7.5	± 0.9	3.8–10	6.2	± 1.5	4.4–7.8	5.7	± 1.0	4.4–10	6.7	± 1.5	5.0–8.4	5.0	± 0.9
3	5.8–10	7.9	± 1.3	6.4–10	7.4	± 1.0	4.4–10	6.2	± 1.4	4.2–7.6	5.7	± 0.9	4.4–10	6.6	± 1.5	5.0–8.0	6.0	± 0.8
4	4.0–10	6.3	± 1.4	4.4–9.2	6.0	± 0.9	2.0–9.4	5.3	± 1.8	1.6–7.0	4.2	± 1.5	6.2–10	8.2	± 1.3	6.2–10	8.1	± 1.0
5	4.6–10	6.7	± 1.4	5.2–9.4	6.3	± 0.9	2.0–8.6	4.2	± 1.6	1.8–5.6	3.7	± 1.0	4.2–9.4	6.0	± 1.3	3.8–8.0	5.6	± 0.9
6	4.4–10	6.2	± 1.3	4.0–9.2	5.9	± 1.0	3.4–9.4	5.1	± 1.4	2.6–7.6	4.3	± 1.1	4.4–9.4	6.1	± 1.4	4.2–7.8	5.6	± 0.9
7	3.8–9.6	5.9	± 1.3	4.0–7.4	5.6	± 0.9	3.6–9.6	5.5	± 1.4	4.0–8.8	5.2	± 1.1	3.8–9.4	5.7	± 1.3	3.8–7.4	5.1	± 0.8
15	3.0–9.2	4.9	± 1.5	2.8–6.0	4.4	± 1.0	3.4–10	5.7	± 1.7	2.8–7.8	5.2	± 1.4	3.4–8.2	5.4	± 1.2	3.2–7.2	4.9	± 0.9

# Indicates marker number.

(in order to avoid smoothing of the data through higher order finite differences (Pezzack *et al.*, 1977)), were used to calculate the higher derivatives. The optimal cut-off frequency was calculated by minimising root mean square error (RMSE) between the filtered and reference signals in every derivative domain (0th, 1st and 2nd) (Corradini *et al.*, 1993; Vint and Hinrichs, 1996).

The four strategies applied are briefly described in Fig. 2. Refer also to Fig. 2 for explanation of abbreviations. The cut-off frequencies were calculated separately for each data file. Procedure 1 represents the conventional strategy used. The displacement was filtered for optimal displacement. The higher derivatives were then computed from the filtered displacement signals. In Procedure 2, the displacement was filtered with a different cut-off frequency depending upon optimal 0th, 1st and 2nd derivatives. In Procedure 3, the technique used by Vaughan (1982) was tested, i.e. double filtering – differentiation. The displacement was filtered and differentiated for optimal velocity; the velocity data were then filtered with a new cut-off and differentiated for optimal acceleration. Procedure 4 simulated the strategy used by ORFOS. In the first part [Procedure 4(i)], the displacement data were differentiated once, and were then filtered in the velocity domain for optimal velocity or along with differentiation for optimal acceleration. In the second part [Procedure 4(ii)], the displacement data were differentiated twice and then were filtered in the acceleration domain for optimal acceleration.

RESULTS

Table 1 presents the calculated cut-off frequencies (range, mean and standard deviations) for the Y, X and Z direction of each marker when 'random' and 'white' noise is superimposed, using the conventional method (Procedure 1). This table shows how important the original signal is for the selection of optimal cut-off frequency. For example in the anterior–posterior direction, the cut-off frequencies required for the three lowest markers (#1, #2 and #3) were clearly higher than the cut-off frequencies required for the three highest (#6, #7 and #15).

In all the results and figures presented below, the 30 levels of 'random' noise only are shown; similar results, however, were obtained by the 30 levels of 'white' noise. Fig. 3 shows the mean RMSE for the velocity and acceleration when 'random' noise is superimposed onto the reference data. The lower limit for the stochastic error in each estimated derivative was also calculated as a function of measurement noise variance, sampling interval, signal bandwidth (cut-off frequency) and derivative order using the formula given by Lanshammar (1982). It is evident that the level of noise affects dramatically the effectiveness of the filter. The lower limit for the stochastic error increases as noise level increases.

The RMSE values in the velocity domain of Procedure 2 were always lower than those of Procedure 1 and lower than those of Procedure 4 in

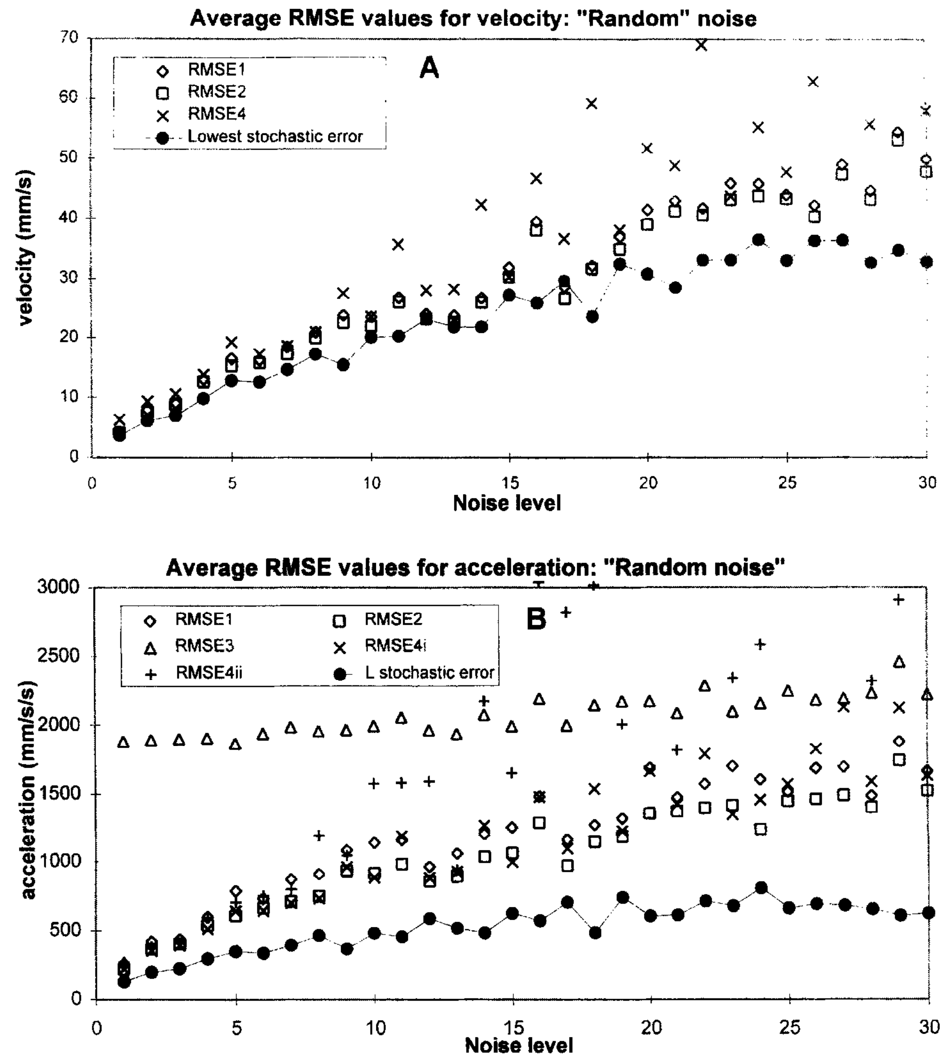


Fig. 3. Mean RMSE for the velocity (A) and acceleration (B) when 'random' noise is superimposed in the reference data. Refer to Fig. 2 for the RMSE abbreviations. The lower limit for the stochastic error was calculated using the formula of Lanshammar (1982).

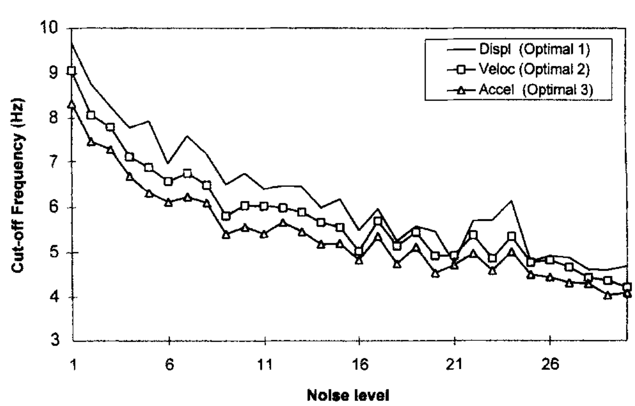


Fig. 4. The cut-off frequencies required for procedure 2 for 'random' noise. *Optimal 1* represents the cut-off frequency used by the conventional strategy. Refer to Fig. 2 for the abbreviations.

55 out of 60 cases. Concerning the accelerations, the RMSE produced by Procedure 2, in most cases, was lower than all the other procedures. The RMSE from procedures 3 and 4(ii) produced in most cases the highest RMSE values in the acceleration domain (Fig. 3).

Generally, the calculated cut-off frequencies decreased as noise level increased. Procedure 2 produced the best results and therefore the associated cut-off frequencies calculated are only displayed (Fig. 4). It is apparent that the cut-off frequencies required for optimal acceleration were lower than those required for optimal velocity which in turn were lower than those required for optimal displacement.

DISCUSSION

Four different filtering strategies (including the conventional filtering–differentiation strategy), were applied and compared in terms of the root mean square error (RMSE) between the reference and smoothed signal. The criterion was the minimum RMSE. Two of the strategies applied, Procedures 3 and 4(ii), aiming to calculate the acceleration (Fig. 3), were found to be inappropriate because they produced very high RMSE values.

The RMSE values increased with noise level. It is evident from Fig. 3A (velocity) that the RMSE was approximately as high as the lowest stochastic (theoretical) error calculated by the formula found in Lanshammar (1982) especially for the low levels of noise. However as the noise level increases some systematic error is included and the RMSE is much higher than the theoretical error. This is more evident in the acceleration domain (Fig. 3B).

The strategy of double filtering (Procedure 3) produced unacceptable results in the acceleration pattern. It is remarkable that the RMSE is high even at the very low levels of noise (Fig. 3). The double filtering forced the acceleration data to follow a very smooth pattern with a tendency to become flat, i.e. it oversmoothed the data. The fact that this strategy produced the best results in digital filtering when used by Vaughan (1982), can be explained from the nature of the data used. Vaughan's data had a flat second derivative (gravitational acceleration), and therefore the results obtained were ideal as explained above. The acceleration patterns in this procedure presented boundary problems as well. This is probably because of the boundary effects of recursive digital filters and the application of this type of filtering twice. It is therefore suggested that this strategy should be avoided for general purpose filtering: however, whenever applied it has to be used with caution especially at the boundaries of the signal.

The 'reverse' strategy of differentiating and then filtering (Procedure 4) did not improve (decrease) the RMSE of the higher derivatives. It seemed that the noise was substantially amplified after differentiation (ill-posed problem), resulting in a dramatic decrease of the signal-to-noise ratio and, consequently, ineffective filtering. However, the acceleration calculated from Procedure 4(i) was better than the one obtained by Procedure 1 and sometimes slightly better by Procedure 2. This phenomenon was observed at the low levels of noise. Therefore, it might be an alternative strategy that can be applied when the quality of data (signal-to-noise ratio) is high and the acceleration is the only derivative required. However, it has to be used with caution because there are also edge effects especially in the last 3 to 5 points at the boundaries.

Generally, Procedures 1 and 2 produced consistently the best results for both velocity and acceleration. There was a systematic improvement in the higher derivatives when Procedure 2 was applied compared to Procedure 1. The application of the second strategy improved the average RMSE values by 4.4% in velocity and 13.5% in acceleration compared to the first strategy.

The cut-off frequency required in Procedure 2 for the acceleration, was lower compared to the one required for the velocity, and that was lower than the one required for the displacement data (Fig. 4). This occurred because noise is random in the time and frequency domains and therefore it is distributed across the range of frequencies. Filtering a signal with a cut-off frequency f_c will allow the noise that is just below f_c to be magnified through the differentiation procedures thus affecting the reference signal. Therefore, a lower cut-off is required to obtain a 'smoother' higher derivative. The comparison between Procedures 1 and 2 in average terms suggested that the cut-off frequency for optimal velocity and acceleration was 0.46 ± 0.28 Hz and 0.86 ± 0.36 Hz lower, respectively, than the one estimated for optimal displacement. With this filtering strategy the calculation of the higher derivatives was more effective for the data used by the present study.

The cut-off frequency in digital filtering of biomechanical data is usually over- or under-estimated. In the case of over-estimation, i.e. using a higher cut-off frequency, more noise will be allowed to pass through the filtering process; by using the conventional method (Procedure 1) this noise will be amplified leading to erroneous higher derivatives. The application of the strategy proposed by the present study (Procedure 2) will decrease the high-frequency components in the calculation of the velocity and acceleration. In this case, the improvement in the higher derivatives will probably be higher compared to those reported above. However there might be cases in which consistence is needed between position, velocity and acceleration; in these cases the conclusion of the present study does not apply and the conventional method should be used.

In conclusion, the results of the present study show that the standard protocol of filtering displacement data and then, by differentiation, calculating the higher derivatives should be avoided. Better results can be obtained when the displacement data are filtered with different cut-off frequencies depending on the derivative of interest. The cut-off frequency for filtering the displacement data when the second derivative is required, should be lower than the cut-off required for the first and this should be lower than the cut-off frequency when only the displacement data are required.

Most automatic techniques based on digital filtering estimate an optimal cut-off by retrieving information only from the displacement data. The results of the present study indicate that information from the higher derivatives must also be obtained and considered Therefore further investigation on these topics is required. The strategies examined by the present study need to be applied to other filtering techniques.

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