The detailed study of electromyograms (EMGs) is a course (or career) in itself. In this set of notes we will focus on understanding a few points about the analysis of the signals.

### Stages in EMG signal processing

1. Raw signal amplification (differential mode)
2. Analog band pass filter
3. Analog to digital conversion at a steady sampling rate
4a. Rectify and digital low pass filter ('linear envelope')
   or
4b. Root mean square (RMS) filter
5. On, off time determination
6. Time-frequency analysis

#### 1. Raw signal amplification

Record EMG in differential mode, i.e. measure difference in voltage between two electrodes, which may be surface or needle electrodes.

#### 2. Analog filtering

Analog filtering, usually band pass, is applied to the raw signal before it is digitized. Band pass filtering removes low and high frequencies from the signal. Low frequency cutoff of band pass filter removes baseline drift sometimes associated with movement, perspiration, etc., and removes any DC offset. Typical values for the low frequency cutoff are 5 to 20 Hz. If the mean value of the signal is not zero before high pass or band pass filtering, it will be afterward, because these filters remove low frequency components of a signal, and so they force the mean value to be zero or nearly zero.

High frequency cutoff of band pass filter removes high frequency noise and prevents aliasing from occurring in the sampled signal. The high frequency cutoff should be quite high so that rapid on-off bursts of the EMG are still clearly identifiable. Typical values are 200 Hz – 1 kHz.

Seniam recommendations for surface EMG: high pass with 10-20 Hz cutoff, lowpass “near 500 Hz” cutoff, in most cases.

ISEK recommendations for surface EMG: high pass with 5 Hz cut off, low pass with 500 Hz cutoff.

ISEK recommendations for intramuscular and needle EMG: low pass with 1500 Hz or higher cutoff; high pass not specified.

### Table of analog filtering and sampling parameters for EMG signals from selected journal articles.

<table>
<thead>
<tr>
<th>Muscle</th>
<th>( f_{\text{low}} )</th>
<th>( f_{\text{high}} )</th>
<th>SR</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>ta, pl, mg, so, tp</td>
<td>40</td>
<td>400, ~80</td>
<td>250</td>
<td>Chen &amp; Shiavo 1990</td>
</tr>
<tr>
<td>ta, mg</td>
<td>6</td>
<td>400</td>
<td>1000</td>
<td>Abraham in LVSP, ed Chugani, 1998</td>
</tr>
<tr>
<td>Bb</td>
<td>5</td>
<td>1000 (2p Cheb)</td>
<td>2000</td>
<td>Burden &amp; Bartlett 1999</td>
</tr>
<tr>
<td>Bb</td>
<td>10</td>
<td>450 (2p)</td>
<td>1024</td>
<td>Rainoldi et al 1999</td>
</tr>
<tr>
<td>mg, ta</td>
<td>20?</td>
<td>800?</td>
<td>1000</td>
<td>Lamontagne et al 2002</td>
</tr>
<tr>
<td>vl, vm, st, bf</td>
<td>20</td>
<td>450</td>
<td>1000</td>
<td>Burden et al 2003</td>
</tr>
<tr>
<td>vl, vm, st, bf, rf, mg</td>
<td>5.5</td>
<td>600</td>
<td>1000</td>
<td>Benoit et al 2003</td>
</tr>
<tr>
<td>mg, ta</td>
<td>16</td>
<td>600</td>
<td>1000</td>
<td>Roetenberg et al 2003</td>
</tr>
</tbody>
</table>
### Table: electrode abbreviations

<table>
<thead>
<tr>
<th>Electrode Abbreviations</th>
<th>Flow, fhigh</th>
<th>SR</th>
</tr>
</thead>
<tbody>
<tr>
<td>vl, ra, d, lmf</td>
<td>10</td>
<td>500</td>
</tr>
<tr>
<td>st, sm, bf</td>
<td>5</td>
<td>1000</td>
</tr>
<tr>
<td>st, bf, gm, rf, ta, so, al</td>
<td>150</td>
<td>1000</td>
</tr>
</tbody>
</table>

**Notes:**
al=adductor longus, bb=biceps brachii, bf=biceps femoris, d=deltoid, gm=gluteus maximus, lmf=lumbar multifidus, mg=medial gastrocnemius, ra=rectus abdominis, rf=rectus femoris, sm=semimembranosus, st=semitendinosus, ta=tibialis anterior, tp=tibialis posterior, vl=vastus lateralis, vm=vastus medialis

All electrodes are surface electrodes except Mulroy et al., Onishi et al., (both fine wire) and Hodges & Hui (surface and fine wire).

**f**\textsubscript{flow}, \textit{f}\textsubscript{high} = low, high frequencies of the analog band pass filter used before the signal is sampled; **SR**=sampling rate. Values for **f**\textsubscript{flow}, \textit{f}\textsubscript{high}, and **SR** are in Hz.

### 3. Analog to digital conversion

Seniam and ISEK unwisely recommend a sampling rate of at least twice the frequency of the cutoff frequency of the analog low pass filter used, in other words, sampling rate of at least 1000 Hz if the low pass filter cutoff frequency is 500 Hz. I recommend a higher sampling rate (at least five times the nominal low pass filter cutoff frequency) to avoid aliasing, because analog low pass filters roll off slowly, so there can be significant power at frequencies well above the cutoff frequency. Thus, if the high frequency cutoff is 500 Hz, a sampling rate of 2.5 KHz or more is recommended.

#### 3a. Digital high-pass filter

Some authors apply a digital high-pass to the signal to remove movement and other artifacts. Example: Solnik et al., “6th order, high pass filter at 20 Hz”, for surface EMG from vastus lateralis.

#### 4a. Rectify and digital low pass filter

Take the absolute value of the signal. This is also called full wave rectification. The rectification step is essential for getting the shape or “envelope” of the EMG signal. One might think the envelope could be captured by simply low-passing the un-rectified signal to smooth it. The reason this doesn’t work well by itself is that the EMG signal is naturally nearly zero mean, with fast oscillations that swing quickly and more or less equally on either side of zero. If you smooth such a signal you just get zero – not very useful. If one first rectifies, the negative swings turn into positive swings.

The rectified signal is low pass filtered, with in the 5 – 100 Hz range, and the result looks like the “envelope” of the original signal. One way to low pass filter a signal is to simply take the mean value, in a window which “slides” along the signal. Some authors advocate this “rectify and mean” approach. Such a moving–average window is an example of a finite impulse response (FIR) filter. If the window is symmetric and centered, then it will not alter the phase, or timing, of the signal. Filters that do not alter the phase are said to have “zero phase shift”. Another way to low pass filter the rectified signal is to use a discrete version of a traditional low pass filter such as Butterworth or Chebyshev. These are “infinite impulse response” (IIR) filters. An IIR filter is often applied in both the forward and backward directions, because this results in zero phase shift.

The combination of rectification and low pass filtering is also called finding the “linear envelope” of the signal, since the filtering operation meets the mathematical definition of linearity (although the absolute value operation does not), and, because it is low pass, it captures the “envelope” of the signal.

#### Pseudo-code to compute the “linear envelope” with a zero-phase Butterworth filter

Assume raw data=x(t), \(x_m=\text{mean}(x(t))\).

If, as is usually the case, the raw data was high-pass filtered or band-pass filtered, then its mean will be zero already.

\[ y(t) = |x(t)-x_m| \]

\[ z_{\text{Butter}}(t) = y(t) \text{ filtered with Butterworth filter in forward and reverse directions} \]
Pseudo-code to compute the envelope with a rectify-and-mean approach (window width=$T_w$ seconds=$N_w$ points)

\[ y(t) = |x(t) - x_m| \]

\[ z_{MA}(t) = \frac{\text{Sum of } y(t) \text{ from } t-T_w/2 \text{ to } t+T_w/2}{N_w} \]

**4b. RMS value (alternative to 4a)**

An alternative way to capture the EMG envelope is to compute the root mean square (RMS) value of the signal within a window which “slides across” the signal. The RMS value of the signal that is in the window is plotted at the center of the window, to avoid time shifts in the envelope relative to the signal. This approach is mathematically only slightly different from the rectify-and-lowpass approach. For example, De Luca CJ, J Appl Biomech, 1997, shows results using a 25 ms wide RMS window.

Pseudo-code for envelope detection with a RMS window (width=$T_w$ seconds=$N_w$ points)

\[ y(t) = (x(t) - x_m)^2 \]

\[ z_{MA}(t) = \sqrt{\frac{\text{Sum of } y(t) \text{ from } t-T_w/2 \text{ to } t+T_w/2}{N_w}} \]

I can think of at least two reasons for the popularity of RMS methods of analysis: 1. if the signal values are normal random deviates, then the RMS approach can be shown to be an optimal method for estimating the standard deviation of the underlying normal distribution. 2. If the signal were a voltage applied across a resistor, the mean square method correctly predicts the power (heat) that will be dissipated in the resistor - which has an intuitive appeal as a measure of "strength of signal".

A reason to be wary of the RMS method is that if the signal values are NOT normally distributed, particularly if outliers occur more often than predicted by the normal distribution (which, by the way, seems to be the case in many laboratory measurements), then the RMS method is liable to make significant estimation errors. Mean-of-the-absolute-value methods are not as sensitive to outliers, and are less likely to make big errors when the data is non-normal. Median-based methods are even more robust, but I have not seen them used in EMG analysis.
Figure 1 Frequency response of “envelope detectors” designed for 20 Hz cutoff.

Figure 1 shows the frequency response of the envelope detection for Butterworth, moving average, and root mean square (RMS) methods. The x axis indicates the frequency of sinusoidal modulation of a simulated raw EMG. The value on the y axis is the ratio of the amplitude of the envelope, to the amplitude of the modulation the raw signal. A 2nd order Butterworth filter was used twice for zero phase. The adjusted cutoff frequency of the Butterworth was 25 Hz, in order to achieve a net cutoff frequency (attenuation=0.71) of f_c0=20 Hz after both filter passes. The moving average window width set to T_w=22 ms, in order to achieve a cutoff frequency f_c0=20 Hz for the moving average filter. The figure shows that the Butterworth and moving average filter attenuate by 0.71 at 20 Hz, as expected. The theoretical response of the moving average filter is also shown and is the same as the moving average filter response measured with simulated EMGs. The moving average and RMS filters have very similar frequency responses, with rather poor lowpass filter characteristics: non-monotonic in the stop band, and attenuation≈0.2 at f≈3*f_c0. The zero-phase Butterworth filter has a response closer to unity in the passband, and closer to zero in the stopband, than either the moving average or the RMS filter. The cutoff frequency for the RMS filtered (defined as the frequency where attenuation=0.71) is very slightly lower than for the moving average filter with the same width. (Results computed and plotted with linear_envelope_filter_analysis.m; image file emg_fig1.jpg.)

Figure 2 Frequency response of “envelope detectors” designed for 5 Hz cutoff.

Figure 2, like Figure 1, shows the frequency response of the envelope detection for Butterworth, moving average, and root mean square (RMS) methods. The desired final cutoff frequency for the Butterworth filtering is now 5 Hz instead of 20 Hz, and as a result a window width of 88 ms has been chosen (four times longer window to get four times lower cutoff frequency). In this case a 4th order Butterworth was used instead of a second order Butterworth. The results in the figure show that the Butterworth and moving average filters do attenuate by 0.71 at 5 Hz, as expected. The 4th order Butterworth’s frequency response is closer to unity in the pass band and cuts off more sharply than the 2nd order Butterworth in Figure 1. A key point is that a wider window results in a lower cutoff frequency. (Image file emg_fig2.jpg)
Figure 3 Biceps brachii EMG envelope detection with 20 Hz cutoff.
The application of these filters to a real EMG (obtained from biceps brachii during a brief strong voluntary
contraction) is shown above. Figure 3 shows envelopes detected with a 20 Hz, 2 pole Butterworth and with 22
ms wide moving average and RMS filters, as in Figure 1. A zoomed in view of the start of muscle activity is
shown on the right side. The envelopes are very similar. The envelope detected with the Butterworth is
smoother than the moving average or RMS envelope. (Image file emg_fig3.jpg)

Figure 4 Biceps brachii EMG envelope detection with 5 Hz cutoff.
Figure 4 shows envelope detection with a 4th order Butterworth with a net cutoff frequency of 5 Hz, and with 88
ms wide moving average and RMS windows, with a zoomed-in view on the right. The three envelopes are
quite similar. The Butterworth envelope detected with a 4th order filtered applied twice shows undershoot
before it rises. (Image file emg_fig4.jpg)

Envelope detection using cutoff frequencies of 5 to 20 Hz (which correspond to window widths of
approximately 22 to 88 ms) are reasonable for many situations.

Matlab code to find linear envelope of a signal
Lowpass filter frequency = 50 Hz. Sampling rate = 10000 Hz.
When specifying frequencies for digital filters in Matlab, the frequencies should be normalized by the Nyquist frequency.

```matlab
>> x=load ('wcr0301164a.emg');
>> Fs=10000;
>> Fnyq=Fs/2;
```

(Line above is Nyquist frequency. Next line rectifies signal after removing mean value.)

```matlab
>> y=abs(x-mean(x));
```

(Next line defines desired final cutoff frequency, in Hz.)

```matlab
>> fco=20;
```

Next line creates 2nd order Butterworth low pass filter. The cutoff frequency is adjusted upward by 25% because the filter will be applied twice (forward and backward). The adjustment assures that the actual -3dB frequency after two passes will be the desired \( f_c \) specified above. This 25% adjustment factor is correct for a 2nd order Butterworth; for a 4th order Butterworth used twice, multiply by 1.116.

```matlab
>> [b,a]=butter(2,fco*1.25/fnyq);
```

Next line filters the rectified data forward and backward.

```matlab
>> z=filtfilt(b,a,y);
```

Next lines create a vector of time values for plotting, plot the data versus time: raw data in blue, offset vertically for visibility, rectified data in green, and linear envelope in red. Axis labels and a legend are also added.

```matlab
>> plot(t,x-mean(x)-4,'b',t,y,'g',t,z,'r');
>> xlabel('Time (s)'); ylabel('EMG (V)');
>> legend('Raw (offset)','Rectified','Linear envelope');
```

Matlab code to compute and plot spectra

Next lines creates vector of frequencies present in the spectra, up to the Nyquist frequency.

```matlab
>> N=length(x);
>> freqs=0:SR/N:fnyq;
```

Next: compute fft and plot the amplitude spectrum, up to the Nyquist frequency.

```matlab
>> xfft = fft(x-mean(x));
>> figure;
>> plot(freqs,abs(xfft(1:N/2+1))); 
```

Next: compute and plot the power spectrum, up to the Nyquist frequency.

```matlab
>> Pxx = xfft.*conj(xfft);
>> figure;
>> plot(freqs,abs(Pxx(1:N/2+1)));
```

5. On/off times

One often uses the EMG to determine the times at which muscles “turn on” and “turn off”. An automatic on/off detection algorithm with graphical display, and which allows user override, may be used.

5a. A standard approach for on and off time estimation is to determine the times at which the envelope of the signal (determined using methods above: remove mean value (if any), rectify, digital low pass filter) exceeds a threshold. The threshold may be given by

\[
\text{Threshold} = \mu + J\sigma
\]

where \( \mu \) and \( \sigma \) are the mean and standard deviation of the envelope during a period of inactivity, and \( J \) is a constant. Di Fabio used a 3\( \sigma \) threshold, i.e. \( J=3 \). Hodges and Bui (1996) required that the mean of the points in a sliding window of width \( T_w \) exceed the threshold. The time of the first point in the window was considered the threshold crossing time. They used a digital 6th order elliptic low pass filter with cutoff frequency \( f_c \). They did not specify whether it was a zero phase (forward and backward) implementation. They tried three values for \( J \) (1, 2, 3), three values for \( T_w \) (10, 25, 50 ms), and three values for \( f_c \) (10, 50, 500 Hz). They used all 27 possible methods on 300 EMGs (surface and fine wire) and compared the on-times to the on-times found by an experienced examiner. They found good agreement (no statistically significant difference) between the computerized and human-determined on-times, for all traces and for low-and high-background traces, with two
of the twenty-seven combinations: \((J, T_w, f_c) = (3, 25 \text{ ms}, 50 \text{ Hz})\) and \((1, 50 \text{ ms}, 50 \text{ Hz})\). See also Di Fabio (1987), Bogey et al. (1992).

5b. A generalized likelihood ratio method for determining EMG on and off times has been described by Staude (2001). In this method, the raw (zero mean) recorded EMG, \(x(k)\), is presumed to be a Gaussian white noise signal which has been altered due to filtering by tissues, electrodes, etc. The filtering is assumed to be autoregressive with transfer function \(H(z)\):

\[
H(z) = \frac{1}{1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_p z^{-p}}
\]

The user estimates \(a_1\) through \(a_p\) from the spectrum of the “quiet” signal. The measured signal is then filtered with “whitening filter” \(H_w\), which is the inverse of \(H\), to give a signal with a white spectrum:

\[
H_w(z) = 1 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_q z^{-q}
\]

where \(p=q\) and \(b_i=a_i\). The whitened signal \(y(k)\) is therefore given by

\[
y(k) = x(k) + b_1 x(k-1) + b_2 x(k-2) + \cdots + b_q x(k-q)
\]

The signal \(y(k)\) is used for detection of on-time. Staude et al. (2001) used \(p=8\) with sampling rate=1 kHz. The theory behind the following method is that \(y(k)\) is presumed to have a constant variance up to time \(t_0\), after which the variance changes. After \(t_0\), the variance is presumed to step to a new level or to ramp up (increase linearly with time). We will describe the step-change detection algorithm, since it is slightly simpler, and gives results similar to the ramp detection algorithm (Staude et al., 2001).

A maximum likelihood estimate of variance before change is computed from \(M\) samples of \(y(k)\) during a quiet initial period:

\[
\sigma_0^2 = \frac{1}{M} \sum_{i=1}^{M} y^2(i)
\]

Then a window of constant width \(W\) is moved across \(y(k)\). At each position of the window, a log-likelihood ratio \(g(k) = S_j^k = \frac{W}{2}(\hat{\rho}(j, k) - \ln \hat{\rho}(j, k) - 1)\) is computed and compared to a threshold \(h\).

\[
S_j^k = \frac{1}{W} \sum_{i=j}^{k} y^2(i)
\]

where

\[
\hat{\rho}(j, k) = \frac{\sum_{i=j}^{k} y^2(i)}{\sigma_0^2}
\]

Note that \(\hat{\rho}(j, k)\) is the ratio of the estimated variance in the window to the baseline variance estimate. The alarm is sounded at time \(t_a=k\), the first time when \(g(k) \geq h\), and corresponds to a window covering \(y(j=k-W+1)\) to \(y(k)\). Then the transition time \(t_0\) is estimated more precisely by

\[
t_0 = \arg \max_{W \leq j \leq t_a} S_j^{t_a + \Delta}
\]

“where \(\Delta\) is an appropriate dead zone which ensures that a minimum number of observations is available for parameter estimation” (Staude et al. 2001). This means \(t_0\) equals the value of \(j\) which maximizes the log likelihood \(S_j^{t_a + \Delta}\), which is computed using a variable-width window going from \(j\) (which moves from \(W\) to \(t_a\)) to \(k\) (which is fixed at \(t_a + \Delta\)). Staude et al. (2001) obtained good results with \(M=200\) (width for estimation of baseline variance), \(W=25\) (moving window width), \(h=10\) (threshold for log likelihood alarm), and \(\Delta=100\) (number of points in the dead zone, used after alarm sounds), where sampling rate=1 kHz.
Li et al. (2007) report that the Teager-Kaiser energy (TKE) operator gives more accurate EMG onset times than a conventional threshold method or a generalized likelihood ratio method, especially when signal to noise ratio is poor. They used simulated EMGs. Solnik et al (2008), using real EMGs, found more accurate onset times with the TKE operator than with a conventional threshold method. Neither Li et al. (2007) nor Solnik et al. (2008) report about off time accuracy. The TKE operator gives a transformed signal \( y(n) \), as follows:

\[
y(n) = x^2(n) - x(n+1)x(n-1)
\]

where \( x(n) \) is the EMG and \( n \) is the sample number. Threshold crossing detection was done on signal \( y(n) \). The threshold was defined as

\[
y_{\text{thr}} = \mu + J\sigma
\]

where \( \mu \) and \( \sigma \) are the mean and standard deviation of \( y(n) \) during a period of inactivity, and \( J \) was set to 8 (after testing different possible values of \( J \)). The alternate, “conventional”, processing was full wave rectification followed by threshold crossing detection with a threshold of three times the standard deviation of the rectified signal during inactivity. They did not say whether they low-pass filtered the rectified signal or whether they required a certain number of consecutive points to exceed threshold for detection. Solnik et al. (2008) report that the TKE operator gives more accurate EMG onset times than conventional threshold detection, using surface EMG from vastus lateralis. They high-pass filtered the raw EMG before applying the TKE operator (“6th order, high pass filter at 20 Hz”) They low-pass filtered the TKE signal \( y(n) \) (“6th order, zero-phase low-pass filter at 50 Hz”) before threshold detection. They used \( J=15 \). The alternate, “conventional”, processing was (apparently) high-pass filtering, then full wave rectification, then low pass filtering. For the conventionally processed signal, a threshold of 3 standard deviations was used \((J=3)\). For conventional and TKE operator signals, threshold crossing was defined as the first point to cross the threshold and stay above threshold for 25 or more samples, at a 1 kHz sampling rate.

6. Time-frequency analysis

The frequency content of the EMG may convey information that is different, and perhaps diagnostically distinct from, the information in the linear envelope of the signal. The problem here is that the “classical” approach to frequency analysis is to find the power spectrum of the entire signal, from start to finish. If the muscle is not at a steady level of activation for the entire recording period, then taking the power spectrum of the whole signal is not appropriate. Instead, we need to calculate the frequency content of the signal for short, perhaps overlapping, time segments. This is what time-frequency analysis does. A moving window (which can be rectangular, Gaussian, Hamming, Hanning, etc.; see the discussion of windowing) “slides” across the recording, and at each position, the power spectrum of the signal inside the window is computed. This generates a whole family of power spectra – one for each time position of the window. The power spectrum describes the frequency content of the signal at the time corresponding to the “center” of the window, plus some time (the window half-width) before and after that time. The user must decide what window to use, how wide to make it, and the step size for moving the window. Any smooth window - not rectangular! - is a reasonably good choice: Gaussian, Hamming, or Hanning, for example. The width of the window determines the frequency resolution: wider window = more frequency resolution, but wider window means averaging together a longer signal segment, which is more likely to include dissimilar portions. A wider window also means that there is poorer resolution in the time domain of changes in the frequency content of the signal. Example: a 50 msec wide window will do a good job of resolving a change in frequency content that occurs over 200 msec, and which takes another 200 msec to change back again. However, that same 50 msec wide window will do a poor job of resolving a change in frequency content that occurs over 20 msec, and in another 20 msec changes back again. A step size of one quarter to one tenth of the window width is reasonable. Smaller step sizes yield little new information, since successive window steps will “see” virtually the same input data. The frequency resolution (in Hz) is \( \Delta f = 1/T_w \), where \( T_w \) is the window width in seconds.

\[1\quad \text{Theory predicts that } f_{0,\text{net}} = f_{0,\text{Butter}} \times 0.802, \text{ for a second order Butterworth used twice. Therefore we set } f_{0,\text{Butter}} = f_{0,\text{net}} / 0.802. \text{, where } f_{0,\text{net}} \text{ is the desired final corner frequency of 20 Hz.} \]
2 Theory predicts that a moving average filter will have a cutoff frequency (3 dB attenuation) $f_{co,MA}=0.443/T_w$.

3 The theoretical response of a discrete time moving average filter is $H(f)=\frac{\sin(\pi f T_w)}{[N_w \sin(\pi f dT)]}$ where $N_w=$ window width in points, $dT=$ sampling interval in seconds, and $T_w=N_w dT=$ window width in seconds.

4 The numerical tests shown here indicate that the RMS filter has a cutoff frequency $f_{co,RMS}=0.42/T_w$.

References


Sources for more information

SENIAM is a European project to establish EMG standards. Seniam 8 (1999) is a book with the final recommendations. Seniam 9 is a CD ROM; I suspect contains the same information as the Seniam 8 book. A short summary by Stegeman & Hermens is available online (see below). I have not found the Seniam 9 CD ROM contents online.

The International Society for Electrophysiology and Kinesiology (ISEK) published standards for EMG recording in 1999, available online (see below).


