Now you see it, now you don't: A unit to support eighth grade conceptualizations of size and scale through scientific notation applied to optics and light along the electromagnetic spectrum

Allison Gantt

Introduction

There are 100 thousand million stars in the Milky Way[1], and approximately 100 thousand million galaxies beyond our own. The smallest object visible under an optical microscope measures 200 nanometers[2]. While these facts may sound fascinating, many of us encounter challenges in perceiving these quantities beyond the descriptors "very large" and "very small". However, as topics in optics emphasize, the distinctions between these "very large" and "very small" quantities can make critical differences in how our world behaves. The speed of light in a vacuum is roughly 671 million miles per hour, while the Earth orbits at just 67,000 miles per hour[3], or approximately 10,000 times slower. On the opposite end of the scale, differences between wavelengths of light just nanometers big can make the difference between visible light and an ultraviolet or infrared wave of light. A sense of scale and perspective is critical to conceptualizing and applying these quantities.

Students in eighth grade in the United States encounter the challenge of understanding scale from a mathematical lens. Proficiency with scientific notation is a Common Core standard for eighth graders (8.EE.A.3 & 8.EE.A.4), applied both as a tool for dimensioning size and simplifying calculations. However, students often encounter difficulty in this first formal exploration of size and scale of this magnitude because it is beyond what can be immediately visualized. Research shows that students in middle school use their person-based characteristics as reference, struggling particularly with measurements beyond one million meters, as this extends beyond their concrete references. Similarly, students tend to encounter more difficulty in perception of small quantities over large quantities[4]. However, fluency with size and scale in mathematics has been shown to be positively correlated with performance in science classes[5], indicating that mastery during this time in students' academic careers is critical to future success in other areas, as well.

As a teacher of eighth grade mathematics, the question that guides this unit is the following: How can an optics perspective support students to reinforce concepts of size and scale presented through the lens of scientific notation? In painting a picture about the numbers that guide our day-to-day lives--visible and otherwise--I hope to help students to develop new references and relevance to their study of scale and number that they can utilize in future mathematics courses and beyond.

Rationale

This unit has been motivated by my participation in the Delaware Teachers' Institute seminar "What is Light?". Through this seminar, we have explored the applications of light across the electromagnetic spectrum, the functioning of visible light, and even sources of visible light within our world. The seminar has illuminated new perspectives around the connections between mathematics and optics, and the theme of large and small numbers became particularly relevant to the needs of my current teaching context.

I teach sixth and eighth grade mathematics at an International Baccalaureate school in Wilmington, Delaware. The International Baccalaureate program values high-level mathematical thinking as well as an interdisciplinary approach to learning, strengthening the focus on context and application throughout the unit. Additionally, the program values the development of approaches to learning skills and character traits that support students to become responsible global citizens. These influences have also shaped the objectives for the unit that follows.

As a magnet school, our student population is diverse in identities, community memberships, and needs. Students attend our school from several different school districts, and though they must apply to enter the program, academic achievement is not a criterion for entry. Our program celebrates this diversity, and students are not put into fixed ability tracks during their time as a student. I am seeking to develop a unit that is accessible to and meaningful for students of a wide range of ability levels within the same classroom.

Content

Mathematical Content

Beyond a concept, scientific notation is a format in which quantities can be expressed that quickly conveys the magnitude of the value as a power of ten. To be considered "scientific notation," the number expressed must include two key components: 1) a coefficient greater than or equal to one and less than 10, and 2) a power with a base of ten and an integer exponent. For instance, the value 34,800,000 can be expressed as 3.48×10^7 in scientific notation. Similarly, very small numbers, such as 0.00065 can be expressed as 6.5×10^{-4}.

Indeed, though we often take notational choices for granted in our day-to-day work with number, the ways in which we express quantities often embed new perspectives that allow for new advances and computational fluency. Scientific notation is no exception, and its widespread usage has catalyzed advances in work with number both in mathematics and across a variety of scientific fields of study.

History of Scientific Notation

To illustrate, consider the case of the Hindu-Arabic numeral system that we employ today. Prior to the thirteenth century A.D., Western Europe still made use of Roman numerals, which expressed quantities as a summation of various “benchmark” values, each represented with a distinctive letter. For example, the quantity “123” would be expressed as “CXXIII,” translating directly as “one hundred, two tens, and three ones”. Very large quantities, however, could be even more complex to express. The value “1,234,567” is illustrated in Figure 1.

$\overline{\text{MCCXXXIV}}\text{DLXVII}$	
One thousand times one thousand, two hundreds, three tens, and one less than four	Five hundred, fifty, ten, and five and two ones.

Figure 1

Certainly, if you were asked to total the quantities of “123” and “1,234,567” using Roman numerals, the task would be challenging but possible. However, consider then the task of multiplication problem with these values. The lack of consistent place value, representations of “special values” such as fifty or four, and the limits on expressing novel characters in the Roman numeral system would make the task particularly challenging. With the Hindu-Arabic numbering system used today, place values align with ease and allow for faster and more efficient multiplication. For these reasons, when Leonardo of Pisa published his book “Liber abbaci” that presented the Hindu-Arabic system to Italian merchants, today’s numerals became widely accepted and resulted in what is dubbed an “arithmetic revolution” in Europe[6].

Once Hindu-Arabic numerals had been widely adopted, representations of number continued to evolve. Modern-day decimal representations emerged in Europe around 1585; exponents followed in the next century. Scientific notation as it is defined today, however, only became popularized around the 1960s, first appearing in a dictionary in the year 1961[7]. This notational invention became employed extensively within modern calculators and computing, aligning with the emergence of the handheld pocket calculator in the 1970s[8]. Indeed, scientific notation continues to be used by computers today to express values that are very large and very small and exceed the display capabilities of our calculating devices, as a quick Google search of a calculation like “789,000,000 times 34,000,000” can show.

A Tool to Make Comparisons

When written in the format of scientific notation, numbers can be quickly compared using powers of ten, given that these values explicitly capture the place value of the number. For instance, comparing the values 0.000234 and 0.0000056 is not intuitive upon first glance. However, when written as the values 2.34×10^{-4} and 5.6×10^{-6}, not only is it easy to distinguish which value is larger or smaller (2.34×10^{-4} must be larger than 5.6×10^{-6} given that it has a larger power of ten attached to it), but other more detailed comparisons become possible. We can claim that 0.000234 is approximately 50 times larger than 0.0000056 given that their powers of ten show a difference of 2 place values (10^{-4} divided by 10^{-6} is 10^{2}) and that their coefficients have a ratio of about one half (2.34 divided by 5.6 is roughly 0.5, given that 2.5 divided by 5 is exactly 0.5).

Scientific notation might help a student to realize that the difference between a conversation about one million (1×10^{6}) and one billion (1×10^{9}) is a factor of 10^{3}, or that one value is 1,000 times as large as the other. Indeed, comparisons involving a billion are a critical foundation for making sense of objects in the inverse--the nanoscale--as well[9]. Developing intuition around size and scale are enhanced through scientific notational views of number.

A Tool to Perform Calculations

Scientific notation also enables new calculation strategies that are rooted in conceptual understandings of place value. Consider the sum of the numbers 6,170,000,000 and 520,000,000. Traditional algorithms of addition in standard notation emphasize that we should align and add values according to place value. However, a scientific notation perspective emphasizes this idea further. If we first consider 6,170,000,000 as 6.17×10^{9} (6.17 billions) and 520,000,000 as 5.2×10^{8} (5.2 one hundred thousands), we can also see 6,170,000,000 as 61.7×10^{8}, or 61.7 hundred thousands. 61.7 and 5.2 hundred thousands would indicate that we have 66.9 hundred thousands, which is 66.9×10^{8}, also known as 6.69×10^{9} or 6,690,000,000. In emphasizing the connections to place value and the relative relationships between numbers, students are encouraged to perform calculations flexibly and fluently with scientific notation structures.

Optics Content

Optics is a unique field in that foundational concepts involve work with numerical values that are both very small and also very large. Thus, scientific notation is advantageous and frequently applied within this area.

Very Small Numbers: Differentiating Light Across the Spectrum

"Light" is the term used to describe a form of energy that exists across what we call the electromagnetic spectrum, though the term "light" most typically applies to the small subsection of the spectrum that is visible to the human eye. Indeed, categories of "light" across the spectrum are differentiated in large part by their wavelengths, or the distances between peaks in a wave of light, which can range in measurement from readily perceivable to incredibly small.

Longer and shorter wavelengths have different properties that make them uniquely suited for a variety of functions within our world and universe. Radio waves, for instance, are among those with the longest wavelengths, ranging from 1 millimeter to up to 100 kilometers[10]. Their great lengths, variety, low energy consumption, and ability to penetrate the Earth's atmosphere make them ideally suited for applications in communications, global positioning, and radar. On the opposite end of the spectrum, gamma waves have wavelengths that measure less than 10^{-9} centimeters, and these high energy waves travel long distances across our universe and are applied on Earth in some radiation therapies to treat cancer and even to pasteurize some foods without the use of heat[11].

The visible light spectrum, in fact, includes a very limited span of wavelengths between 7×10^{-5} centimeters and 4×10^{-5} centimeters[12]. However, within this band of the spectrum, variations in wavelength are perceived by the human eye as distinctive colors, most typically defined as red, orange, yellow, green, blue, and violet[13]. Materials in our world each have unique properties that enable certain wavelengths to be reflected back to the eye, thus determining the colors of the objects around us. Our eyes work as a lens and camera that perceives light waves in these wavelengths to generate the images of our world.

Indeed, for students to understand the mechanisms behind light--visible and invisible--that power our world, they must be able to distinguish and describe variations of wavelength beyond "very small". The differences here between nanometers, micrometers, and centimeters can mean the difference between light we can perceive and light we cannot, or light that is dangerous to human contact and light that is safe for application. Supporting fluency with scientific notation can enable students to truly visualize size and scale in this important field.

Very Large Numbers: Frequencies and The Speed of Light

In addition to unique wavelengths, light waves also have unique frequencies, measured as the number of times a peak of a wave passes a particular point in some interval of time[14].

Waves with longer wavelengths, such as radio waves, also have lower frequencies. Conversely, waves with shorter wavelengths have higher frequencies. Importantly, the product of wavelength and frequency is always constant, and calculates to the speed of light[15].

The fastest runner on Earth still travels about 25,000,000 times slower than the speed of light through a vacuum[16]. Clocking in at exactly 299,792,458 meters per second, which we will describe as "constantly 3×10^8 meters per second" throughout the rest of this piece, the speed of light in a vacuum is known as the "speed limit" in our modern world. For this reason, light is leveraged for a variety of functions beyond what is simply visible, particularly in the fields of communication and technology. Though light can be slowed, reflected, or absorbed as it encounters different materials, light unimpeded travels at this constant, critical pace.

Importantly, too, to the travel of light is not only its speed, but its straight line trajectory. Indeed, unless light encounters another transparent material that bends its direction (or a reflective material that also could change its path), light will continue to follow a straight line path indefinitely. In fact, the "bending" of light occurs precisely due to the sudden change in its speed. As light is refracted through a transparent material, it slows, thus also changing its path at an angle determined by the properties of the material and the properties of the light's wavelength[17].

Objectives

Mathematical Content Objective: Scientific Notation

This unit will be developed to supplement curricular materials from Illustrative Mathematics to support the following two Common Core Standards:

> "Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as 3 times 10^8 and the population of the world as 7 times 10^9, and determine that the world population is more than 20 times larger." (CCSS.MATH.CONTENT.8.EE.A.3)[18]

> "Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology." (CCSS.MATH.CONTENT.8.EE.A.4)[19]

Illustrative Mathematics, a free curricular resource that our school district adopted last year, includes the study of scientific notation as part of a unit titled "Exponents and Scientific Notation". The unit launches with a review of exponents, and then follows with several lessons that focus on developing the exponent rules for multiplying powers with the same base, powers of powers, and dividing powers with the same base. Importantly, throughout this development, students focus first on developing fluency with the rules in the context of powers of ten; only then do they generalize to the cases of other bases[20].

Following this first part of the unit, Illustrative Mathematics provides a clear map of how students should move from conceptual to procedural fluency with scientific notation through their lesson structure[21], shown in Figure 2.

	Lesson
9	Describing Large and Small Numbers Using Powers of Ten
10	Representing Large Numbers on the Number Line
11	Representing Small Numbers on the Number Line
12	Applications of Arithmetic with Powers of Ten
13	Defining Scientific Notation
14	Multiplying and Dividing with Scientific Notation
15	Adding and Subtracting with Scientific Notation
16	Is a Smartphone Smart Enough to Go to the Moon?

Figure 2

As is shown in the table above, the curriculum spends significant time building student understanding of powers of ten and visual representation of such powers using number lines before even introducing the term "scientific notation". However, once scientific notation is introduced, the operations are covered in just two lessons. A follow-up lesson emphasizes comparisons using scientific notation, but the focus during this lesson is on work with powers of ten where the exponent is greater than one.

To address the guiding question of the unit (How can an optics perspective support students to reinforce concepts of size and scale presented through the lens of scientific notation?), it is here that I intend to construct supplemental activities that serve the following purposes: 1) to ground students' work at the beginning of the study of scientific notation in measurable benchmark quantities with the speed of light and measurements of wavelengths and 2) to support fluency with scientific notation as a tool to solve problems involving light and its applications. Through these objectives, the goal will be to build students' conceptual understanding of size and scale within a concrete context that will also bolster procedural fluency.

Mathematical Practice Objective: Criterion D

The International Baccalaureate program also outlines additional goals for mathematics instruction, made explicit to teachers and students through what are known as assessment criteria. These expected outcomes are consistent throughout grades 6 through 10, but the level of abstraction and complexity at each grade level steadily increases as students mature. This unit will culminate in an assessment aligned with the International Baccalaureate's Criterion D, Applying mathematics in the real-world, which evaluates the ability to identify relevant elements of authentic real-life situations, select appropriate mathematical strategies and apply them successfully to reach a solution, and explain the degree of accuracy of a solution and whether it makes sense in the context of the authentic real-life situation. Using optics as a lens will not only contribute to meeting the goals of the International Baccalaureate program but will also further deepen students' understanding and fluency as they are supported to develop these mathematical learning practices.

Teaching Strategies

To meet the objectives of the unit in developing student fluency with size and scale through scientific notation and optics, teaching strategies will need to place emphasis on proportional and relational reasoning skills[22]. While proportionality refers to direct scaling of quantities and the multiplicative comparison of one quantity to another, relational reasoning embodies the cognitive tools through which comparisons are made. Strategies such as analogies that liken quantities to human-experienced values will be critical to developing flexible understanding in the topics at hand[23].

Beyond the tools specific to the topics of size and scale, mathematical reasoning activities will need to be carefully implemented with structures that elicit student thinking and encourage the revision of ideas. One such structure is known as "I Notice, I Wonder"[24], where students are presented with a scenario, often devoid of a specific problem, and the class collects observations about things noticed and things wondered. The structure provides a critical point of entry for all learners, as no contributions are off-limits, and thus engages all students in gathering initial ideas before exploring a task.

To sustain exploratory learning of the mathematical content that eventually leads to mastery, all tasks and activities to follow embed values of rough-draft talk and revision of ideas to draw new conclusions[25]. Through the construction of open and collaborative tasks as well as focuses on consensus, discussion, and multiple solution strategies, an environment that is accepting of mistakes and focused upon building new ideas will be promoted and include all students in the learning.

To facilitate high-level analysis of student ideas and connections in full-class discussions, the five steps from “5 Practices for Orchestrating Productive Mathematical Discussions” will be used regularly in each activity. These five steps—anticipating, monitoring, selecting, sequencing, and connecting—describe the core activities in which a teacher must engage to ensure that discussions of student ideas productively arrive at the learning goal and highlight student ideas in that process[26]. The first step, anticipation of student solutions to problems, has been addressed briefly within each activity that utilizes full class discussion techniques. From there, the teacher will need to closely monitor student work as it occurs in the classroom, select strategies and solutions to share during discussion moments, sequence appropriately to reach the desired learning outcome of each activity, and finally ensure that supports are in place for students to make the desired connections within each task.

Activities

Activity 1: How Big Are Light Waves?

This activity is intended to encourage students to develop some points of reference around sizes expressed in nanometers that describe wavelengths within the visible light range of the electromagnetic spectrum. The activity will replace and build upon Activity 11.3 in the section of the unit about scientific notation[27]. The original activity focuses on studying decimal representations of number, expressing values using notation that applies powers of ten, and then visualizing them along a number line[28]. These key mathematical skills will be embedded within the activity, and visualizations will be further enhanced through the direct connections to the electromagnetic spectrum.

Prior to beginning the mathematical part of the activity, students will be prompted to consider a big question in a discussion called “Is this light?”. Through this activity, which could be presented most easily as a quick warm-up activity, students will be asked to classify the following statements as true or false using their prior knowledge.

- Light is a form of energy.
- Light is also known as electromagnetic radiation.
- Light determines our perception of colors.
- Light is a wave and particle.
- Light powers our cell phones and Internet.

- Light heats food in our microwaves.
- Light allows x-rays to function.
- There are types of light that we cannot see with human eye.

After this warm-up, students will be asked to select one response in which they are confident and one response in which they are not sure with a partner. The teacher can choose to have a few student volunteers to share some of these opinions with the class. Then, students will be told to prepare to revise their thinking as they watch a quick video that will give them more information on the nature of light. The teacher can show the five-minute film from NASA called “Tour of the EMS”[29]. Then, students can take a final minute to revise their original decisions to the true/false set. The teacher can prompt the students to consider, “Were there any statements on the list that you found to be false?” Discussion around these ideas should lead students to recognize that all statements are, in fact, true about light if defined to refer to any form of electromagnetic radiation.

The teacher should then introduce the definition of a wavelength to students, which can also be printed and distributed to students for reference throughout the activity, as shown in Figure 3.

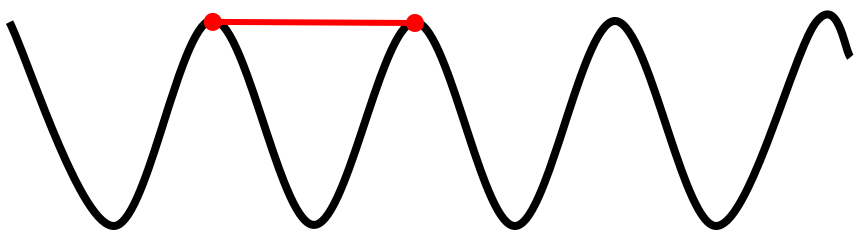

Figure 3

A poster-sized version of the wave in the figure should be printed or projected, and the teacher should go through the process of measuring the real-life projected wavelength of the beam of light. Students should be told that this actual measurement would mean that the beam of light would fall into the category of a radio wave in our world, the range of wavelengths used to transmit information to and from our cell phones, Internet, and, of course, car radios. Its long wavelength makes it such that it exists in our world, but our human eye cannot detect it.

The teacher can then begin a conversation around light that can be seen, emphasizing that wavelengths determine whether or not light is visible and the color that it appears. For this conversation, all students should have a ruler that includes centimeter measurements. Students can be asked to guess the wavelength of a beam of red light in centimeters. A red light beam can be projected from an optics kit for reference. As students make their guesses, the teacher should consistently ask students to reference the rulers in front of them and write the guessed measurements out in decimal form on the board. Students can be encouraged to refine their guesses to smaller and smaller values.

As students' guesses become smaller than what the ruler can show, the teacher can encourage students to imagine the one centimeter space on the ruler divided into hundreds and thousands of pieces. Visual supports such as pictures of millions of dots on a page could help students to visualize what this might mean[30]. Finally, the teacher can reveal that the beam could have a wavelength anywhere between 0.00007 cm and 0.0000635 cm. Colors within this band we perceive as "red"[31].

Next, students will begin the task, first considering the table of values of the wavelengths of each of the colors[32] below in Figure 4. It should be noted that centimeter measurements are applied to continue students' visualization process, though meters are more typically applied to describe wavelengths and will be used in later activities. Students should first write each wavelength as a multiple of a power of 10 in a blank copy of the table.

	Longest Wavelength (cm)	Shortest Wavelength (cm)
Red	0.00007	0.0000635
Orange	0.0000635	0.000059
Yellow	0.000059	0.000056
Green	0.000056	0.000052
Cyan	0.000052	0.000049
Blue	0.000049	0.000045
Violet	0.000045	0.00004

Figure 4

Next, students should be given the instruction of constructing a number line that can show the relationships between the wavelengths of the colors listed. A blank copy of this number line can be found in Figure 5.

Figure 5

Students should be assigned to work in teams of two to complete this activity. Colored pencils or markers may should be provided, and printing the number line image on 11-

by-17-inch paper is recommended. The following instructions may be helpful as students work:

- Label each of the long tick marks of your number line using multiples of powers of ten. You may wish to consider the highest and lowest values of your number line first.
- Plot the shortest wavelength and longest wavelength of each color band on the number line. Then, label the space in between with the color that the range defines.

Figure 6 illustrates a sample of the final completed product:

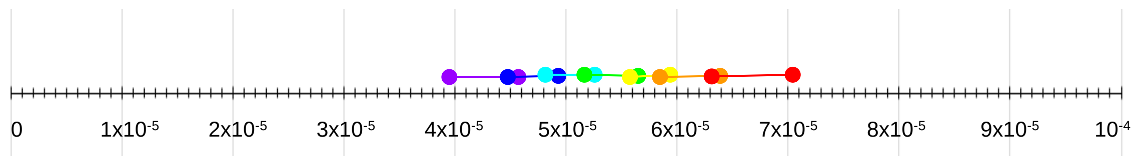

Figure 6

From here, the teacher can facilitate questions around the ideas the number line helps to reveal. For example:

- Are all of the colors you know represented on this number line? Why do you think that some of the colors are missing?
- Does light with a wavelength of $8x10^{-5}$ exist? Why is it not marked on our number line with a color?
- Which color do you think has the most energy?

The teacher can use understanding informed from the content background sections of this paper to help facilitate this discussion to connect the mathematics and science content. To finish this activity, teachers are encouraged to have students complete the final problem in the original Illustrative Mathematics activity, which asks students to use the number line to consider the wavelength of an x-ray[33].

Activity 2: Frequency and The Speed of Light

The following activity is intended to replace and/or supplement the activities in Lesson 14 of the Illustrative Mathematics curriculum[34]. At this stage in student learning, students have defined scientific notation, and they work in this activity to apply multiplication, division, and comparisons to draw conclusions with this form of numerical expression[35].

To launch this activity, students will need to define the concept of frequency of light, and develop some intuition around how frequency might be related to calculations of speed. To demonstrate, the teacher will open the "Wave Interference" simulation[36]. The teacher should begin with the settings of the visualization focused on the extreme of red light. The electric field graphing tool will also need to be pulled out from the toolbar at the right.

The teacher, first, should define the concept of frequency of light for students. Frequency is the number of times a wave passes through a position within some interval of time. For the purposes of illustration, students will be asked to measure the frequency of red light in the visualization tool during a ten-second period. Students should count approximately 12 peaks within this period of time. Importantly, the teacher should draw attention to the scale of measurement applied in this visualization--every interval on the graph is one femtosecond, or 10^{-15} seconds, of time. The reality, then, is that the frequency of red light is actually, at its lowest value, 4.3×10^{14} peaks in a single second[37]. The unit used to describe this measurement is known as Hertz, which is defined as the number of cycles per second.

The teacher will then switch the conversation to think about colors of light along the spectrum, changing the frequency in the visualization to study violet light. The teacher should ask students again to count the number of peaks that pass a certain position in the chart in a ten-second period, and students should find a value close to 22 or so. The teacher can ask students how the frequencies of red and violet light compare, using calculations to support their conclusions. Students should find that violet light's frequency is approximately 1.83 times that of red light. The teacher can then connect to the actual frequency of violet light, which is approximately 7.5×10^{14} Hertz[38]. Students will be encouraged to compare these values expressed in scientific notation using a division, finding a similar result as their own comparison from the visualization.

At this stage, students will be introduced to some data about particular waves of light, and the teacher should conduct a brief, open discussion using an "I Notice, I Wonder" framework around the data in the activity[39], shown in Figure 7. It should be noted that the data collected for this table of values comes from a variety of resources on light waves and their features across the spectrum[40]. This structure will likely elicit some baseline comparisons of values, such as the fact that radio waves have the longest wavelengths but lowest frequencies, and so on. Calculators should not be permitted during this conversation, but should be allowed for the activity that follows.

Type of Wave	Wavelength (in meters per cycle)	Frequency (In Hertz, or cycles per second)
Red Light	7×10^{-7}	4.3×10^{14}
Violet Light	4×10^{-7}	7.5×10^{14}

Microwave Oven	1.2 x 10^{-1}	2.45×10^{9}
Channel 93.7 on the Radio	3.2 x 10^{1}	9.37×10^{7}
Garage Door Opener	1×10^{0}	3×10^{8}
X-Ray Imaging Machine	2×10^{-10}	1.5×10^{18}
Sunburn-Causing UV Light	2.9×10^{-7}	1×10^{15}

Figure 7

Then, students should be given a few index cards or quarter sheets of paper with the following instruction: Write an interesting question from the data that could be answered through a calculation. Students should write the original question on one side of the card, and then show the calculation and solution to answer the question on the other side. Students may be shown the example below in Figure 8:

Question: How does the wavelength of light from a microwave oven compare to the wavelength of violet light?	Solution: $\frac{1.2 \times 10^{-1}}{4 \times 10^{-7}} = 0.3 \times 10^{6} = 3 \times 10^{5}$ The microwave's wavelength is 30,000 times longer than a wave of violet light.

Figure 8

The teacher should allow students some time to construct questions on the quarter sheets, collecting these cards into small stacks around the room facing question-side up. Once the teacher has given enough time for every student to construct at least one question and answer, the teacher can then ask students to draw a card from the stack, answer the question on the worksheet with the table of values, and check their answer to the question with the solution on the back. Once they have finished with a card, they can return it to the bottom of the stack and draw another card.

Finally, to close the activity, the teacher will return students to the visualization of red and violet light, posing a single question for debate in the room: If we shot a beam of violet light toward an object, would it arrive faster than a beam of red light? Students should be encouraged to debate some perspectives on this topic in small groups and then as a full class. It is hoped that discussion emerges that demonstrates that despite higher frequency, red light has a longer wavelength. This can be seen in the visualization as the outer band of each circle of color arriving to the edge of the screen in the same period of time. In fact, students should be told that the speeds of red and violet light are exactly the same, and that speed of light (measured in meters per second) can be calculated as a product of wavelength (measured in meters per cycle) and frequency (measured in cycles per second).

Students will be set on a final investigation with their table of data: How do the speeds of light compare for each type of wave in our table? If desired, the teacher can assign each group of students a separate type of wave with which to calculate, therefore ensuring that all waves are analyzed across the class. In follow-up discussions, students should share the speeds they found for each entry, all arriving to values that approximate the constant speed of light of $3x10^8$ meters per second, thereby illustrating that light, no matter what its wavelength and frequency, travels at a constant speed.

Activity 3: How Long Would That Take?

The following activity is intended to make final connections to the context of light and fluency with scientific notation after the Illustrative Mathematics Activity 15.3: A Celestial Dance. This activity currently asks students to consider the distances between planets and the Sun as well as combining their diameters. In this follow-up activity, students will consider what implications these distances might have in terms of our perception of objects from Earth, as well as the influences of the Sun. The content basis for these questions are motivated in large part by the BBC video "What is a light year?", which should be shown to students after the activity is completed[41].

The teacher should remind students that light travels at constantly 3×10^5 kilometers per second. The following set of questions can then be set up as stations around the room, allowing students to visit in the order of their choice and answer questions that might be interesting to them. For each question, students should be guided to identify the information they would need to answer each question first and then solve the problem.

- The Sun, which is one of the many stars in space, is one of the main sources of light that reaches the Earth. Approximately how long does it take for a beam of light from the Sun to reach the Earth?
- Voyager 1 is a space probe that sends information from far depths of space back to the Earth through radio signals. Voyager 1 is currently about 1.4×10^{10} miles away[42]. How long does it take information gathered on Voyager 1 to arrive back to Earth?
- Proxima Centauri is the nearest star to Earth after the Sun, approximately 40.14 trillion kilometers away. How long would it take light to travel from Proxima Centauri to the Earth so that we can see it?
- In operating the Mars rovers, commands are sent from Earth to the rovers via radio signals[43]. At its shortest distance, Mars and Earth are about 54.6 million

kilometers apart. How long would it take the Mars rover to respond to instructions sent from Earth?

Showing the video “What is a light year?” will support students to check their answers and also discuss the implications of what they discovered through these activities[44]. The teacher can emphasize that the speed of light is the “speed limit” for phenomena in our world, and so this is a fundamental limit that governs our reality. The example of the difficulties in sending information to Mars rovers or in perceiving objects within and outside of our solar system can serve as examples to illustrate this very important point to students.

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Appendix A: Implementing District Standards

The following Common Core Standards for grade 8 are addressed in this unit:
CCSS.MATH.CONTENT.8.EE.A.1
Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3\text{-}3 = 1/3^3 = 1/27$.

CCSS.MATH.CONTENT.8.EE.A.3
Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3 times 10^8 and the population of the world as 7 times 10^9, and determine that the world population is more than 20 times larger*.

CCSS.MATH.CONTENT.8.EE.A.4
Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Notes

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