Developing Word Problem Solving Skills through Relational Representations

Marci Agnew

Introduction

> *"Cognitively Guided Instruction is an inquiry-based approach to teaching mathematics that was developed at the Wisconsin Center for Education Research (Carpenter et al, 1999). This extensively researched approach provides teachers with knowledge about the developmental stages of children's mathematical reasoning. This knowledge enables teachers to plan mathematics instruction based on their students' understanding and guide them toward greater mathematical reasoning and concept mastery."*

These days, it is not enough for students to look and an equation and solve using any "old-school" strategy that works for them. Today's students are challenged to explain their thinking and show conceptual understanding of any and all math problems that they may be faced with. Students need to be able to explain the why and the how, and this is usually done through mathematical reasoning, often using word problems. Reasoning and understanding what a problem is truly asking of you is the foundation of math, beyond the basic operational skills students are taught. Students who truly understand and make sense of math ideas can apply them in problem solving and unfamiliar situations and can **use** them as a foundation for future learning (Battista,2018) This is a critical skill, that without, students may struggle to understand math and math problem solving.

My unit will address the need for problem-solving skill-building when it comes to mathematical reasoning, using cognitively guided instruction. It will cover 2 of the 4 critical areas for 2nd grade math Common Core standards; 1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction.

In this unit, I hope to develop lessons using various problems, sorts, and hands-on activities, including mini-lessons that teachers can use during whatever time they have available. Lessons will be created for a thirty-minute block, but can easily be chunked to smaller time frames if needed. The plans will be written with small group instruction in mind, and will be best used for re-teach groups, math RTI, or special education small group settings. I am currently a Special Education teacher - and work with students in grades 2 and 3. This unit will be best suited for students working at that instructional level in math.

Prior to starting this unit, I will give my students one of each of the word problem types that will be further explained in this unit. There are 11 word problem types involving addition and subtraction (Empson, 2014). While my students will all receive

the same word problem/situation - I will change the numbers in the problem based on their needs and IEP goals. While it is always beneficial for students to have the additional practice of solving math equations in addition and subtraction, students will be allowed to use calculators for the bulk of the unit. The main focus and desired outcome for the students is to understand how to problem solve and develop reasoning.

Demographics

I am a special education teacher in the Red Clay Consolidated School District, which is located in New Castle County, Delaware. The district serves over 17,000 students in grades k-12, with students coming from many different socioeconomic backgrounds. At the school where I will implement my unit, Cooke Elementary, there are approximately 650 students in grades K-5. In the 2017-2018 school year, the student population included 13% English language learners, 15% low income, and 7% special education. The school is somewhat diverse - with 66% Caucasian, 15% Hispanic/Latino, 8% Asian, and 6% African American. When compared to state and district averages for behavior referrals, suspensions, and expulsions - Cooke is below the average. When compared to state and district averages for the percentage of students scoring proficient on our state test, Smarter Balance, Cooke is above the average. In the 2016-2017 (most recent public data) school year, 70% of students were found proficient in math, and 63% were found proficient in reading.

This is my first year teaching at Cooke Elementary, and I will work with approximately 10 students in grades 2 and 3 that have been identified as a student who requires an Individual Education Plan (IEP). I have found that not just with this group of students, but with many of the special education students I have worked with, that problem solving (word problems) is often an area of need on IEPs. I have found that many students really struggle to understand the true meaning of what a question is asking them, and once they have an answer they have a hard time deciding if the answer is in fact reasonable. In addition to problem solving and word problem goals - my students' goals range from addition/subtraction within 20, addition/subtraction within 200, to the basics of multiplication. I pull each of my math groups 5 times a week for 30 minutes per day. This unit will take one full week, but may be extended into more time as based on student need.

Learning Objectives

- Students will be able to read a word problem and decide from a list (including examples) what type of Cognitively Guided Instruction (CGI) problem it is.
- Students will be able to choose the appropriate "formula" or operation to use based on problem type

- Students will engage in math talk, with problem unpacking and questioning, to improve mathematical reasoning.
-

Essential Questions

- What is the relationship between the numbers in a word problem?
- How can I use drawings and number sentences to represent this word problem?
- How can I use real life/real word situations and experiences to solve this problem?

Rationale

The Common Core State Standards expect students to reason with mathematical ideas and be able to discuss and explain their reasoning. In most cases, visual representation is a clear and easy way for students at the primary grade level to show their thoughts and support their work. I have found that when students are not getting the right answer for math, I am best able to support them when I ask them to draw out their thinking. Student diagrams often make things so clear, that I am able to find exactly what is causing them to solve the problem incorrectly. I am purposefully designing many of the activities and lessons for this unit to provide the opportunity for students to have "math talk" or discussions around the problems they are solving. Our math curriculum, Math Expressions, refers to math talk as "a discussion tool that makes students thinking visible". In this unit we will have time to discuss different strategies that were used to solve a problem with an emphasis on why a particular strategy was used. Each lesson and activity will focus on using both visual representation and math talk in order to strengthen my student's math reasoning skills. With stronger math reasoning skills, it is my hope that my student's math confidence will improve and they will be able to solve story problems with more accuracy, and also grow in their ability to explain how and why they solved a problem the way that they did.

Throughout my unit, students will be able to use calculators to solve problems after using a visual model in order to check their work. Many may ask, why is it that I am developing a math unit in which students are just going to use calculators. My answer to that is that because this is what real life is - at any given moment I have access to a calculator. My students will not be required to use the calculator - but it will be available. Personally, I believe that technology is a necessity that students need to be comfortable with in order to thrive in the 21st century. Students need to be prepared for careers of the future, this is where computational thinking comes in. The goal of this unit is for students to truly understand what the word problem is asking them to, and having them decide how they can represent the problem using pictures. I do not want the focus of this unit to be based on the student's math fluency and computation, but more so on the actual

problem solving. In addition, I work with students with learning and developmental disabilities who are provided many accommodations and modifications in the classroom.

Research behind the Unit

When I began researching Cognitively Guided math Instruction (CGI) most, if not all, of the major research came from the University of Wisconsin - Madison , and their National Center for Improving Student Learning and Achievement. This center (NCISLA) describes CGI below:

"Cognitively Guided Instruction (CGI) (Carpenter, Fennema, Franke, Levi, & Empson, 1999) is a professional development program based on an integrated program of research focused on (a) the development of students' mathematical thinking; (b) instruction that influences that development; (c) teachers knowledge and beliefs that influence their instructional practices; and (d) the way that teachers' knowledge, beliefs, and practices are influenced by their understanding of students' mathematical thinking"

Once I had a general understanding of CGI and what it entailed, I also looked at the research that was provided by Carpenter on the developmental thinking processes that children go through when learning to solve word problems. In his research, Carpenter looked at how students at different developmental abilities. The strategies that students often intuitively used to solve range from direct modeling (using counters) at developmental level I, counting on or counting back (using fingers or mental math) at developmental level II, to using math equations or mental math to solve the problem, level III.

The research also looked at the problem-solving skills and strategies students used at each of these levels. I used this information when creating and structuring my unit. It was very eye-opening (and relieving) to see that I could place many of my current and former students right into the examples that were provided in the research for both computation and problem solving strategies! Finally, the story problem type framework and the rationale behind many people who put their own versions of this together was another major part of the research that went into this unit.

There is a very organized framework for the 11CGI problem types, which are broken down into levels of difficulty. It is important, just with any skill you are teaching in your classroom, that these problems are taught within sequence of easiest to most difficult. The easiest of the problems; join: result unknown, separate: result unknown, and part/part/whole: whole unknown, are problems that can be easily represented through pictures and drawings. Within these problem types however, there are problems that can easily be acted out or represented by a student, and problems that require a little more thought to act or draw out. The example below featuring a separate result unknown problem type shows this in more detail.

a) Mark had 7 cookies. He gave 5 to Ryan. How many cookies does Mark have now?
b) Mark gave 6 cookies to Ryan. He had 9. How many cookies does Mark have now?
c) Problem A is much easier for a child to visualize and then either represent through actions or drawings. Problem B requires much more thought by the student, and may pose more of a challenge and a delayed "start time".

The research also states that when students are first learning to solve story problems, that they often approach them in the order in which they hear them, instead of starting with the question and working backwards. It is because of this, and their unfamiliarity with problem solving that the location of the unknown number in a story problem makes a difference. When it comes to the level of difficulty, students have the hardest time with problems where the "change" or "start" in unknown, in comparison to when the result is unknown. We will look at the 3 examples below to further demonstrate this, using "join" problems.

a) Kasey had 5 apples. Jen gave her 3 more apples. How many apples does Kasey have now?
b) Kasey had 5 apples. Jen gave her some more apples. Kasey now has 8 apples. How many apples did Jen give Kasey?
c) Kasey had some apples. Jen gave her 3 apples. Now Kasey has 8 apples. How many apples did Kasey have to start with?

All 3 of these problems essentially require the same math fact and skill set, however, problem A is the only problem in which I see my students being able to easily act out and/ or visually represent with little thought. In the next section of my unit, I will discuss how I will use the developmental levels, based on research, to tackle the types of problems that are listed above. The developmental stages when solving word problems are; 1) direct modeling 2) counting on/back, 3) deriving.

Developmental Stages of Problem Solving

Level 1: Direct Modeling

When all the quantities in a story problem refer to complete sets of physical objects of amounts, the problem can be modeled directly. This means that students are represent this in a very concrete way using manipulatives such as counters or their fingers, making a drawing such as circles or a math mountain, or tally marks. In this stage you will see students making two groups in order to combine, or setting out a full amount of counters and physically removing in order to subtract. For these types of strategies, it is best to

work with the CGI problem types where the start in not an unknown. It is very difficult for students to use direct modeling strategies where the start is unknown, and may be frustrating for students. In the example below, a student is encouraged to use a real life example of our school currency for our Positive Behavior Support program. The student was able to use the actual manipulative in order to solve.

Emily had 14 Cardinal Code cards in her wallet this morning. Today, she received 4 more Cardinal Code cards. How many Cardinal Code Cards does Emily have now?

Emily, a special education and English Language Learner student, had to physically make two piles to solve this problem. In one pile she set out her "wallet" cards of 14, in the other pile she set out her 4 cards that she had recently earned. Emily counted 1,2,3 all the way to 14, she then counted the second pile, 15,16,17,18 – to get her answer of 18.

If I were to change the location of the unknown in this problem, and tell Emily that she had 14 cards this morning, and has 18 cards now, she would be able to use the direct modeling to solve. Emily would more than likely need her wallet of 14 cards, along with a pile of extra Cardinal Code cards. She would then count her 14, and take one card from the extra pile until she got to 18.

Level 2: Counting On/Counting Back

When a student is able to successfully use the direct modeling strategy for some time, they will next move onto the counting on or counting back strategy. This type of student is able to hold a number in his or her head, and then count back or forward in order to solve a problem. In my case with special education students, I do allow them to use a number line for this type of problem solving. Other methods that we use for counting on and counting back are called cave man or simply "in my head". Let's use the same story problem from above with Emily and the Cardinal Code Cards to explain how to solve this problem using level 2 thinking.

"Caveman" or "In my head" Counting On: The student knows that we are starting with 14 code cards and need to add on 4 more Code Cards. The student will make a fist, and pound their chest stating the number 14. Once the students hand comes off his/her chest they can use fingers to count on 4 more, 15,16,17,18.

A student can also draw out this type of problem, perhaps he or she will write down the number 14 and then draw out 4 tally marks or circles. The student can point to the 14, or keep that number in his or her head, and then count on from there.

This counting on strategy is of course much more efficient than the direct modeling strategy, and a step to gaining a deeper understanding to the relationships between numbers.

Level 3: Deriving Strategies

In developmental level 3, the student is able to show that they possess number sense and the ability to solve story problems in flexible ways. Students at this level are able to break down numbers and recombine them using known facts. We will once again use the same Cardinal Code Card problem, this time showing how a student may use their number sense to solve this problem.

At this point the student knows that he/she has 14 Code Cards and needs to add 4 more to that number to find a total. In this level the students thinking may sound like this " I know that 4+4 equals 8, and that 8+10 equals 18, so 14+4 would total 18 code cards". At this level, the student will most likely not be using any type of visual representation, other than the mathematical expression.

CGI Classroom

What should the classroom look like and sound like when teaching CGI practices? Should a specific curriculum or activity set be used? These practices can be used with just about any curriculum at any age or grade level, as long as there are story problem s present that can elicit student thinking. Teachers that use CGI introduce a topic using a story problem and then tell students to solve the problem any way that they can. Students are encouraged to explore and figure out ways to solve, usually using manipulatives or visual representations at first, and then told that they should be able to explain their thought process and how they solved to a classmate or the teacher. Students will begin engaging in math talk as they explain their thought process, and often times once they start explaining will either spark a thought in a classmate's head or will realize a mistake that they have made. Once a student has solved and explained a problem using a visual representation, they should be asked to write a number sentence to represent the story problem.

The teacher's role in the CGI classroom is to observe the problem-solving strategies that students are using, determine which developmental level the student is with problem solving, and use that information to help the child extend their understanding. Even with using a text book problem, the teacher with the CGI mindset is able support a student in solving a problem in a way that works best for him or her using the five principles for unpacking a problem, which is a list created by Carpenter et al.

1. make sure that all students understand the problem.
2. align the story and the math.
3. support students to unpack the problem independently.
4. do not solve the problem for students.
5. remove barriers so that all students are able to participate in the discussions.

The goal of unpacking the problem is to support sense making around the context, not to help students come up with a strategy or an answer. By focusing unpacking on having the students explain what the story is about, you can avoid doing too much of the work for the students I will use an example problem to explain what the unpacking of a problem may look like in the second grade special education classroom.
Grace has 21 seashells. Maggie has 4 more shells than Grace. How may seashells does Maggie have?

If I were presenting this problem to my students, I would first focus on comprehension (step 1 – make sure that all students understand the problem) of the problem by reading the story several times. For the first read, I would read the story to the students, having them follow along. For the second read, the students and I would read the problem together. Finally, I would ask the students to read the problems to themselves, really thinking about what is taking place in the story and what the question is asking us. Next, I would try to create the connection between the story and the math (step 2 – align the story and the math) by asking probing questions. I would ask if any of the students have ever collected seashells and what the label would be in our answer. To support students in unpacking the problem, I would ask them questions such as "who has more seashells, Grace or Maggie?" and have them think to themselves about what type of drawing or representation they could use to solve. It is important to follow the last of the 5 principles, which is removing barriers so that all students are able to participate. Being in a special education classroom this is particularly important for me. In order to achieve this, I will often allow the students the flexibility to solve and explain the problem using any form of expression they feel comfortable with. This may mean that some students have manipulatives and paper, one student may be on the white board, and another student may need to explain their thoughts to be and then I voice their ideas to the rest of the students for him or her.

Once students have unpacked and solved a problem, the students should be asked a series of questions to better understand the steps they took, which of course will then help the teacher decide if they are using direct modeling, counting on/back, or are using true number sense to solve the problem. There is no script of questions to be used, since each student and situation will be different, but the student should be asked questions that promote math talk within the classroom. Some question ideas include:

- What did you do first?
- Can you tell me why you chose this strategy to solve?
- Explain the picture that you created to solve.
- What did you find most difficult about this problem?
- Was there any way you were able to check your work?
- Did anyone solve this problem a different way?

It is a good idea to allow the student to provide answers and really explain their thoughts, before asking additional questions based on the modeling they provide. It is also vital that when a student is solving a problem incorrectly, to still allow them to model their thinking out loud so that you can guide them back on track without providing them with a way to solve the problem.

Teaching Strategies: CGI in Special Education

While we already have a focus on problem solving using story problems in my classroom, driven mostly by student IEP goals, I would like to take this instruction even further by using these CGI practices daily. I will use the following strategies in the small group setting in which I teach math.

Problem of the Day Interactive Notebook

Each of the students that I work with have a math notebook that we put a problem of the day in at the beginning of the week. The paper has 5 "tabs" that each have one problem to be completed each day. I will continue this practice with a more purposeful intent. I will give the students less direction in solving after I check-in for understanding of the problem. Students will be encouraged to discuss the problem solving strategies that they are using with peers, and will be encouraged to use math talk to explain their thinking after solving.

CGI Framework Grid (Appendix 3)

I plan on having a "student-friendly" framework grid posted in my classroom, and also a mini version for the students to put in front of their interactive notebooks. Prior to posting, the types that we will be using will be reviewed and students will be taught which each means. Although having this visual may not be beneficial for all students, it will be a good reminder and visual for students who need prompting to think about prior knowledge and problems that they have solved in the past that they can use to solve a new problem.

Collaborative Learning

Students will have the option to work along side peers while in the special education classroom. Once students are familiar and confident with math talk and explaining or modeling their thinking, I will also talk to the general education teachers about pairing them up with a non-disabled student in the general education setting that they can use CGI strategies and learning with.

Unit Activities

Lesson One: Pancake Pre-test

Objective: Teacher will gain an understanding of students' ability to problem solve using story problems. Student will model their thinking using manipulatives, drawings, and explanations for teacher to understand which developmental level a student is working.
Materials: CGI Pre-test Questions (also found in appendix A), Manipulatives (number line, counters, 100s chart), student recording sheet

Procedure/Instruction: Students will work with teacher 1:1 in the small group setting (para-educator can work with other students or they can be doing independent work) to solve 9 word problems, that are each of a different type from the CGI Framework.

1) Join: Change Unknown
    a. Jack has 5 pancakes. Jill gives him some more. Now Jack has 17 pancakes. How many pancakes did Jill give to Jack?
2) Join: Start Unknown
    a. Liam had some pancakes. Bella gives him 6 more pancakes. Now Liam has 8 pancakes. How many pancakes did he have before Bella gave him any?
3) Separate: Change Unknown
    a. Jazmine had 11 pancakes. She gave some to Henrik. Now she has 2 pancakes left. How many pancakes did she give to Henrik?
4) Separate: Start Unknown
    a. Lawson had some pancakes. He gave 3 to Peter. Now he has 6 pancakes left. How many pancakes did Lawson have to start with?
5) Part-Part-Whole: Whole Unknown
    a. Martin has 4 chocolate chip pancakes and 6 blueberry pancakes. How many pancakes does he have altogether?
6) Part-Part-Whole: Part Unknown
    a. Khloe has 18 pancakes. 11 of the pancakes are chocolate chip, and the rest are banana. How many pancakes are banana?
7) Compare: Difference Unknown
    a. Billy has 7 pancakes. Mary has 2 pancakes. How many more pancakes does Billy have than Mary?
8) Compare: Quantity Unknown
    a. Elliot has 5 pancakes. Lily has 3 more pancakes than Elliot. How many pancakes does Lily have?
9) Compare: Referent Unknown
    a. Sarah has 11 cookies. She has 4 more cookies than Sam. How many cookies does Sam have?

Assessment: I will use the information collected from this pretest to drive the rest of the lessons and activities. I will focus on the question types that students performed the worst on, and decide if I should use the direct modeling method or counting on/back for each student as we dive further into this topic.

Lesson Two: Comparison Problems – Part 1

Objective: Students will learn how to use manipulatives (direct modeling) in order to solve comparison word problems

Materials: Bag of 20 counters that are different colors on each side (red/yellow), blank paper or recording sheet, shaker cup

Procedure/Instruction: Students will start off by counting their total number of counters in their bag to ensure that they do in fact of 20, and so that they know how many they have. Students will then place their counters into the shaker cup, and put the cup aside. The teacher will demonstrate the following activity to students.

Activity – Teacher will shake up the cup of 20 counters and carefully dump onto the table. The teacher will then ask the students "are there more red counters or yellow counters?". If needed, the teacher and students can count the chips together – if not the students should answer which color has more. If there are more red, the teacher will then ask "How many more red do we have than yellow?" This will elicit student discussion on how we can determine how many more red that we have. After a few moments of discussion, the group should come up with a few ideas of how to solve this type of problem. The students should then get their own shaker cups and practice this activity. On the recording sheet they should record how many of each color, and how many more/how many less red or yellow counters.

Assessment: The teacher will collect the shaker cups from the students. She will give them a new recording sheet, Appendix B, and then use the teacher shaker cup as done before for the modeling. The students will use the recording sheet to check their understanding of the days lesson.

Lesson Three: Comparison Problems – Part 2

Objective: Students will use visual representations to solve word problems

Materials: shaker cup full of 20 counters that are different colors on each side, blank paper, teacher question guide

Procedure/Instruction: Teacher will start off the lesson the same way as the previous lesson. The students will be told that they will need to solve the same type of problem as the day before, however this time they will not have their own manipulatives. Teacher will "shake out" the counters and ask students to count how many of each color. They will then have to draw a picture or some sort of representation to find the difference

between the two numbers. Students will record their thoughts on paper, and then be engaged in math talk as a group.

Activity: After the math talk discussion for the first portion has ended, the students should have a slight to moderate understanding of how to find the difference between two numbers. The teacher will then give the following two problems to the students to solve with a partner.

1) Addison has 5 blocks. Easton has 4 more blocks than Addison. How many blocks does Easton have?
2) Reese has 7 blocks. She has 2 more blocks than Taylor. How many blocks does Taylor have?

Students will again be asked to model this thinking using direct modeling strategies followed by counting on/counting back strategies. The teacher will ask students which of these two problems did they find more difficult, and if one was more challenging, why? Students will be encouraged to relate these problems to the problems that we completed the day before. Students will also be introduced to comparison bars, which is what they are encouraged to use in their Math Expressions curriculum.

Assessment: Students will be given two problems, similar to the problems above, with the numbers changed. The problems will be reviewed by teacher, if the student has success we will continue with unit. If the students are struggling, there will be a reteach lesson.

Conclusion

As mentioned before, I will incorporate the CGI framework for the different problem types in our daily problem of the day. At the end of our problem of the day activity, we will refer to the framework chart and decide what type of problem we just solved. As students grow familiar with these concepts I am hopeful that they will be able to apply the problem solving skills and strategies to their real life situations. The lessons and activities focused on in my unit were based on my pre-test data that showed students mainly struggled with comparison problems. After teacher these lessons, my plan is to focus on problems in which the start is the unknown, which was the second weakest skill that my students showed. It is my hope that at the end of the school year I am able to get the same pre-test to the students and see tremendous growth.

Appendix A: Pretest

1) Join: Change Unknown
    a. Jack has 5 pancakes. Jill gives him some more. Now Jack has 17 pancakes. How many pancakes did Jill give to Jack?
2) Join: Start Unknown
    a. Liam had some pancakes. Bella gives him 6 more pancakes. Now Liam has 8 pancakes. How many pancakes did he have before Bella gave him any?
3) Separate: Change Unknown
    a. Jazmine had 11 pancakes. She gave some to Henrik. Now she has 2 pancakes left. How many pancakes did she give to Henrik?
4) Separate: Start Unknown
    a. Lawson had some pancakes. He gave 3 to Peter. Now he has 6 pancakes left. How many pancakes did Lawson have to start with?
5) Part-Part-Whole: Whole Unknown
    a. Martin has 4 chocolate chip pancakes and 6 blueberry pancakes. How many pancakes does he have altogether?
6) Part-Part-Whole: Part Unknown
    a. Khloe has 18 pancakes. 11 of the pancakes are chocolate chip, and the rest are banana. How many pancakes are banana?
7) Compare: Difference Unknown
    a. Billy has 7 pancakes. Mary has 2 pancakes. How many more pancakes does Billy have than Mary?
8) Compare: Quantity Unknown
    a. Elliot has 5 pancakes. Lily has 3 more pancakes than Elliot. How many pancakes does Lily have?
9) Compare: Referent Unknown
    a. Sarah has 11 cookies. She has 4 more cookies than Sam. How many cookies does Sam have?

Pre-test Recording Sheet

1)
2)
3)
4)
5)
6)
7)
8)

9)

Appendix B – Lesson Two : Comparison Problems

Total Counters: 20

Red Counters: _____

Yellow Counters: _____

Circle the correct statement:

I have more RED counters.

I have more YELLOW counters.

I have (#) _____ more (color) _____ than (color) _____ counters.

Appendix C – CGI Grid

PROBLEM-SOLVING SITUATIONS

JOINING PROBLEMS

Join: Result Unknown (JRU)	Join: Change Unknown (JCU)	Join: Start Unknown (JSU)
♦ Grandmother had 5 strawberries. Grandfather gave her 8 more strawberries. How many strawberries does Grandmother have now? 5 + 8 = □	♥ Grandmother had 5 strawberries. Grandfather gave her some more. Then Grandmother had 13 strawberries. How many strawberries did Grandfather give Grandmother? 5 + □ = 13	♣ Grandmother had some strawberries, Grandfather gave her 8 more. Then she had 13 strawberries. How many strawberries did Grandmother have before Grandfather gave her any? □ + 8 = 13

SEPARATING PROBLEMS

Separate: Result Unknown (SRU)	Separate: Change Unknown (SCU)	Separate: Start Unknown (SSU)
♦ Grandfather had 13 strawberries. He gave 5 strawberries to Grandmother. How many strawberries does Grandfather have left? 13 - 5 = □	♥, Grandfather had 13 strawberries. He gave some to Grandmother. Now he has 5 strawberries left. How many strawberries did Grandfather give Grandmother? 13 - □ = 5	♣ Grandfather had some strawberries. He gave 5 to Grandmother. Now he has 8 strawberries left. How many strawberries did Grandfather have before he gave any to Grandmother □ - 5 = 8

PART -PART -WHOLE PROBLEMS

Part-Part-Whole: Whole Unknown (PPW:WU)	Part-Part-Whole: Part Unknow (PPW:PU)
♦ Grandmother has 5 big strawberries and 8 small strawberries. How many strawberries does Grandmother have altogether? 5 + 8 = □	♥, Grandmother has 13 strawberries. Five are big and the rest are small. How many small strawberries does Grandmother have? 13 - 5 = □ or 5 + □ = 13

COMPARE PROBLEMS

Comp. Difference Unknown	Comp. Quantity Unknown	Comp. Referent Unknown
♦♥, Grandfather has 8 strawberries. Grandmother has 5 strawberries. How many more berries does Grandfather have than Grandmother? 8 - 5 = □ or 5 + □ = 8	♣ Grandmother has 5 strawberries. Grandfather has 3 more strawberries than Grandmother. How many strawberries does Grandfather have? 5 + 3 = □	♣ Grandfather has 8 strawberries. He has 3 more strawberries than Grandmother. How many strawberries does Grandmother have? 8- 3 = □ or □ + 3 = 8

MULTIPLICATION & DIVISION PROBLEMS

Multiplication	Measurement Division	Partitive Division
♦ Grandmother has 4 piles of strawberries. There are 3 strawberries in each pile. How many strawberries does Grandmother have? 4 x 3 = □	♦ Grandmother had 12 strawberries. She gave them to some children. She gave each child 3 strawberries. How many children were given strawberries? 12 ÷3 = □	♦♥, Grandfather has 12 strawberries. He wants to give them to 3 children. If he gives the same number of strawberries to each child, how many strawberries will each child get? 12 ÷ 3 = □

Problem chart based on Cognitively Guided Instruction Problem Types (Carpenter et al., 1996)

References

Battista, Michael. 2018. "Reasoning And Sense Making In The Mathematics Classroom: Grades 3–5". *Nctm.Org*. Accessed December 12. http://nctm.org.

Carpenter, Thomas P., Elizabeth Fennema, Megan Loef Franke, Linda Levi, and Susan B. Empson. Children's Mathematics: Cognitively Guided Instruction. Second Edition ed. Portsmouth, NH: Heinemann, 2015.

Empson, Susan. "What Does a CGI Classroom Look Like?" Professional Development: TR40-R41. macmillanmb.com.

Fennema, E., Carpenter, T. P., Levi, L., Franke, M. L., & Empson, S. (1997). Cognitively guided instruction: Professional development in primary mathematics. Wisconsin, Madison:Wisconsin Center for Education Research.

Fuson, Karen. Math Expressions Common Core. Teacher Edition ed. Orlando, Florida: Houghton Mifflin Harcourt Publishing Company, 2013. Xviii-1mm.