Fostering Students to Reason Mathematically for Developing Their Problem-Solving Skills in Addition and Subtraction

Katie Zimmerman

Introduction and Rationale

> *"It's important to help students make connections among mathematical ideas so they do not see these ideas as disconnected facts. It is important to build students' new understanding on the foundation of their prior learning."*
> Marilyn Burns, founder of Math Solutions Professional Development

Addition and subtraction first appear in the Common Core State Standards (CCSS) in Kindergarten and continue through the Third Grade Operations and Algebraic Thinking strand. However, addition and subtraction skills continue to be developed and used on well into adulthood.

Addition and subtraction do not just begin in elementary school. The ideas of wanting more of something or losing one item that they were using begin even as babies. If they are eating some Cheerios and there are five on their tray and they eat three of them they notice that there are less of the Cheerios on their tray. The same applies as an adult in their life adds more of the Cheerios to their tray when there was only two of them left just a few minutes prior. Adding and subtracting is integrated into many parts of our life from very young until we are very old.

Exposure obviously starts way before Kindergarten and continues to be used well into old-age. Addition and subtraction set up the foundation for multiplication and division, are used in fractions and measurement, and definitely used in real life. Addition and subtraction are something that needs to be used daily in many different areas of life from balancing a checkbook to cooking in our kitchens. This is why it becomes imperative for our younger students to have a solid foundation in these two concepts so that they will then become successful and productive adults.

Common Core State Standards is set up from Kindergarten where the concepts are introduced and scaffolded consistently through Third Grade. However, even with the introduction, scaffolding, and progression that has been set up, many students continue to struggle even into middle and high school. At the beginning of the school year the elementary schools in Colonial School District begin using the mathematics computer program Dreambox Learning to access and practice math skills during the school year. Dreambox Learning is a K-8 digital math program that research has found to raise student achievement and better prepare children for future math experiences. Students use Dreambox to work independently on meaningful math learning activities. Teachers then

use Dreambox in large- and small- group instructional settings, and on a 1-to-1 basis, to help all students excel in math. As of the August/September 2018 Dreambox data, only 3 out of 20 of my second-grade students were proficient in second grade standards.

As a teacher I begin to ask myself why are so many of my students lacking addition and subtraction fluency? I think a lot of it comes back to the "rigor" required of the Common Core State Standards. Too often the students are moved through the expectations without reaching mastery. They are being introduced to the concepts and practice the concepts but then have to move onto the next concept without mastering the addition and subtraction that is required for the next grade level.

In addition to the conceptual hurdles needed to be met within the standards. Our current math curriculum does not meet the needs of many of our district students. It is currently 10 years old and is not aligned with Common Core. Teachers need to find supplemental materials on their own so this means that they vary from teacher to teacher and school to school. This then means that not all students are receiving the same rigorous instruction.

I have also noticed through my teaching that there is just not enough emphasis placed on the mastery of being fluent in basic one -digit facts. Many of my students do not know their combinations of 10 (2+8=10, 5+5=10, etc) or any other facts for that matter. They struggle with these basic facts which means multi-step, multi-digit equations and story problems become too difficult for them to do.

It is my hope that this unit could be used as an intervention option for those students that need help with addition and subtraction. It could be used after a unit was taught and give more practice in the concepts that are needed. I hope to provide more practice and strategies to help them when doing multi-digit, bigger number addition and subtraction problems.

Demographics

My school, Wilmington Manor Elementary School, is located in New Castle, Delaware. It is part of the Colonial School District which is composed of a variety of students in terms of race/ethnicity, socioeconomic status, English language ability, and special education classification. This is a small school, with about 331 students in grades K-5. There are two classrooms of second grade with my classroom being a Single Approach to Mastery (SAM). This year I have 20 students in my classroom with two special education students who have behavior as well as academic IEPs. My class also has a full time Para-educator for the one-on-one requirement of the IEP. All subjects are taught within my classroom, specifically, ELA (Reading and Writing), Math, Social Studies, and Science. Our schedule includes two intervention periods, one for Reading and one for Math. As

with other schools in the district, we use the Response to Intervention (RTI) model for both reading and math.

There is a very high population of English Language Learners (ELL). 37.7% of students at Wilmington Manor are classified as ELL. Many of our students are their family's only English speaker. Many parents cannot read my notes or information/instructions that are sent home in English, and they are unable to help their students with work that is sent home in English. Within my own classroom 10% of students are labeled Special Education. They are serviced within my classroom by me and a paraeducator. 90% are Regular Education students that are also in my classroom, although many of these "regular" education students are reading way below grade level. With such a diverse group of students my unit will include activities for all different levels of learners. Those students that work above grade level, on grade level, and below grade level all have equal opportunities to work toward proficiency in this unit.

Learning Objectives

- Students will represent and solve problems involving addition and subtraction.
- Students will understand and use place value along with the properties of operations to add and subtract.
- Students will fluently add and subtract within 20 using mental strategies.

Delaware has adopted the Common Core State Standards for ELA and Math and the Next Generation Science Standards. I will be addressing CCSS Math Operations and Algebraic Thinking and Numbers and Operations in Base 10.

Common Core Standards addressed:
- CCSS.MathContent.2.OA.A.1: Use addition and subtraction within 100 to solve one- and two- step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing with unknowns in all positions, eg. by using drawings and equations with a symbol for the unknown number to represent the problem.
- CCSS.MathContent.2.OA.B.2: Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.
- CCSS.MathContent.2.NBT.B.5: Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

My first objective is that my second graders will use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking

from, putting together, taking apart, and comparing with unknowns in all positions, eg., using drawings and equations with a symbol for the unknown number to represent the problem.

My second objective is for students to be able to fluently add within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

The third and last objective will have the students fluently add and subtract within 20 using mental strategies. By the end of second grade students should know from memory all sums of two one-digit numbers.

In this unit, my goal is to help my students become more proficient in their mathematical reasoning. In second grade it is important that students focus on extending their understanding of the base-ten notation and build their fluency with addition and subtraction. I hope to enable them to do this by the end of our year together.

Essential Questions

- How can numbers be composed and decomposed into place value parts to help us add and subtract multi-digit numbers efficiently?
- How can I determine whether the operation is addition or subtraction if it is a story problem?
- What strategies are most efficient for solving this problem?

Research Behind the Unit

Starting my research for this unit I had to first take a hard look at what my students were struggling with in my classroom. I had to look for common errors that they make and misconceptions that they may have brought with them from prior grades.

"Cognitively Guided Instruction is an inquiry-based approach to teaching mathematics that was developed at the Wisconsin Center for Education Research" [1]This approach gives teachers knowledge about the developmental stages of a child's mathematical reasoning. This in turn allows teachers to plan instruction based on their students' understanding and guide them toward greater mathematical reasoning and concept mastery. Development of mathematical reasoning using word problems are organized within a framework. The framework then enables the teacher to distinguish among problems in terms of reasoning difficulty. One important aspect when teaching is placing

---

[1] (Carpenter, Loef Franke, Levi, & Empson, 2015)(Carpenter, Loef Franke, Levi, & Empson, 2015)

the word problems from easiest to the most difficult. These problems were used with my second grade students for me to analyze and create a way to help fix their misconceptions through intervention. A goal for this approach is for students to become independent problem solvers who are able to develop a method for solving word problems without needing to depend on a teacher for telling them how to do it. [2]

One of the key components of Cognitively Guided Instruction (CGI) is having students make explicit their mathematical thinking by asking, "Can you tell me how you solved that?". By doing this, students have a productive way of explaining the strategies that they use to solve a problem. "Research shows that when students are expected to describe their strategies in detail with the teacher and with each other, they demonstrate higher mathematical achievement" [3]

Teachers need to elicit student thinking during problem solving in order to seek out how students solved their problems or to support the students in completing the problem or correcting the strategy or even extending the child's mathematical thinking. Teachers need to listen carefully during a child's explanation so that they can respond to what the students have shared. There are no "best" questions to use rather the most productive questions following "tell me what you did" is to refer specifically to what the child has shared during the discussion or within their writing. Carpenter et al (2015) explains that " following up on student explanations not only helps the teacher and other students learn more about the student's thinking, it also helps the student…enhance his understanding because as he verbalizes he can make and deepen connections among mathematical ideas…sharing allows the student and the class to learn what counts as a complete mathematical explanation"[4]

Supporting students to extend their mathematical ideas encourages them to go beyond that they have shared with the class in some way. "Eliciting student thinking starts with regularly asking students to explain their thinking. Consistently asking students to share their thinking, expecting each student to do so, and accepting whatever ideas students can provide will help students come to understand how to do math in your class, what it means to explain their mathematical ideas, and that everyone is capable of explaining mathematics."[5]

The instructional focus in second grade should shift to helping students develop quick recall of addition and related subtraction facts, as well as fluency with multi-digit addition and subtraction.[6] A critical step in establishing a deeper understanding is

---

[2] (Carpenter, Loef Franke, Levi, & Empson, 2015)
[3] (Carpenter, Loef Franke, Levi, & Empson, 2015)
[4] (Carpenter, Loef Franke, Levi, & Empson, 2015)
[5] (Carpenter, Loef Franke, Levi, & Empson, 2015)
[6] (Caldwell, 2011)

unpacking ideas related to addition and subtraction, for example, addition determines the whole of the parts and subtraction determines the missing part. Two big ideas to consider is when to use each operation and the second is how to get answers efficiently and accurately. [7]

Children need to learn that many different types of problems can be represented by addition and subtraction. The important part is to learn how to recognize these situations and represent them symbolically.[8] Addition and subtraction can be considered as either counting forward or backward. It is important for students to understand the relationship between addition and subtraction as this will reduce the number of facts students must know. It will help them by giving them a reliable strategy for subtraction which would be using a related addition fact. Many times, students have a bigger issue with subtraction due to the language that is used. It would then be helpful if students begin to think of them in terms of parts and wholes. In this way students do not become confused by having to distinguish the differences among the meanings of phrases, and the confusion that leads the children to make frequent reversal errors. Strategies can be taught in order to help students make sense of what needs to be done when dealing with word problems.

So, the question then becomes, why is part-part-whole reasoning important. This strategy helps students think about how numbers can be split into parts. "It allows students to see the relationship between a number and its component parts."[9] Typically, addition and subtraction story problems ask student to identify a missing value. Students who can identify the missing or unknown value as a missing part or missing whole will often be successful in choosing the appropriate operation. Students can use materials, drawings, or diagrams to help them solve problems. They will develop two simple but highly valuable rules for solving word problems; "1. If both parts are known and the whole is unknown, use addition. 2. If the whole and one part are known, use subtraction"[10]

In the <u>Educational Leadership</u> Article *Nine Ways to Catch Kids Up* Marilyn Burns stated that she found nine strategies to be essential to successful intervention. Those nine strategies are: 1. Determine and scaffold the essential mathematics content, 2. Pace lessons carefully, 3. Build in a routine of support, 4. Foster student interaction, 5. Make connections explicit, 6. Encourage mental calculations, 7. Help students use written calculations to track thinking, 8. Provide practice, 9. Build in vocabulary instruction. It was stated in the article that "many students needing math interventions have weak mathematical vocabularies". [11]

---

[7] (Caldwell, 2011)

[8] (Caldwell, 2011)

[9] (Queensland Government, 2014)

[10] (Queensland Government, 2014)

[11] (Burns, 2007)

Three issues to teaching mathematics were shared in the article "Nine Ways to Catch Kids Up": 1. It's important to help students make connections among mathematical ideas so they do not see these ideas as disconnected facts. 2. It's important to build students' new understandings on the foundation of their prior learning. And 3. It's important to remember that students' correct answers, without accompanying explanations of how they reason, is not sufficient for judging mathematical understanding.[12] Our struggling learners need more help than just extra practice. "Only when the basics include understanding as well as skill proficiency will all students learn what they need for their continued success."[13]

Strategies

This unit will be based on the struggles my second graders have encountered in addition and subtraction. I will utilize the activities presented in my math intervention group with students who have been struggling with problems with sums and differences within 20. I gave a preassessment, see Appendix B, to my whole class. Then used that to pull my students into small groups during intervention time. We used the journal to write down our thoughts and problem-solving strategies that were discussed with partners and within the group.

I relied on several teaching methods to teach my unit, which included direct instruction within a small group and collaborative pairs/partners. Collaborative pairs offer a great way to ensure that all students become engaged in an activity.

Unit Activities

These are activities designed to be used in small groups during intervention time. They could be used as a pre-teaching before starting a unit or as a re-teaching after lack of mastery on a formative or summative assessment

In the appendices there is a pre-assessment that was created that can be used to group students. Again, I gave the pre-assessment to 18/20 of my students. Below is a table showing the results of the pre-assessment with the number of students who got the problems wrong:

JOINING PROBLEMS:		
Join: Result Unknown	Join: Change Unknown	Join: Start Unknown

---

[12] (Burns, 2007)
[13] (Burns, 2007)

5+8=___	5+___=13	___+8=13
# of students: 4/18	# of students: 12/18	# of students: 8/18
SEPARATING PROBLEMS:		
Separate: Result Unknown	Separate: Change Unknown	Separate: Start Unknown
13 – 5=___	13 - ___=5	___ - 5=8
# of students: 14/18	# of students: 13/18	# of students: 17/18

As you can see the majority of the students struggled with the separating problems. All of these problems refer to the part-part-whole thinking. These problems refer to how numbers can be split into parts. It allows students to see the relationship between a number and its parts. Students should then be able to generalize the connections between addition and subtraction. This thinking is supposed to develop before children even start formal schooling. When students come to story problems they find them difficult to solve because they are unsure of the correct operation to use. Part-part-whole reasoning can assist my students when solving story problems.

Before beginning each of the activities below you will need to create one Journal (Appendix C) for each student to use in each activity. By creating a journal for each student you will have a place for students to write and think about the types of problems that you will be working on with the students.

Activity 1

This first activity is to help students gain an understanding of the part-part-whole reasoning. These three problems should help the students discuss and interpret the problems so that they can see them as addition and subtraction and then how to select the appropriate strategies to solve them.

Read each situation to the group. Have students talk with a partner before filling in chart. Then have students explain their thinking to the group.
Students will determine the value of parts in the following situations:
- Problem 1:   7 students are in the classroom. 12 students are out in the hallway. How many students will be in the classroom when everyone arrives? (Solved by adding: 7+12)
- Problem 2:   There are 19 children in the classroom. 12 went out to lunch. How many are left? (Solved by subtracting: 19-12 or adding: 12 + __ = 19)
- Problem 3:   19 children are in the classroom. 7 stayed in the room. How many went to lunch? (solved by subtracting: 19-7 or adding: 7 + _ = 19)

While discussing each situation students should understand that 1. Addition and subtraction are related. 2. The same situation can often be represented by addition and subtraction.

Activity 2

The second activity requires students to focus on addition story problems. Students need to understand that addition is commutative and that the same situation can often be represented in different ways.

Joining problems: change unknown and start unknown.
- Problem 1:   Grandmother had 5 strawberries. Grandfather gave her some more. Then Grandmother had 13 strawberries. How many strawberries did Grandfather give Grandmother?
- Problem 2:   Grandmother had some strawberries, Grandfather gave her 8 more. Then she had 13 strawberries. How many strawberries did Grandmother have before Grandfather gave her any?

Discuss how we could use part-part-whole to help us solve these two problems., one at a time. Have students solve on own first then talk with a partner, and finally explain to whole group.

Activity 3

The third activity requires the students to focus on subtraction story problems. Students should learn that subtraction is not like addition, it is not commutative. They should also learn that the same situation can often be represented by either addition or subtraction. The students should have help remembering that addition and subtraction story problems require them to identify a missing value. Students who can identify the missing or unknown value as a "missing Part" can be more successful in choosing an appropriate operation.

Separating Problems: Result Unknown, Change Unknown, Start Unknown
- Problem 1:   Grandfather had 13 strawberries. He gave 5 strawberries to Grandmother. How many strawberries does Grandfather have left?
- Problem 2:   Grandfather had 13 strawberries. He gave some to Grandmother. Now he has 5 strawberries left. How many strawberries did Grandfather give Grandmother?
- Problem 3:   Grandfather had some strawberries. He gave 5 to Grandmother. Now he has 8 strawberries left. How many strawberries did Grandfather have before he gave any to Grandmother?

Discuss how we could use part-part-whole to help us solve these problems one at a time. Have students solve on own first then talk with a partner, and finally explain to whole group.

Conclusion

In conclusion, I found this unit very informative and helpful for my second-grade classroom. The seminar topic interested me and my seminar leader really showed me which direction I needed to go in my research and really helped me to understand mathematics at a deeper level.
It helped to understand that many of my students misconceptions and misunderstandings can be helped with deeper discussions and breaking things down into parts.

Appendix A

Common Core: Second Grade Math

- CCSS.MathContent.2.OA.A.1: Use addition and subtraction within 100 to solve one- and two- step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing with unknowns in all positions, eg. by using drawings and equations with a symbol for the unknown number to represent the problem.
- CCSS.MathContent.2.OA.B.2: Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.
- CCSS.MathContent.2.NBT.B.5:Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

Appendix B

Name _____ Pre-Assessment

Problem:	Strategy:
5+8=___	
5+___=12	
___+4=10	
15-8=___	
12-___=7	
___-8=6	

Appendix C

Journal for use by each student during each activity

_____'s

Math Journal

Addition

Subtraction

Problem Solving

Activity 1

Problem 1

Problem 2

Problem 3

Activity 2

Problem 1

Problem 2

Activity 3

Problem 1

Problem 2

Problem 3

Bibliography

(n.d.).

Burns, M. (2007). Nine Ways to Catch Kids Up. *Educational Leadership*, 1-6.

Caldwell, K. B.-W. (2011). *Developing Essential Understanding of Addition and Subtraction for Teaching Mathematics in Pre-K - Grade 2.* NCTM.

Carpenter, T. P., Loef Franke, M., Levi, L., & Empson, S. B. (2015). *Children's Mathematics: Cognitively Guided Instruction 2nd edition.* Portsmouth: Heinemann.

Queensland Government. (2014). *Hilder Roads School.* Retrieved from Hilder Roads School: https://hilderroadss.eq.edu.au/Supportandresources/Parentresources/Pages/Math