Model-Based Approach to Develop Special Education Students Problem-Solving Skills in Elementary School

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Introduction

When I think about math, I think about math word problems and how difficult they for my special education students. I have seen my students take a word problem and just pull the numbers out of problem and add them. These students will add the numbers in the problem, no matter what the operation is that they are supposed to use. If the problem says, "there are 2 classes with 300 students in each class and I need to know how many students there are in all," The students will add 300+2 and that will be their answer. It takes a lot of time to convince students that the answer is not 302 but is 600. When working as a class on another problem the students could not understand that when you are giving them the cost of something and wanting to know how much money you need as in this example, "A box of paper costs $24 and I need to buy 2 boxes of paper, how much money do I need?" My students looked at this problem and added 2+24. I then turned to another adult in the room and asked her if she thought that she had enough money to go and buy the paper. She said that she could only buy one box of paper because she didn't have enough money. When she said she didn't have enough money the students started to question why she didn't have enough. By relating the problem to real through the conversation between the two of us made the students start to think about what they had done wrong in solving the problem. This problem was documented by Brian Greer, in 1992 in his publication *Multiplication and division as models of situations*. He states, "Observation of students' WP (word problem) solving during the baseline condition indicated an immature impulse to grab numbers found in the problems and apply operations to produce answers for the solution. It is clear that these students bypassed conceptual model–based representations and moved "directly to mathematical expression on the basis of syntactical, surface clues"[1]

When my students are asked to break down a problem without adult assistance, they will sit there and stare the paper until I come back and help them. Thinking about what a word problem is asking causes my students to break down and do nothing or shake their heads and say they understand until they are asked to do a problem on their own. One reason that students act this way is that they are not able to reason and think about a problem as parts, that when solved separately will solve the whole problem. One thing that has helped with this problem is that we discuss every problem as a group, and reason about what the problem is asking and how we are going to get to the answer. One way to get the students to understand math is to convince them that math is everywhere, and that it is important to know how to solve math problems. We have had many discussions about why math is important, and that life is really a lot of word problems and understand them will help you in life.

This unit will focus on getting students to not only break down a problem in to parts and then solve the parts but will also help them to develop their computational skills. The goal of this unit will be to have a set of steps to solving a word problem, that will include asking questions to get to the bottom of what the question is asking. All my students have a sheet that has math problem solving key words. These sheets of key words are supposed to help them to decide what operation to use in a problem. Unfortunately, students are not able to look at the sheet and say, "I see that the problem says how much more, so I need to subtract." The other problem that arises with using keywords is that if the problem has more than one step the key words don't work. When a problem has both addition and subtraction students are confused by the key words. For their birthday John and his sister received candy. John had thirty-two pieces of candy while his sister had forty-two. If they ate thirty-five pieces the first night, how many pieces do they have left? Most of us would know that you have to add the amounts of candy that each child received and then subtract 35, but students may be confused because the problem says, "How many are left?" Students will take 32 and try to subtract 35 from it and come up with 3 as the answer or they may take 42 and subtract 35 and get the answer 7. Using the "key words," method can confuse more than it will help with solving problems.

I want to address the idea of breaking a problem into parts and then solving it. Once my students are able to solve math word problems then they will be able to reason out other problems, by breaking a problem into parts and to think about the parts instead of the whole. The strategy that I am going to focus on is Conceptual Model-Based Problem Solving. This strategy breaks problems into different categories following a text structure that word problems follow, by generalizing the text structure and story grammar, students will be able to answer word problems with more success.

Figure 1: [2]

PROBLEM-SOLVING SITUATIONS

JOINING PROBLEMS		
Join: Result Unknown (JRU)	**Join: Change Unknown (JCU)**	**Join: Start Unknown (JSU)**
◆ Grandmother had 5 strawberries. Grandfather gave her 8 more strawberries. How many strawberries does Grandmother have now? 5 + 8 = □	♥ Grandmother had 5 strawberries. Grandfather gave her some more. Then Grandmother had 13 strawberries. How many strawberries did Grandfather give Grandmother? 5 + □ = 13	♠ Grandmother had some strawberries, Grandfather gave her 8 more. Then she had 13 strawberries. How many strawberries did Grandmother have before Grandfather gave her any? □ + 8 = 13
SEPARATING PROBLEMS		
Separate: Result Unknown (SRU)	**Separate: Change Unknown (SCU)**	**Separate: Start Unknown (SSU)**
◆ Grandfather had 13 strawberries. He gave 5 strawberries to Grandmother. How many strawberries does Grandfather have left? 13 - 5 = □	♥,Grandfather had 13 strawberries. He gave some to Grandmother. Now he has 5 strawberries left. How many strawberries did Grandfather give Grandmother? 13 - □ = 5	♠ Grandfather had some strawberries. He gave 5 to Grandmother. Now he has 8 strawberries left. How many strawberries did Grandfather have before he gave any to Grandmother □ - 5 = 8
PART -PART -WHOLE PROBLEMS		
Part-Part-Whole: Whole Unknown (PPW:WU)	**Part-Part-Whole: Part Unknow (PPW:PU)**	
◆ Grandmother has 5 big strawberries and 8 small strawberries. How many strawberries does Grandmother have altogether? 5 + 8 = □	♥,Grandmother has 13 strawberries. Five are big and the rest are small. How many small strawberries does Grandmother have? 13 - 5 = □ or 5 + □ = 13	

Figure 1(continued): [2]

COMPARE PROBLEMS		
Comp. Difference Unknown	**Comp. Quantity Unknown**	**Comp. Referent Unknown**
◆♥,Grandfather has 8 strawberries. Grandmother has 5 strawberries. How many more berries does Grandfather have than Grandmother? 8 - 5 = □ or 5 + □ = 8	♠ Grandmother has 5 strawberries. Grandfather has 3 more strawberries than Grandmother. How many strawberries does Grandfather have? 5 + 3 = □	♠ Grandfather has 8 strawberries. He has 3 more strawberries than Grandmother. How many strawberries does Grandmother have? 8- 3 = □ or □ + 3 = 8
MULTIPLICATION & DIVISION PROBLEMS		
Multiplication	**Measurement Division**	**Partitive Division**
◆ Grandmother has 4 piles of strawberries. There are 3 strawberries in each pile. How many strawberries does Grandmother have? 4 x 3 = □	◆ Grandmother had 12 strawberries. She gave them to some children. She gave each child 3 strawberries. How many children were given strawberries? 12 ÷3 = □	◆♥,Grandfather has 12 strawberries. He wants to give them to 3 children. If he gives the same number of strawberries to each child, how many strawberries will each child get? 12 ÷ 3 = □

Problem chart based on Cognitively Guided Instruction Problem Types (Carpenter et al., 1996)

“Because WPs of a specific problem type (e.g., PPW) share a common underlying structure involving the same key elements (e.g., part, part, and whole), a set of WP story grammar questions can be generated to serve as prompts in guiding students when they organize information and express mathematical relations in WP conceptual models. For instance, in the PPW problem types, basic WP story grammar questions such as "Which sentence tells about the whole or combined quantity?" and "Which sentence tells about one of the small parts that makes up the whole?" can assist in the comprehension and representation of the underlying structure of a WP in the conceptual model (i.e., part + part = whole), therefore facilitating solution planning. Emphasis on the meaningful representation of mathematical relations in problem solving is consistent with contemporary approaches to story problem solving that emphasizes conceptual understanding of the story problems before decision making on the *choice of operation*. In addition, an emphasis on representing mathematical relations in conceptual models facilitates algebraic reasoning and thinking that involves symbolic expressions of mathematical relations in equations."[3]

Figure 2: [4]

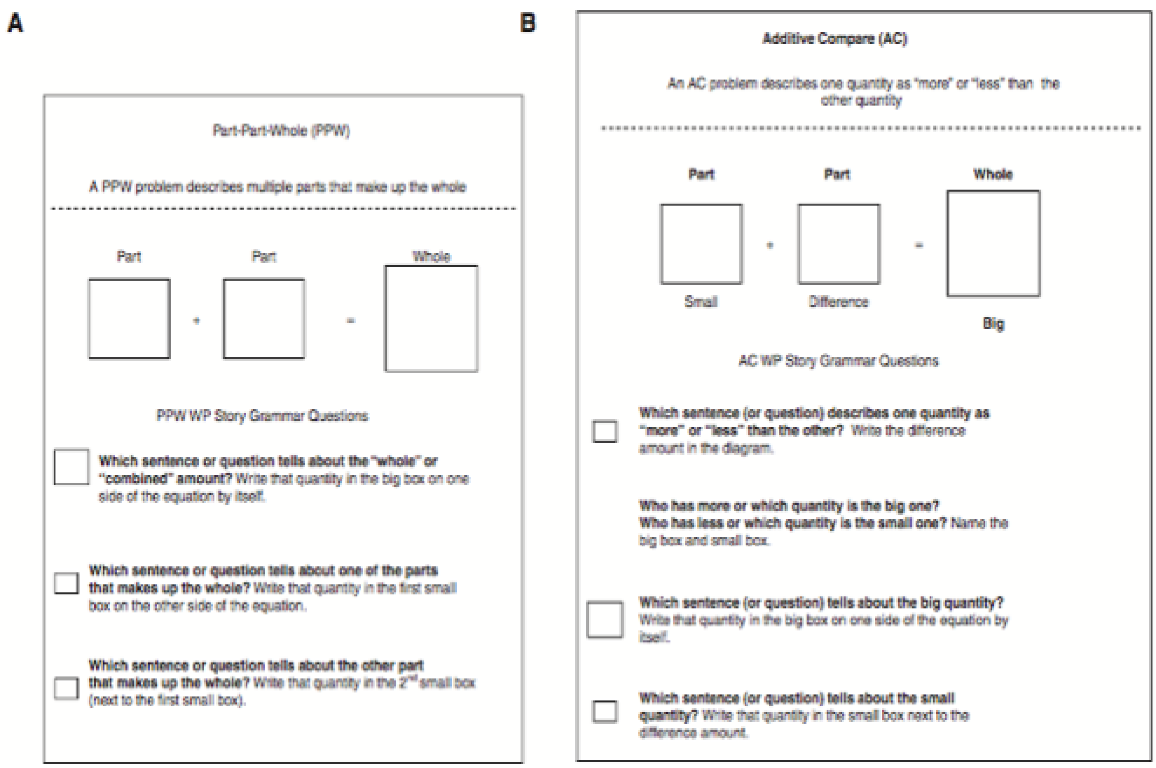

Figure 3: [5]

Table C2-1. Variations in Addition Word Problems (from Xin et al., 2008)

Problem Type	*Sample Problem Situations*
Part-Part-Whole	
	Combine
Part (or smaller group) unknown	1. Jamie and Daniella have found out that together they have 92 books. Jamie says that he has 57 books. How many books does Daniella have? OR Jamie and Daniella have found out that together they have 92 books. Daniella says that she has 35 books. How many books does Jamie have?
Whole (or larger group) unknown	2. Victor has 51 rocks in his rock collection. His friend, Maria, has 63 rocks in her collection. How many rocks do the two have altogether?
	Change-Join
Part (or smaller group) unknown	1. Luis had 73 candy bars. Then, another student, Lucas, gave him some more candy bars. Now he has 122 candy bars. How many candy bars did Lucas give Luis? 2. A girl named Selina had several comic books. Then, her brother Andy gave her 40 more comic books. Now Selina has 67 comic books. How many comic books did Selina have in the beginning?
Whole (or larger group) unknown	3. A basketball player ran 17 laps around the court before practice. The coach told her to run 24 more at the end of practice. How many laps did the basketball player run in total that day?

Table C2-1. Continued

Problem Type	*Sample Problem Situations*
	Change-Separate
Part (or smaller group) unknown	1. Davis had 62 toy army men. Then, one day he lost 29 of them. How many toy army men does Davis have now? 2. Ariel had 141 worms in a bucket for her big fishing trip. She used many of them on the first day of her trip. The second day she had only 68 worms left. How many worms did Ariel use on the first day?
Whole (or larger group) unknown	3. Alexandra had many dolls. Then, she gave away 66 of her dolls to her little sister. Now, Alexandra has 63 dolls. How many dolls did Alexandra have in the beginning?
Additive Compare	
	Compare-more
Larger quantity unknown	1. Denzel went out one day and bought 54 toy cars. Later, Denzel found out that his friend Gabrielle has 56 more cars than what he bought. How many cars does Gabrielle have?
Smaller quantity unknown	2. Tiffany collects bouncy balls. As of today she has 93 of them. Tiffany has 53 more bouncy balls than her friend, Elise. How many bouncy balls does Elise have?
Difference unknown	3. Logan has 117 rocks in his rock collection. Another student, Emanuel, has 74 rocks in his collection. How many more rocks does Logan have than Emanuel?
	Compare-less
Larger quantity unknown	1. Ellen ran 62 miles in one month. Ellen ran 29 fewer miles than her friend Cooper. How many miles did Cooper run?
Smaller quantity unknown	2. Kelsie said she had 82 apples. If Lee had 32 fewer apples than Kelsie, how many apples did Lee have?
Difference unknown	3. Deanna has 66 tiny fish in her aquarium. Her dad Gerald has 104 tiny fish in his aquarium. How many fewer fish does Deanna have than Gerald?

I also want to explore ways for students to solve the computation part of the problem. My hope is that if the students have a set of steps to solve a problem, they will experience triumph. “If you solve it by your own means, you may experience the tension and enjoy the triumph of discovery.”[6] If the student is able, they will be encouraged to work the problems out on paper, but also, they can use a calculator or computer to solve the problem.

Demographics

Red Clay Consolidated School District is in Northern New Castle County in Delaware, the district includes urban and suburban settings within its borders. The district has 28 schools that service about 15,741 students. 12.7% of the students in Red Clay Consolidated School District are Special Education Students.

Cooke Elementary is the newest school in the district and has about 650 students. Demographics for the school year 2016-17, 6.2% of students were African American, 0.2% of students were American Indian and Hawaiian, 8.6% of students were Asian. Hispanic/Latino students account for 15.3% of students and Multi-Racial Students comprise 3.7% of the student body. 66.5% of the students in the school are White/Caucasian. Other student characteristics include 12.4% English Language Learners, 15% Low-Income students, and 7.2% Special Education students.

I am a Special Education Teacher and primarily teach students in 4th grade. For the 2018-19 school year, there are 10 special education students in that grade. Approximately 80% of those students are African American or Hispanic/Latino. I pull students out of their regular education classroom for small group instruction, but I collaborate with the grade level teachers, so I can teach many of the same concepts that are being taught in the general education classroom, as well as helping them make progress toward their Individual Education Plan goals.

Rationale

Many special education students are convinced that they cannot do math word problems, they have either seen that the problems are above what they can do, so they give up before they start so they can be done as quickly as possible. This situation is discussed in the article Understanding Word Problems in Mathematics, “For many students who struggle with mathematics, word problems are just a jumble of words and numbers. However, you can help students make sense of these problems by teaching them problem-solving processes. Indeed, as students move forward in their mathematical learning, they will need to apply problem-solving processes to more and more complex situations, so they become college and career ready.”[7] Working math word problems is especially difficult because there is a reading component and since reading is difficult, word problems are difficult. Since reading and math are both difficult for the students, most of them have given up trying by the time they have reached 4th grade. They are not convinced that math is part of everyday life and is important. Using the Cognitively Model Based approach to Problem-Solving my students will be able to be more successful with problem solving, and their test scores will improve. The fact that students were behind is seen not only in state testing scores, but also in their scores on their math exams. With the Special Education students that I teach, I have had no success with getting the students to be proficient on the Smarter Balanced Assessment and have also had difficulty getting them to make their IEP goals for math. The students are frustrated and have a lack of desire to succeed. So, I want to change the way my students think about math and make it more fun for them to do, and maybe they will be able to problem solve with more consistency. I think that by learning how to take apart or decompose a problem and working on the parts to get a solution, these students can transfer problem solving to other parts of their lives.

The fact that students move around, have different teachers each year, and different curriculum undermines the desire for the student to succeed in math. I feel that even though there is consistency with math instruction that my students receive, if they have been at my school for more than one year, because of math curriculum builds on itself and that there are a lot of professional development opportunities for teachers, these students are still suffering and falling behind. For example, my students will say that they are “done” when they have added two numbers together. Many students have a hard time understanding that every problem is different, but that they follow the same patterns, and they may have to use different strategies for each one. They also cringe at the thought of having to use any operation that is not adding, and will often refuse to do so, because they think it is to hard or they don’t understand how to subtract, multiply, or divide. My hope is that by giving the students a check list to follow will help them to be able to problem solve. “For instance, successful problem solvers (a) quickly and accurately identify the mathematical structure (e.g., compare) of a problem that is generalizable across a wide range of similar problems, (b) remember a problem’s structure for a long time, and (c) distinguish relevant from irrelevant information” [8]

Most of my students have math goals on their IEP’s, and because of that I spend a lot of time working with them on solving word problems and computation. By using the Cognitive Model Based Problem-Solving approach my students will be able to understand how to solve word problems. “The theme that ties together our analysis of students' mathematical thinking is that children intuitively solve word problems by modeling the action and relations described in them. By developing this theme, we are able to portray how basic concepts of addition, subtraction, multiplication, and division develop in children and how they can construct concepts of place value and multi-digit computational procedures based on their intuitive mathematical knowledge.” [9]

Standards to Be Addressed

The introduction to the Common Core State Standards states that students will be able to read a math word problem and solve it by focusing on the Common Core State Standards of Mathematical Practices. Students will be able to make sense of problems and persevere in solving them, and they will be able to reason abstractly and quantitatively and look for and express regularity in repeated reasoning. They will construct viable arguments and critique the reasoning of others, use models, and attend to precision.

In my unit I will be focusing on Common Core State Standard CCSS.Math.Practice.MP1 Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

CCSS.MATH.PRACTICE.MP4 Model with mathematics. Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

CCSS.MATH.PRACTICE.MP5 Use appropriate tools strategically. Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Organization

When having students work on math problems it is important for them to use graphic organizers to organizer their thoughts and keep themselves focused. There are many different types of graphic organizers that can be used but I need to have one that has a check list formula for solving problems. The graphic organizers for the Cognitively Model Based approach to Problem-Solving will be used in conjunction with an organizer/checklist based on the article *Understanding Word Problems in Mathematics*[10]. See Appendix B for checklist. In the article it is stressed that the students should start by reading the problem, then reread the problem and underline the parts of the problem that are important. Next, the students will try to draw a picture, so they can "see" what they are working on, this picture can and should be very simple, for example the pictures should consist of circles or stick figures or other items that are easy to draw. Then they will decide what they are to do to solve the problem, by establishing a strategy or writing an equation to represent the picture. If they are comfortable with estimating an answer that will be the next step, but this is a concept that will have to be taught separately so that students understand why estimating is a useful tool. Solving the problem is the next step in the process, this can be done either by hand or with a calculator if the students are struggling with adding or subtracting numbers. The last thing that the students will do is explain how they got their solution. Some problems will be easy for them to explain, and others will take teacher prompting and questioning.

Math students I have found need to be questioned and challenged with math problem solving daily. When you have many students in a classroom it is impossible to meet with a small group of students daily, so having the students pair up is a solution to the problem of not being able to meet with the students every day. Once the students have had time to work together, I bring them back to the carpet for a whole group discussion about the problem and their solutions. When the students have worked together and thought about what the problems are asking them then they are more invested in learning.

I ask the students questions when I meet with them to help them make sure that they understand what the problem is asking them to do. Some of the questions are based on the Cognitively Model Based approach to Problem-Solving "Which sentence or questions tells about the "whole" or "combined" amount? Which sentence or question tells about one of the parts that makes up the whole? Which sentences tells about the other part that makes up the whole?"[11] All the questions that I use when clarifying content and ideas are open-ended questions some examples are: What words I don't understand this part, what is the problem asking you to do? What was this person doing, did they take away something or add something to the whole? Who else was there, and how do they fit into the solution? What operation do you need to use to solve the problem? Is there another step to get the solution, or do you have the solution?

I also have the students' pair with each other and ask each other to clarify concepts by asking questions. They can use the same questions that I do, or they can make up their own. I have found that it is easier for the students to think of the questions if they are written out for them in advance. While the pair is talking, they should be recording the answers to the questions, so they can share out information the larger group.

Teaching Activities

The activities can either be used in a small group setting or with the whole group in a larger classroom.

To do this I will use mentor texts, *Safari Park, by Stuart J. Murphy and illustrated by Steve Bjorkman*, this book is part of the Math Start book series. I will use Queen Arithma's Party by Mary Goral to illustrate another strategy to work on adding strategies. These will be the main stories that I will use. These stories are great for getting students think about math in different ways, and to understand that math is everywhere and if you understand it your world will make more sense. As we have read the books, the students will start to ask questions about the problems in the story and we can discuss problem solving. I want the students to understand what it takes to logically think about solving problems, and how they can use techniques from math to solve other problems. "Although storytelling is not considered to be a "model", it is considered as another pedagogical technique that can enhance the understanding of abstract mathematics concepts. Furthermore, NCTM (2000) states in their Communication Standard that, communicating, talking, listening, and writing about mathematics are essential components in learning mathematics."[12]

Safari Park is a great book for teaching problem solving. The story is about a family that decides to go to an amusement park. Each child is given 20 tickets to ride the rides and then one of them loses his tickets. The grandfather tells the other children that they will have to take the child on a ride with them. Each child must figure out what rides and games they can do after they take their cousin on a ride. There is a set of problems for each child with an unknown number, and the child must figure out how many tickets are needed for the final category of rides, games, or treats. The book is good for problem solving because you can use it to model the problems. You can draw out the problem and discuss why different configurations of tickets for each child will not work.

Activity one
Safari Park, by Stuart J. Murphy and illustrated by Steve Bjorkman. Before reading, set up the story by asking the students if they have ever been to an amusement park. What kinds of rides did they go on? Did they play games or get anything to eat? On the whiteboard/smartboard draw four circles, assign a name to the circles and an amount of tickets to ride the rides in the circle there are four different rides, games or snacks in each circle, assign values from one to four. Ask the class if you give them 20 ticket each to ride rides what rides would they ride? Will they get to play all the games they want? Will they get a snack? When you are asking the students questions focus on Part (smaller group) unknown, write out on the white/smartboard (part + part + part = whole). In this case you know they whole amount and you need to find the parts that will make up the whole.

Split the group into pairs and have them talk about what they would do. The students are required to ride at least one ride from each group. How will they break up the 20 tickets? Have the students draw the circles on a whiteboard? Ask the students, will you be able to ride all the rides you want to ride with 20 tickets? Start reading the story, *Safari Park*. While you are reading, write part + part + part = whole on the board, have the students figure out the what part is missing before you go to the page with the answer.

Activity two
Continue reading *Safari Park*. While you are reading, write part + part + part = whole on the board, have the students figure out the what part is missing before you go to the page with the answer. When you have finished the book give the students the same type of problem to solve but change the amounts that the rides cost. Then give the students another type of problem and see if they can generalize the skill part (smaller group) unknown, that was focused on the story. John and Jack have 75 marbles together. John has 29 marbles. How many marbles does Jack have? Ask students to explain how they came up with their answer. Did they subtract? Did they add on to 29?

Activity three:

Write on the whiteboard or smartboard: Part + Part=Whole, Whole= Part + ☐, Part + Part = ☐ Share with the class that if you are missing the whole part of the problem, they you should be adding the parts together, but if you have the whole but are missing a part then you should subtract to find the answer. Read a word problem to the class and have them discuss what it is that they need to find. Is the unknown a part or the whole? How will they find the number?

When I am teaching my students how do figure out word problems, I will assign a letter or name to the part that I know. An example would be: Sara has 100 red balls and Jack has 25 red balls. How many balls do they have altogether? I would assign an S =100 to Sara and J = 25 to Jack. The whole would be R=☐ Another example would be, Evan has $250 and needs $395 to buy a bike, how much more money does he need? The equations for this problem would be E + ☐ = $395. This helps the students keep the information straight and will help them in the future when they must write equations for problems. There are many examples of word problems that can be used for theses example, they can be found online or in textbooks, but the best thing to do is have the students come up with their own word problems that follow the pattern.

Activity four

Read Queen Arithma's Party by Mary Goral. See Appendix A for the story. As you are reading the story, have the class think about how they would count the invitations. Would they count by 1's, like Queen Arithma did at first? Or are there other ways to count, that would make it easier? When you read the part about how many she counted the first 3 times, ask the class if those numbers should be added together? Try adding the numbers together modeling part + part = whole. What was the whole? Discuss with class if this is a reasonable answer based on what the Queen was doing? Do the parts need to be added together, or were they numbers put in the problem to throw us off?

Before reading the story draw a blue circle (hundreds), a red circle(tens), a green circle(ones) and a black box (whole) on the board. Above the circles and box write part + part + part = whole, or if students are familiar with the concept you can use ☐+☐=☐. When you get to the end of the story fill in the circles and boxes, what is the answer?

Next, model on the board counting by 10's and then grouping the items in groups of 100, then how many 10s are there that don't add up to 100, and how many single invitations are there? After reading the story split the class into small groups of 3-4 and give them something to count. An example of this would be counting cubes, Legos, pennies, or a mixture of items. Have the groups count out the items by 10s and then grouping them into 100s, then how many 10s are that don't make 100, and how many singles are left over. Once the groups have done this, they can add the items up using part + part + part = whole, or if students are familiar with the concept you can use ☐+☐+☐=☐. Give the groups several different numbers to model.

Conclusion

After implementing the unit on Conceptual Model-Based Problem Solving, for math word problems students should be able to solve word problems that involve addition and subtraction. Students should be able to look at a word problem and decide if the operation needed to solve the problem is an addition problem using the Part + Part =Whole method or a subtraction problem using the Whole = Part + Part. Once they have mastered this concept for solving addition and subtraction problems, they should be able to generalize the formula and solve multiplication and division problems.

Bibliography

BaBerry, Miles, and Andrew Csizmadia. "Computational Thinking and Mathematical Reasoning – An Open Mind." An Open Mind. Accessed January 14, 2019. http://milesberry.net/2016/11/computational-thinking-and-mathematical-reasoning

Cai, Jinfa, Mary S. Jakabcsin, and Suzanne Lane. "Assessing Students' Mathematical Communication." The Canadian Journal of Chemical Engineering. March 17, 2010. Accessed January 13, 2019. https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1949-8594.1996.tb10235.x.

Candler, Laura. How to Introduce Decimals with Base Ten Blocks. Accessed January 14, 2019. https://www.lauracandler.com/math-word-problems/.

"Computational Thinking How Do We Think about Problems so That Computers Can Help?" CAS Computational Thinking - A Guide for Teachers Computing At School.

Accessed January 14, 2019. https://community.computingatschool.org.uk/resources/252

Greer, B. (1992). Multiplication and division as models of situations. In D. Grouws (Ed.), Handbook of research on mathematics teaching and learning (pp. 276–295). New York: Macmillan.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). Adding it up: Helping children learn mathematics. (Washington, DC: National Academy Press) Moses, B (1997) Algebra for a new century. Teaching Children Mathematics, 3, 264–265.

"Mathematics Glossary." Common Core State Standards Initiative. Accessed January 13, 2019. http://www.corestandards.org/the-standards/mathematics.

Mulvahill, Elizabeth. "Are Your Students Struggling With Math Word Problems?" WeAreTeachers. July 17, 2017. Accessed January 14, 2019. https://www.weareteachers.com/struggling-with-math-word-problems/.

Ahoskin4. "Over Fifty Problem-Solving Strategies Explained." Simple Strategies Academics Can Use to Help Students Improve Their Writing Skills - University of New England (UNE). November 13, 2018. Accessed January 14, 2019. https://www.une.edu.au/about-une/faculty- of-humanities-arts-social-sciences-and-education/bcss/future-students/study- areas/psychology/research/psychology-community-activities/over-fifty-problem-solving-strategies-explained.

Pólya, George. How to Solve It: A New Aspect of Mathematical Method. Garden City, NY: Doubleday & Company, 1957.

Weintrop, David, Elham Beheshti, Michael Horn, Kai Orton, Kemi Jona, Laura Trouille, and Uri Wilensky. "Defining Computational Thinking for Mathematics and Science Classrooms." SpringerLink. October 08, 2015. Accessed January 14, 2019. https://link.springer.com/article/10.1007/s10956-015-9581-5.

Wing, Jeannette M. "Computational Thinking." Communications of the ACM49, no. 3 (2006) 33. doi:10.1145/1118178.1118215.

Xin, Yan Ping. "Conceptual Model-Based Problem Solving Conceptual Model ..." Accessed January 13, 2019. https://www.sensepublishers.com/media/1424-conceptual-model-based-problem-solving.pdf.

Zorfass, Judy, Tracy Gray, and Power Up What Works. "Understanding Word Problems in Mathematics." College or Training Programs: How to Decide | LD Topics | LD OnLine. Accessed January 14, 2019. http://www.ldonline.org/article/62401/.

Notes

1. Greer, B. (1992). Multiplication and division as models of situations. In D. Grouws (Ed.), Handbook of research on mathematics teaching and learning (pp. 276–295). New York: Macmillan.
2. Carpenter, T., Fennema, E., Franke, M.L., Cognitively Guided Instruction: A Knowledge Base for Reform in Primary Mathematics Instruction (Chicago: The University of Chicago Press), 3-20
3. Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). Adding it up: Helping children learn mathematics. (Washington, DC: National Academy Press) Moses, B. (1997). Algebra for a new century. Teaching Children Mathematics, 3, 264–265.
4. Xin, Yan Ping. "Conceptual Model-Based Problem Solving Conceptual Model ..." Accessed January 13, 2019.
5. Ibid.
6. Polya, G., How To Solve It, (Princeton: Princeton University Press, 1945)
7. Zorfass, Judy, Tracy Gray, and Power Up What Works. "Understanding Word Problems in Mathematics." College or Training Programs: How to Decide | LD Topics | LD OnLine. Accessed January 14, 2019.
8. Krutetskii, V. A. (1976). The psychology of mathematical abilities in school children. (Chicago: University of Chicago Press.) Quilici, J., & Mayer. R. (1996). Role of examples in how students learn to categorize statistics word problems. Journal of Educational Psychology, 88, 144–161.
9. Carpenter, T., Fennema, E., Franke, M.L., Cognitively Guided Instruction: A Knowledge Base for Reform in Primary Mathematics Instruction (Chicago: The University of Chicago Press), 3-20
10. Zorfass, Judy, Tracy Gray, and Power Up What Works. "Understanding Word Problems in Mathematics." College or Training Programs: How to Decide | LD Topics | LD OnLine. Accessed January 14, 2019.
11. Carpenter, T., Fennema, E., Franke, M.L., Cognitively Guided Instruction: A Knowledge Base for Reform in Primary Mathematics Instruction (Chicago: The University of Chicago Press), 3-20
12. Barr Goral, Mary & Meyers Gnadinger, Cindy. (2006). Using Storytelling to Teach Mathematics Concepts. Australian Primary Mathematics Classroom.
13. Ibid.

Appendix A

Queen Arithma's Party [13]

By Mary Goral

Once upon a time a long, long time ago in the land of Htam, lived a queen. Her name was Queen Arithma. Queen Arithma was a good leader. Her loyal subjects loved her and knew her well. In fact, there were three things about Queen Arithma that everyone knew. The first thing was that she loved mathematics and was an excellent problem solver. Second, she loved parties — the bigger the better. And finally, she hated winter. The ice, snow and blowing winds made her sad and depressed.

Upon awakening one cold and dreary winter's day, the queen knew she needed to do something to lift her spirits. She thought and thought and finally came up with a brilliant idea. She would have a party! This would not just be any party. Queen Arithma decided that this party would be a costume ball and that she would invite everyone she knew. She wanted this to be the biggest party she had ever given.

Early the next morning, the queen called her assistants to her study. There she told them about the party. The first task was to make a list of everyone the queen knew. The assistants wrote and wrote and finally by the end of the day, they had finished the list. This list was so long it trailed off the big writing desk and went all the way to the door.

The next morning the queen's assistants arrived in the study. Their task for the day was to begin making invitations. This was hard work, because all the invitations were made by hand. The assistants worked all day. By the end of the day they had finished making invitations for only half the list. On the third day, the assistants arrived early in the queen's study to finish the work. Again, they wrote and wrote. By the end of the day they had finished all of the invitations and the queen congratulated them on a job well done.

The following morning the queen began to count the invitations. They were heaped on her study table, but she knew she must begin, because her trusted friend Gwendolyn was to arrive at noon to collect the invitations and deliver them. Queen Arithma began to count. She reached 52 when someone knocked on the door. After the queen had attended to the person at the door, she went back to counting, but unfortunately, she forgot where she was and had to begin again. This time she counted to 77 when there was another interruption. The queen was beginning to get very frustrated. Again, Queen Arithma started counting from the beginning. This time she only reached 19 when she was interrupted again.

When the queen went to the door, she was surprised to see Gwendolyn standing there. Could it be 12:00 noon already? The queen immediately told her friend that she was not finished counting and asked if she had any ideas to help. Not surprisingly, Gwendolyn was flattered. Queen Arithma was known throughout the land for her love of mathematics and her ability to solve problems. Gwendolyn thought for a short time, and then asked the queen if she ever counted by 10s, because that is what she did when she had a large number of items to count.

Queen Arithma was delighted and asked Gwendolyn if they could begin counting by 10s at once. However, Gwendolyn said that before they began counting, they needed to bundle the invitations in some way. She asked if the queen had three different colors of ribbon in her study. The queen found red, blue and green ribbon tucked away in her desk. Gwendolyn suggested they tie the bundles of 10s with red ribbon, and when they got 10 bundles of 10, they would bundle those in blue ribbon. The remainder of the envelopes they would tie together in green ribbon, but they would need to indicate the number of individual ones left over by writing that number on a separate piece of paper.

At last they began to count. In no time at all, Gwendolyn and Queen Arithma had counted and bundled all of the invitations on the table. Counting by 10s and grouping the invitations together in bundles made the work so much easier! In the end, there were 5 bundles tied with blue ribbon, 7 bundles tied with red ribbon, and 4 invitations left over tied with a green ribbon. How many invitations did Queen Arithma have in all? (574)

Appendix B

Math Word Problem Work Sheet😁

I read the problem 2 times and underlined the important information 😁😁

Type problems here

I know what the problem is asking me 😁

Decide what operations need to be used Addition, Subtraction, Multiplication or division.

I have written an expression to solve the problem😁

Write your number problem here:

I have solved the problem and then checked to make sure it is correct 😁😁

Did I reread the problem to make sure that I answered all the steps required- if not go back to top of this page and re-read. Think "What was I asked?"

I can explain what I did to solve the problem 😁

Talk to a partner about how you solved the problem. Did you get the same answer? If not compare your answers and see if any mistakes were made. Correct your mistakes