

Developing Understanding and Proficiency for English Language Learners' Addition and Subtraction

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Introduction

The National Council for Teachers of Mathematics (2000) recognizes that

“Addition and its inversely related operation, subtraction, are powerful foundational concepts in mathematics, with applications to many problem situations and connections to many other topics.”¹

The operations of addition and subtraction first appear in the Common Core State Standards in Kindergarten and stretch through the Third Grade Operations and Algebraic Thinking Strand. However, adding and subtracting skills stretch well beyond those bounds.

Before most children even enter school, they are exposed to the idea of adding by wanting more cookies, by putting blocks together, or by counting digits on their fingers. Subtraction may present itself in a like manner. For example, they have 2 cookies but eat 1, they take all the red blocks away, or they put fingers down when showing that they are four years old.

Just as most children are exposed to the operations of addition and subtraction before Kindergarten, all people continue to use these skills beyond the realms of Third Grade. Addition and subtraction set the foundations for multiplication and division, are used when manipulating fractions, negative numbers, calculating chemistry equations, and most importantly, in real life. Adults use addition and subtraction on a daily basis when shopping for groceries, paying bills, playing sports, cooking, purchasing goods, etc. For the rest of their lives, on a daily basis, my students will continue to use addition and subtraction. In research by Cai and Moyer, they determined that the reversal relationship between addition and subtraction is also fundamental for developing students' algebraic thinking in earlier grades². Thus, this was created to foster fluency in their facts and to help them develop models to understand how to problem solve.

Demographics

My school, Pleasantville Elementary School, is located in New Castle, Delaware within the Colonial School District. In this small school of less than 500 students, there is a high population of Low Economic Status Families (LES) and English Language Learners

(ELL). 48% of families are identified as living below the poverty line. 23% of families and their students have identified a language other than English as their first spoken language. These statistics are tied closely to trauma outside of the school.

13% of the school population and 25% of my current classroom population are labeled Special Education. Therefore, they are typically working on personalized goals that are well-below the grade level standards on which they are tested.

Despite some challenges, Pleasantville Elementary School is fortunate to have a Reading Coach, three Reading Interventionists, two Reading Tutors, a Mathematics Coach, and one Math Interventionists. These additional supports assist teachers to reach students identified on benchmark assessments as performing below grade level.

On the most recent Smarter Balanced Assessments, 47% of Third Grade students at Pleasantville were proficient in English and Language Arts, while 35% were proficient in Mathematics. The state of Delaware has set a goal that by 2020 all scores will increase, across the board by 18%.

Learning Objectives

- Students will be able to add fluently and accurately within 100.
- Students will be able to subtract fluently and accurately within 100.
- Given a word problem, students will be able to determine the correct operation.
- Given a word problem, students will be able to use an efficient strategy to solve it.

Essential Questions

- How can I determine if the problem is addition or subtraction?
- How can I best represent this word problem?
- What strategies are most efficient for solving this problem?
- What mathematical tools are appropriate for this task?

Rationale

When teachers look at the Common Core State Standards (CCSS)³, they notice that the Operations and Algebraic Strand is dominated by the operations of addition and subtraction in grades KN-3. Despite the introduction, scaffolding, and progression of foundational skills, many students still enter Third Grade struggling with both operations. Using Dreambox computer mathematics program, August and September 2018 data showed that 0/20 of my students were proficient on current grade 3 standards. Dreambox is a computer program that initially determines a student's mathematical proficiency on all CCSS. From there, students are assigned instruction, tasks to complete, and games to review the standards with which they need review. Students may be working below grade level (as far as Kindergarten), on grade level, or above depending on their needs.

Why is there such a lack of fluency in Third Grade? I believe that many factors have contributed to the detriment of the students. One reason I believe students are struggling is due to the “rigor” introduced by CCSS. The expectations increase too quickly. For example, a Kindergarten student is asked to master Counting and Cardinality and addition to 10. First grade increases the expectation to addition and subtraction within 20. By Second Grade, students are asked to increase tenfold to 100, and also perform one- and two-step equations. Finally, in Third Grade, students must perform the operations of addition and subtraction accurately to 1,000, while computing one- and two-digit equations⁵. The growth expected between grades 1-2 and 2-3 is exponentially more than previous grade levels.

In addition to the conceptual leap present in the standards, our current curriculum falls flat in meeting the needs of both the teachers and students. Our curriculum is outdated (ten years old), so it was purchased before the adoption of CCSS. Students are not presented with ways to represent addition and subtraction beyond drawing sheets of 100, strips of 10, and singles or using a number line. More appropriate tools and strategies are needed for students whom need a visual. Also, teacher supplements are partially present (inserts of additional pages), but more are necessary. Supplements vary from classroom to classroom and school to school, so not all students are receiving the same rigorous instruction.

Aside from the formal evidence, I have also informally observed the need for more practice with addition and subtraction through classwork completion and sometimes, lack thereof. Families also continue to request more resources to practice the operations as they feel their students lack mastery of the basic facts. I agree. In the classroom, I notice that students struggle to accurately add and subtract even one-digit equations, and word problems become much more difficult for them.

It is my hope that this unit could be used to pre-teach addition and subtraction to Special Education students before teaching it whole group or be used as a re-teaching unit for students who did not show proficiency on addition and subtraction after unit completion. Through these lessons, I hope to provide students with more practice, the context of word problems, and give them skills that they can transfer from single-digit problems to the thousands as CCSS Grade 3 requires.

Research Behind the Unit

In an effort to help third grade struggling mathematics students, I began my research by focusing on instruction for K-2 mathematics, as they are not performing on grade level.

In *The Acquisition of Addition and Subtraction Concepts in Grades One Through Three*, Carpenter and Moser identified the three basic levels of addition strategies⁶. The

strategies are (1) direct modeling with fingers and physical objects, (2) based on counting sequences, and (3) recalled number facts. I immediately identified that most of my struggling learners were stuck in strategy 1. They rely on their fingers, struggle when they get past 10, and can have different answers to single-digit addition problems every day of the week.

Whereas there are levels of strategies to solve addition and subtraction equations, there are also a variety of types of problems that students can face. Join, separate, part-part-whole, and comparison are the four categories of addition and subtraction problems. Within the categories, there are eleven types of problems. At the lowest level, students can solve join addition and separate problems but not missing addend (Carpenter & Moser)⁷. Their research also found that in order for students to be proficient in the operations of addition and subtraction, they need to show mastery of all types of addition and subtraction problems that may not be present or available to students in their current textbooks⁸. The image below shows the types of problems that I want my students to master despite their current level of achievement.

	<u>Result Unknown</u>	<u>Change Unknown</u>	<u>Start unknown</u>
Join	Connie had 5 marbles. Jim gave her 8 more marbles. How many does Connie have altogether?	Connie has 5 marbles. How many more marbles does she need to have 13 marbles altogether?	Connie had some marbles. Jim gave her 5 more marbles. Now she has 13 marbles. How many marbles did Connie have to start with?
Separate	Connie had 13 marbles. She gave 5 marbles to Jim. How many marbles does she have left?	Connie had 13 marbles. She gave some to Jim. Now she has 5 marbles left. How many marbles did Connie give to Jim?	Connie had some marbles. She gave 5 to Jim. Now she has 8 marbles left. How many marbles did Connie have to start with?
Part-part-whole	Connie had 5 red marbles and 8 blue marbles. How many marbles does she have?	Connie has 13 marbles. Five are red and the rest are blue. How many blue marbles does Connie have?	
Compare	Connie has 13 marbles. Jim has 5 marbles. How many more marbles does Connie have than Jim?	Jim has 5 marbles. Connie has 8 more than Jim. How many marbles does Connie have?	Connie has 13 marbles. She has 5 more marbles than Jim. How many marbles does Jim have?

As I pinpointed the stage of my learners/audience, I further researched how I can get them to the next level. Carpenter, Moser, & Bebout's research shows that initial strategies are too cumbersome to be effective for larger numbers⁹. Therefore, I need to move my targeted group of learners from counting digits and draw all, as they are not manageable as numbers get bigger. Carpenter, Hiebert, & Moser (1983) also identified that kids fail to

connect the manipulatives used (in early learning) to the number sentences that they are now writing¹⁰.

A NCTM article by Whitacre and Wessenberg states that students are engaged in more complex problems when they draw pictures and find their own ways to solve problems¹¹. With draw all being too tedious, upon talking to Professor Jinfai Cai and other teachers, I found that introducing the tape diagram for addition and subtraction might serve my students the best, and it is similar to fraction bars used in later grades. With tape diagrams in mind, I recalled a quote from Alan Schoenfeld that defined mathematical thinkers.

“Learning to think mathematically means (a) developing a mathematical point of view- valuing the processes of mathematizing and abstraction and having the predilection to apply them, and (b) developing competence with the tools of the trade, and using those tools in the service of the goal of understanding structure-mathematical sense-making.”¹²

With my current students in mind, I determined that my focus should be first on interpretation and representation of addition and subtraction word problems within 20. Once students are able to read the word problems, identify the information present and what is missing, and generate representations and equations, I will move onto numbers within 50, within 100, and so forth. Until students get more practice with smaller, nonthreatening numbers, there is no need to intimidate them with larger numbers.

Unit Activities

Schoenfeld defines mathematically powerful students as those who are “quantitatively literate” and can use mathematics in everyday situations¹³. My goal for all of my students is for them to accurately and efficiently solve addition and subtraction problems not just for the year, but in later grades, in the workforce, and one day when making decisions for themselves and their families.

While approximately 70% of my students are able to demonstrate fluency by the middle of the year, these activities, especially the introduction of the tape diagram, would best serve as pre-teaching lessons for struggling learners before our addition and subtraction unit or as a small group lesson to supplement core instruction. The CCSS says that third graders should be proficient in addition and subtraction to 1,000, so this unit is for students not working at grade level¹⁴.

Lessons are planned for 30 minutes. They include a warm-up, whole-group discussion, and an opportunity for independent practice with an exit ticket.

Activity 1

In the warm-up, the group of students will be presented with two mathematical problems: $5 + 3 = 8$ and $8 = 3 + 5$. Students will share ways that the equations are similar and different. Starting with single digit equations will allow an entry point for all learners. Students may answer that similarities include both are addition problems, both have an equals and addition sign, and that both have the same digits 5, 3, and 8. Differences may include the order of the digits and the placement of the equals sign.

The second part of the warm-up will be a little bit more difficult and will not contain an answer. The first equation is a naked equation, and it will be compared to a second problem using the same numbers in a word problem. The naked problem will be $4 + 7 =$ and the word problem is “Hank found 4 seashells in the sand. He found 7 more in the water. How many seashells did Hank find in all?” Students will have discussion with their elbow partner about how they are similar and how they are different. They will share their findings with the group. Possible similarities include the digits 4 and 7, the answer 11 if solved, and the operation of addition. Differences include that one problem is just numbers while the other is in the context of the word problem. Also, the $4 + 7$ is not necessary seashells, which is the unit of the second problem. (Warm-ups 1 and 2 are included in Appendix A).

Next, I will introduce a box as the missing part of an equation by again posing $4 + 7 =$ and a box for the answer. I will use the think aloud strategy to determine that my answer is what is missing and must go in the box. I will write $\text{box} = 11$. I will try a second problem ($7 + 3 = \text{box}$). Again, I will reiterate that the box represents what’s missing and what we are solving for. We will try one more problem with the students providing more of the think aloud and the box moved ($9 + \text{box} = 12$).

Moving into a group activity, I will use large index cards marked 5, 6, and box. As I read aloud statements, the group will decide where to move the 5, 6, and box. The sign card (addition or subtraction) and equal sign will remain constant on the table. (Appendix B) This movement will be an introduction to determining whether in a word problem we know the part, part, or whole without yet using such labels.

Word Problems

Jen has 6 blocks. She picked up 5 more. How many does she have now?

Tom had 6 cupcakes. He has already eaten 5. How many are left?

Nico has \$6.00 left in his wallet. If he already spent \$5, how much money did he start with?

There were 6 eggs in the first nest and 5 eggs in the second nest. What is the total number of eggs?

Juan used to have 5 trading cards. Now he has 6. How many cards must he have received for his birthday?

Probing Questions

What do we know?

What information is missing?

What are we solving for?

What operation do we use- addition or subtraction?

Will all problems have the same answer?

In a group discussion, we will notice that each of the problems has the same digits, but the context is different. Due to the words, some of the problems are addition and some are subtraction. I will point out that this is why “number grabbing” is rarely an accurate strategy. We will also begin to explore that we knew different information for many of the problems. Sometimes we had both addends and sometimes the sum. By moving the 5, 6, and box cards, we were showing that the information we had for the problem was changing (informal introduction to part-part-whole).

This activity will wrap-up by having each student choose one of the word problems to solve. Before they leave the small group table, they will reread the problem chosen to me, identify the information that they had, and explain to me how they knew what they were solving for.

Activity 2

In Activity 2, students will explore the tape diagram in order to identify parts and whole. Students will use the tape diagram to help visualize what they are answering for in an addition or subtraction problem- the part or the whole. Utilizing visualizations like the tape diagram or a number line allows students to have a tool in their back pocket as opposed to relying the chance availability of a prepared manipulative. I decided to focus on the tape diagram as it is not present in my current curriculum, and it is a tool that students will use in a variety of modalities throughout elementary and middle school.

The CCSS Glossary defines a tape diagram as a drawing that looks like a segment of tape, used to illustrate a number relationship¹⁵. Also known as strip diagram, bar model, fraction strip, or length model.

To start, students will be presented with the equation $5 + 8 = 13$. I will model the creation of a tape diagram and use a think aloud to label part and part above the whole, to emphasize that 5 should be smaller and proportional to 8, and that 13 is the length of both 5 and 8 put together. I will relate the idea of a tape diagram to the length of unifex cubes- 5 cubes and 8 cubes of different colors equal a cube train measuring 13 cubes long. Next,

I will present the same equation in the context of a word problem and again model using the tape diagram. (Appendix D)

After my modeling, the students will work as a group to create and label tape diagrams after determining part and whole. Presented with the problem- Milo picked 5 apples from the first tree and 8 apples from the next tree. How many apples did he pick in all? Students will identify the numbers in the word problem, determine what we know (parts), and create a completed tape diagram on the white board. Pose think aloud wondering if the diagram could change with a different word problem. Pose the problem- Milo picked 5 apples from the first tree. After picking from the second tree, he ended up with 13 apples in his basket. How many apples must he have picked at the second tree? Again, work with students to identify the numbers, determine what we know (part and whole), and figure out how the tape diagram has changed with this particular problem. (Appendix D)

Students will continue this practice with a prepared tape diagram. Given the equation $6 + 12 = 18$, students will work with their elbow partner to fill the numbers in the diagram. After share out, students will work with the same partner to create a model for $\text{box} + 3 = 13$. The teacher will check for proportionality, correct placement of the parts and whole, and completion. A whole group discussion will follow with how students decided where to place the numbers. (Appendix E)

We will continue our work with problems that give both parts or only one including $2 + \text{box} = 10$, $4 + \text{box} = 12$, $11 + 5 = 16$, and $16 + 9$. While working with these problems, students will be able to reference a part, part, whole tape diagram poster behind the table. (Appendix E)

Activity 3

In today's warm-up, students will be given the words part and whole. Independently, they will define the two words in their own language and provide an illustration. After 5 minutes of work time, we will share out definitions and decide on a common one. Part should be defined similarly to "a piece of a whole." Whole is similar to "the entirety composed of parts." We will use student-generated language. Illustrations will be used to show the relationship between part and whole as well as to give students a visualization moving forward. (Appendix F)

By day 3, students should be ready for some guided practice with decreasing support from the teacher. In today's work, we will start as a whole group. I will read a word problem that says "Jody loves baking cupcakes. She made 8 chocolate cupcakes and some vanilla. If she made 24 cupcakes in all, how many must be vanilla?" The small group will work together to determine which representation applies to this problem given three tape diagrams. The correct choice is choice c. C says $8 + \text{box} = 24$. We are missing

a part. By counting up, we can determine that 16 cupcakes must have been vanilla. (Appendix F)

For independent practice, students will use blank tape models to determine the part, part, whole of word problems. They will fill-in their own lines to divide the parts so that they remain proportional. As students work, I will assist reading and provide guidance. (Appendix G)

Activity 4

In the final lesson of the sequence, a warm-up featuring error analysis will be used. Students will be asked to examine and score three addition and/or subtraction word problems. On an 8x10 sheet, I will read aloud a word problem, allow them think time and scratch paper, and they will indicate their score with a thumbs-up for correct, thumbs-down for incorrect, or thumb in the middle if they are not sure. We will then use small group discussion to determine why they each chose their score. If students score a problem as incorrect, we will work as a team to find the right answer. (Appendix H)

Error Analysis Problems

1. Yesterday I was painting in art. My table had black, blue, and silver. Olivia's table had 4 pastel colors. How many paint colors were available? Mock Response: $3 + 4 = \text{box}$. I solved for the box, and $3 + 4$ equals 7 available colors. Answer is correct.
2. Levi loves Legos. He found 15 in his drawer. Once he put them in his Lego bin, he had 29 Legos. How many were already in his bin? $15 + 29 = \text{box}$. I added the 15 Legos that he found to the 29 he had. Levi has 44 Legos. Answer is incorrect. The correct representation would be $15 + \text{box} = 29$. 29 is the whole, not a part.
3. My mom gave me \$15 to go to the mall. When I got home, I had \$3 left. How much money did I spend at the mall? $15 - \text{box} = 3$. Answer is correct.

Following the warm-up, students will solve two, word problems using their own representations. Even if they use a different strategy, I will encourage them to check their work with a tape diagram. The third and fourth problems on the page will ask students to create their own word problems. The third problem will provide students an equation, and they will write the word problem to match. In the final problem, students will write both their own equation and their own matching word problem. (Appendix I) I will monitor student work, ask clarifying questions, push for extended responses and for student to explain their thinking, and assist students in finding the right operation and/or strategy needed to solve.

Follow-Up Lessons

Following these three lessons, students have another mathematical tool that they are adept in using. If students need additional practice, they can watch the free PBS video showing tape diagrams at <https://www.pbs.org/video/good-know-tape-diagrams-2-digit-addition-and-subtraction-grade-2/>, continue solving addition and subtraction word problems, write their own word problems and swap with a teacher/another student to solve, or practice categorizing problems that they are given in their current curriculum¹⁶.

Conclusion

In order to be successful in mathematics, understanding and accuracy of addition and subtraction is essential. Not only is it used in every grade, K-12, but they are two skills that many adults, despite differences in occupation, lifestyle, race, or creed, rely on every day.

These lessons are designed to be flexible in implementation; however, the goal remains the same. By pre-teaching to students who may struggle, I hope to provide a more leveled playing field when the unit is started. By re-teaching to those students who do not show mastery on formative assessments, I hope to provide them with another lens to look at the operations that is different from our textbook. Through these lessons, I hope that I can begin to help students make sense of word problems and come up with a plan of attack whether the numbers be small or large.

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Student Resources

Copied Appendices

White Boards

Dry Erase Markers

Index Cards

Pencil

Blank Paper

Appendices

Appendix A

What are the similarities between these two problems?
What are some differences?

Set A

$$5 + 3 = 8$$

$$3 + 5 = 8$$

Set B

$$4 + 7$$

Hank found 4 seashells in the sand. He found 7 more in the water. How many did Hank find in all?

Set C

$$4 + 7 = \square$$

$$7 + 3 = \square$$

$$9 + \square = 12$$

Appendix B

As I read the word problems, move the 5, 6, and box cards to where they should fall in the problem. You can change the sign card to addition or subtraction depending on the situation.

Word Problems

1. Jen has 6 blocks. She picked up 5 more. How many does she have now?
2. Tom had 6 cupcakes. He has already eaten 5. How many are left?
3. Nico has \$6.00 left in his wallet. If he already spent \$5, how much money did he start with?
4. There were 6 eggs in the first nest and 5 eggs in the second nest. What is the total number of eggs?
5. Juan used to have 5 trading cards. Now he has 6. How many cards must he have received for his birthday?

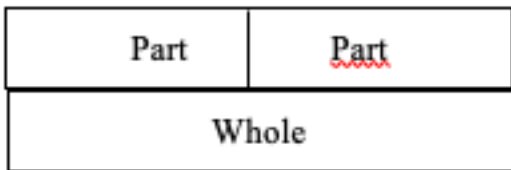
Appendix C

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.

Appendix D

Teacher Models

$$5 + 8 = 13$$



Student Problem

1. Milo picked 5 apples from the first tree. He then picked 8 apples from the next tree. How many apples did Milo pick in all?
2. Milo picked 5 apples from the first tree. After he picked from a second tree too, he had 13 total apples. How many apples must he have picked at the second tree?

Appendix E

Students will complete the following problems with a partner. Create a tape diagram to represent the word problem.

1. $6 + 12 = 18$

2. $\square + 3 = 13$

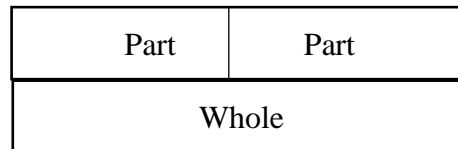
3. $2 + \square = 10$

4. $4 + \square = 12$

5. $11 + 5 = \square$

6. $16 + 9 = \square$

Tape Diagram



Appendix F

Name: _____ Date: _____

Directions: Write your own definition for each mathematical term. In the box below, draw a picture to help you remember each word. Be ready to share out.

Part: _____

Whole: _____

Directions: Read the problem below, and identify the correct equation.

Jody loves baking cupcakes. She made 8 chocolate cupcakes and some vanilla. If she made 24 cupcakes in all, how many must be vanilla?

a. $8 + 24 = \square$

b. $24 - \square = 8$

c. $8 + \square = 24$

Appendix G

Name: _____ Date: _____

Directions: Create a representation to solve for each word problem below. Blank tape diagrams have been provided.

1. Jose has 11 pencils in his desk. There are 3 more on top of his desk. How many pencils does Jose have in all?
2. I had 24 pencils in my pack to start the year. I have used 12. How many pencils are left in my pack?
3. Mrs. Wagner had 10 pencils in her cup to start the day. If there are 3 left at the end of the day, how many must students have borrowed?
4. My mom gave me a quarter to buy pencils at the school store. I found a nickel in my bookbag. How much money do I have now?
5. My pencil used to be 8 inches long, but now it is just 3 inches long. How much of my pencil have I used?

Appendix H

Error Analysis Warm-Up

1. Yesterday I was painting in art. My table had black, blue, and silver. Olivia's table had 4 pastel colors. How many paint colors were available? Mock Response: $3 + 4 =$. I solved for the box, and $3 + 4$ equals 7 available colors.

2. Levi loves Legos. He found 15 in his drawer. Once he put them in his Lego bin, he had 29 Legos. How many were already in his bin? $15 + 29 = \square$ I added the 15 Legos that he found to the 29 he had. Levi has 44 Legos.

3. My mom gave me \$15 to go to the mall. When I got home, I had \$3 left. How much money did I spend at the mall? $15 - \quad = 3. \square$

Appendix I

Name: _____ Date: _____

Directions: Solve the three word problems below. Decide on how to best represent the problem.

1. The penguins at the zoo love fish. Plucky the Penguin ate 6 fish for breakfast. His friend Flipper ate 13 more. How many fish did they eat in all?
2. Yesterday I baked cookies. I had 20 chocolate chips. I used 13 in my first batch. How many chips were left for the rest of my cookies?
3. There are 21 students in Mrs. Bishop's class. If 9 are boys, how many students are girls?

Directions: Given the equation below, write your own word problem.

4. Equation: $6 + \square = 13$

Word Problem: _____

Directions: Write your own equation and matching word problem. Solve to check your work.

5. Equation:

Word Problem: _____

Notes

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⁴Common Core State Standards Initiative. "Grade 3 Introduction." (2010)

⁵Common Core State Standards Initiative. "Grade 3 Introduction."

⁶Carpenter, T.M. & Moser, James. *The Acquisition of Addition and Subtraction Concepts in Grades One Through Three*. (1984)

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⁸Carpenter, T.M. & Moser, J.

⁹Carpenter, T.M. & Moser, J.

¹⁰Carpenter, T.M. Moser, J., & Bebout, H.C. *The Acquisition of Addition and Subtraction Concepts in Mathematics Education*. (1988). pp. 345-387.

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¹⁶Common Core State Standards Initiative. "Mathematical Glossary."

¹⁷PBS. "Tape Diagrams." www.pbs.org (2017)