Algebra Connections in Chemistry and Physics

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Introduction

This curriculum unit is centered on Conservation of Mass and Energy, two laws that govern Chemistry and Physics. My intent is for students to understand that they are applying basic algebraic concepts as they work with conservation laws. After all, if the "Before" is equal to the "After," then there must be a mathematical equation that can be written to satisfy this condition. Along the way, students will learn some basics about chemistry (atomic structure) and nuclear physics (subatomic particles), but don't be scared off! The miniscule size and very brief lifetimes of some particles are the context for learning mathematical concepts from measurement and scientific notation to exponents to solving systems of equations and exponential equations. I am writing this unit for my 9th and 10th grade Algebra students, but the content and activities could also be used for Chemistry and Physics classes.

I have been teaching all levels of high school mathematics for all 23 years of my career. I also taught Chemistry for about 8 years early on, and Physics for the last 5 years. I naturally see the connections between math and science, but my students do not. When they leave math class they don't think they have to do math any more. When I bring up a science concept, they either remind me they're in math class or wonder how I know science. I have wanted to write a curriculum unit about balancing chemical equations by writing and solving systems of equations, and I'm excited to finally figure out how to put it on paper.

In the first few meetings of our seminar, "What Makes the World Around Us?" I heard about subatomic particles that I had never studied when I was in college. Until I did some research, my knowledge of subatomic particles was limited to protons, neutrons and electrons. I had heard of antiparticles, quarks and bosons, but I didn't know what they were, and decided it was time to learn. I actually changed schools and teaching assignment (no more physics) after selecting this topic and doing much of the research. As a result, the emphasis of the unit has changed a bit. However, I am hoping that my math students find the names of the subatomic particles intriguing enough to learn a little bit about them. The background information focuses on the Laws of Conservation of Mass and Energy to show connections between algebraic equations, chemical equations and nuclear reaction equations.

My new high school is one of five high schools in the Red Clay Consolidated School District. There are approximately 850 students in 9th – 12th grade. It is the most diverse school in the district with approximately 30% African-American, 30% Hispanic/Latino, and 30% Caucasian. Nearly half of the students are categorized as low income (all students receive free breakfast and lunch), 15% as English Language Learners, and over 25% with Disabilities. It is a comprehensive high school with an emphasis on both academics and career preparation; there are 10 career and technical pathways offered at the school. The pathways are Animal Science, Audio, Radio, and Video Engineering, Automotive Technology, Culinary & Hospitality Management, Early Childhood Education, Manufacturing Engineering Technology, Marketing Communications, Natural Resources & Environmental Science, Plant & Horticulture Science, and Teacher Academy. Students must complete at least one pathway, but may experience and/or complete two. In addition to the Career Pathways, students can earn College credits from AP and Dual Enrollment classes at the local Community College. Additionally, students are expected to engage in community service projects; Advisory groups with a service focus meet during the school day each Wednesday.

Background Content

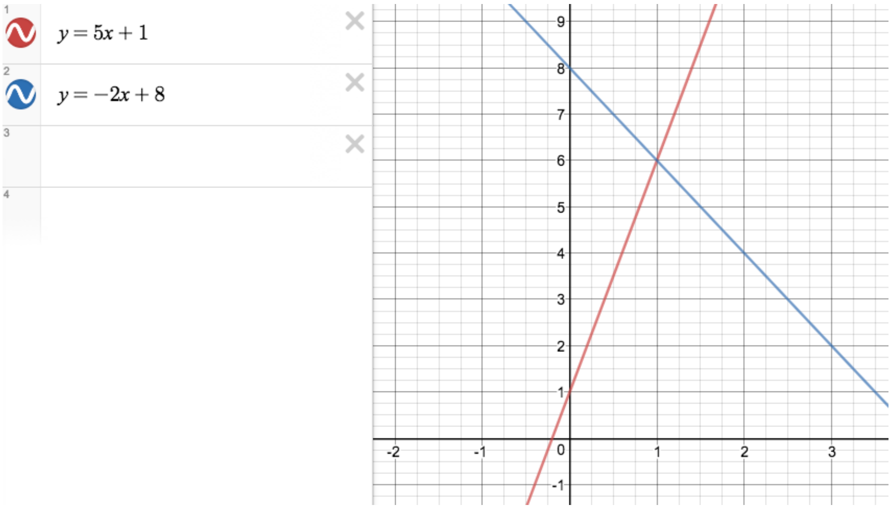
There are two laws of nature that are the backdrop to Chemistry and Physics. They are the Law of Conservation of Mass/Matter and the Law of Conservation of Energy. They allow us to quantify what is happening in chemical and nuclear reactions. The simple fact that something is conserved means that the (amount of mass and/or energy) “Before” must equal the “After.” And so, we have a mathematical equation!

Mathematics – Solving Systems of Equations

Students learn to solve systems of equations in multiple ways. Generally, lower level math classes restrict the systems to two linear equations with two variables. If there are more variables than there are (unique) equations, it is impossible to get a single solution (there would be multiple/infinitely many solutions). If there are more equations than variables, then either some of the equations represent the same line (e.g. $2x + 3y = 9$ and $6x + 9y = 27$, if graphed, contain all of the same points), or there will be no solution (all of the lines would not intersect at a single point). Algebra students learn to solve systems of equations by 1) *graphing* the two linear equations on a coordinate grid and finding the (x, y) coordinates of the intersection of the two lines; 2) *substitution* – replacing one variable in one of the equations with an equivalent expression, usually by rearranging the second equation to solve for one variable; and 3) a process called *elimination* in which one or both equations are strategically rewritten in equivalent forms (using rules of equality) so that when they are added one of the variables is eliminated. A fourth method of solving systems of equations uses *matrices*, and is extremely useful for solving more complex systems of equations (beyond 2 or 3 equations and variables). The process of solving systems using matrices is really very easy, but usually isn’t taught until later grades, if at all. In the CCSS, it is considered a (+) topic not required for all students.

Method 1: Solve the system of equations by graphing

$$\begin{cases} y = 5x + 1 \\ y = -2x + 8 \end{cases}$$



1 $y = 5x + 1$

2 $y = -2x + 8$

3

4

Figure 1 (Desmos graphing calculator screen shot)

The solution to the system is the point (1, 6) because that is the intersection of the two lines. The solution tells us that both equations are true when *x* = 1 and *y* = 6. Check: 5(1) + 1 = 6 and -2(1) + 8 = 6.

Method 2: Solve the system of equations using the Substitution Method

$$\begin{cases} 3x + 5y = 11 \\ x = 5 - y \end{cases}$$

The first step would be to substitute *5 – y* from the second equation for *x* in the first equation:

$$3(5 – y) + 5y = 11$$

Then, using the Distributive Property of Multiplication over Subtraction and combining like terms, the equation simplifies to

$$15 – 3y + 5y = 15 + 2y = 11$$

Subtracting 15 from both sides of the equation, then dividing by 2 on each side, we get

$15 - 15 + 2y = 11 - 15$
$2y = -4$
$y = -2$

Then, we use the value of *y* to find the value of *x* from the second equation: $x = 5 - (-2) = 7$. To check, substitute again into the first equation: $3(7) + 5(-2) = 21 - 10 = 11$.

Method 3: Solve the system of equations using the Elimination Method

$$\begin{cases} 3x + 4y = -5 \\ 2x \ + 5y = -1 \end{cases}$$

This procedure begins with linear equations in standard form: A*x* + B*y* = C, where A, B and C are integers. In order to eliminate one of the variables by adding the two equations, we will need to have coefficients that are opposites of each other. We can make that happen by multiplying each term in the first equation by the opposite of the coefficient of *x* in the second equation AND multiplying each term in the second equation by the coefficient of *x* in the first equation.

$$\begin{array}{ll} -2(3x + 4y = -5) \rightarrow & -6x - 8y = 10 \\ 3(2x + 5y = -1) \ \rightarrow & 6x + 15y = -3 \end{array}$$

Adding the two equations (like terms with like terms), we get $7y = 7$, so $y = 1$. We then substitute $y = 1$ into either of the original equations to find the value of *x*:

$$\begin{array}{lcl} 3x + 4(1) = -5 & \text{or} & 2x + 5(1) = -1 \\ 3x + 4 - 4 = -5 - 4 & & 2x + 5 - 5 = -1 - 5 \\ 3x = -9 & & 2x = -6 \\ x = -3 & & x = -3 \end{array}$$

Check: $3(-3) + 4(1) = -9 + 4 = -5$ and $2(-3) + 5(1) = -6 + 5 = -1$.

As students become more proficient with the Elimination Method, they will begin to see shortcuts. For example, sometimes it will be easier to eliminate the second variable (*y*) by multiplying terms in only one equation. If the coefficients of *x* are opposite signs, then we can multiply all terms in both equations by positive numbers.

Method 4: Solve the system of equations using Matrices

$$\begin{cases} x - 2y + 4x = -4 \\ 3x + y - 2x = 16 \\ \ x + 3y + 3z = 7 \end{cases}$$

Matrices are rectangular arrays of numbers that can be used to simplify repeated operations. The dimensions of a matrix are written as $m \times n$, where m is the number of rows and n is the number of columns in the matrix. Matrices with the same dimensions can be added or subtracted. Each element in a matrix can be multiplied by a scalar (number). Two matrices can be multiplied only if the number of columns in the first is equal to the number of rows in the second matrix. The product has the number of rows in the first matrix and the number of columns in the second matrix. Matrix multiplication is not commutative; the order of the matrices determines the product (or even if there is a product).

For the system of three equations with three variables, given above, we can write a matrix equation in the form [C]*[X] = [P]. Matrix C ([C]) would be a 3x3 matrix containing the coefficients of each variable, one equation per row. Matrix X ([X]) would be a 3x1 matrix containing the variables *x*, *y* and *z*. Matrix P would be another 3x1 matrix containing the values from the right hand side of each equation.

$$\begin{bmatrix} 1 & -2 & 4 \\ 3 & 1 & -2 \\ 1 & 3 & 3 \end{bmatrix} * \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4 \\ 16 \\ 7 \end{bmatrix}$$

We solve matrix equations as we would solve any algebraic equation in the form Cx = P. We multiply both sides of the equation by the multiplicative inverse of C: $\frac{1}{C} * Cx = \frac{1}{C} * P$, so $x = \frac{P}{C}$. Only square matrices (equal number of rows and columns) can have an Inverse. As I explained at the beginning of this section, systems of equations must have the same number of equations as variables in order to have a single solution, so the coefficient matrix will always be square. The notation $[C]^{-1}$ represents the inverse of Matrix C and $[C]^{-1} * [C] = [I]$. Without going into the process of matrix multiplication, The Identity Matrix ([I]) is the matrix equivalent of the number one. Any matrix multiplied by the Identity Matrix (having compatible dimensions) remains the same just like multiplying any number by one keeps the value of that number the same. Therefore, a matrix equation is solved by

$$[C]^{-1} * [C] * [X] = [I] * [X] = [C]^{-1} * [P]$$

Graphing calculators like the Ti-84 make solving systems of equations with matrices incredibly simple. There is a MATRIX button on the calculator. Next, select EDIT and a matrix name ([A], [B], etc.) to enter the dimensions and then the row entries. Arrow keys allow us to move the cursor to any position in the matrix, but hitting ENTER after an entry moves the cursor across the first row, then the second, etc. Repeat for the product matrix. We do not enter the variables into the calculator. When we multiply the product matrix by the inverse of the coefficient matrix, the solution will show on the screen as a 3x1 matrix representing *x*, *y* and *z*.

Back to our example: we will solve the matrix equation by entering the following on the calculator: $[C]^{-1} * [P]$. The result on the screen is $\begin{bmatrix} 4 \\ 2 \\ -1 \end{bmatrix}$. That means x = 4, y = 2 and z = –1. We can check the solution by substituting these values into all 3 equations. Check: (4) – 2(2) + 4(–1) = –4, 3(4) + (2) – 2(–1) = 16 and (4) + 3(2) + 3(–1) = 7!

Chemistry (for non-science people)

One of the first things that students learn when they study Chemistry (often in a Physical Science class) is about the make-up of atoms. They learn about Rutherford's gold foil experiment, which was conducted in 1911 and demonstrated that atoms are made up of a dense nucleus that contains positively charged protons surrounded by a lot of "empty" space containing negatively charged electrons. Using a cathode ray tube, Rutherford shot a beam of alpha particles through a thin piece of gold foil (4×10^{-5} cm thick). Most of the particles were deflected less than 1 degree as they passed through the foil. But, a very small percentage (1 in 20,000) of the alpha particles were reflected backwards at more than 90 degrees with respect to the incident beam. The conclusion was that the particles that were reflected backwards were hitting a dense nucleus that is relatively very small compared to the total size of an atom.[1]

The nucleus of an atom also contains neutrons. Neutrons are particles that are neutral in charge with virtually the same mass as protons. Together, they account for nearly the entire mass of an atom. Electrons are so small in comparison to protons and neutrons, that their mass is considered to be zero in basic Chemistry classes. Atoms are neutral; the number of negatively charged electrons is equal to the number of positively charged protons in the nucleus. The number of protons is specific to each different element that exists. If you look at a copy of the Periodic Table of Elements, it is arranged by atomic number. Hydrogen has an atomic number of 1 and it has one proton in its nucleus. Carbon has an atomic number of 6 and it has six protons in its nucleus. Uranium has an atomic number of 92 and it has 92 protons in its nucleus, etc. Electrons are constantly moving; their position outside of the nucleus has been described in multiple ways. One description is that there is an electron cloud surrounding the nucleus. A simplistic model often taught in basic Chemistry is the Bohr model in which electrons are positioned in circular rings, called shells, at different energy levels, and are filled in order. The first shell, closest to the nucleus, contains two electrons. Moving outward, the next two shells contain up to eight electrons, followed by two shells with up to 18 and two more containing up to 32 electrons each. These orbitals have suborbitals with different shapes designated as *s, p, d,* and *f* orbitals.

Students learn that it is the electrons that determine how elements react. Elements that have full shells are stable; they do not react with other elements. These elements are called Noble Gases (Helium, Neon, Radon, etc.) and are in the last column of the Periodic Table. Elements in the first two columns of the Periodic Table often referred to as Family or Group numbers 1 and 2, have one or two electrons in their outermost shell, respectively. This electron configuration makes the Group 1 and Group 2 elements highly reactive because they "willingly" give up those outermost electrons to become more stable with a completed outermost shell at a lower energy level. Likewise, elements in Family or Groups towards the right of the Periodic Table need only a few electrons to fill their outermost shells. They are most likely to gain electrons to become more stable. When elements lose or gain electrons, the atom is no longer electrically balanced. They become *ions* and carry a positive or negative charge. Elements that lose electrons have more protons than electrons, are positively charged, and are called *cations*. Elements that gain electrons have fewer protons than electrons, are negatively charged, and are called *anions*. Cations and anions join together with *ionic bonds* to form a chemical compound. The elements that lay towards the center of the Periodic Table, often bond together by sharing electrons to fill their outermost shells, and these bonds are called *covalent bonds*.

In a chemical reaction, bonds between atoms in the reactants are broken and new bonds are formed to make the products. The Law of Conservation of Energy tells us that the total amount of energy in all forms (chemical, thermal, kinetic, potential, light, etc.) remains constant throughout the reaction, from reactants to products. If heat (thermal energy) is added to initiate a reaction, that energy may be stored in the product(s) as chemical energy, or released as light or heat after the reaction, for example.

Applying the Law of Conservation of Mass/Matter in Chemical Reactions

The Law of Conservation of Mass tells us that matter can be neither created nor destroyed. I used to tell my Chemistry students that we are drinking the same water and breathing the same oxygen that the dinosaurs did. However, I learned from my seminar and from my research that I was misleading them. I believed that Hydrogen and Oxygen atoms were and always would be Hydrogen and Oxygen atoms. They could break apart and recombine in different ways, but the atoms stayed the same; the protons in the nucleus that defined the element never changed. I have since realized that it is only in chemical reactions that this is true. In nuclear reactions, the composition of the nucleus can and does change to become a different element. More about that in a later section, though.

One part of Chemistry is understanding the properties of chemical compounds and using them to predict the properties of others and/or trying to produce new compounds with given properties. Another part is understanding how atoms combine on a quantitative level. Chemists write chemical equations that describe chemical reactions, showing the reactants on the left side of the equation forming products on the right. Instead of an equal sign, however, reactants and products are related to one another using an arrow (→). Here's where the mathematics come in. Because of the Law of

Conservation of Mass, the number of each type of atom in the reactants must be the same as the number of each type of atom in the products. Anyone that has ever taken a Chemistry course remembers balancing chemical equations. We had to determine the coefficients of each chemical compound in the reaction so that all atoms in the reactants were accounted for in the products. We practiced and practiced and over time started to recognize some similarities that helped us balance equations faster.

Hydrogen peroxide breaks down into water and oxygen according to the equation: $H_2O_2 \rightarrow H_2O + O_2$. The subscripts on individual elements represent the number of atoms per molecule of the compound; no subscript means there is only one atom per molecule. We can make a table to keep track of all of the atoms in the reactants and products.

	Reactants	Products
H	2	2
O	2	1 + 2 = 3

The equation is not balanced since there are more oxygen atoms in the products than there were in the reactants. We can make a guess and double the number of H_2O molecules to get an even number of oxygen atoms. That gives us 4 atoms of hydrogen and 4 atoms of oxygen on the product side: $H_2O_2 \rightarrow 2H_2O + O_2$.

Revising the table, we find that the equation is still not balanced:

	Reactants	Products
H	2	4
O	2	2 + 2 = 4

To balance the equation we need to double the number of H_2O_2 molecules on the reactant side: $2H_2O_2 \rightarrow 2H_2O + O_2$.

	Reactants	Products
H	4	4
O	4	2 + 2 = 4

As I explained the process, I am considering the chemical reaction at the molecular level. For laboratory purposes, chemists would consider chemical reactions in terms of moles of reactants, products, atoms, etc. because a mole of an element or compound can physically be measured. Just like a pair are two, and a dozen is 12, a mole of an element, a mole of a compound, or a mole of doughnuts is the tremendously large number of 6.022×10^{23} of that item. No matter the scale, the subscripts and coefficients indicate the relative amounts of each element within a compound and the relative amounts of each reactant required to produce the products, respectively.

I am proposing that we can apply the mathematics used to solve systems of equations to balancing chemical equations. In a math class, when solving a system of two equations with two unknowns (variables x and y) students learn to solve for x and y values that make both equations true. To balance a chemical equation, we will be solving for the coefficients that make the number of each type of atom in the reactants equal to the number of the same type of each atom in the products.

Returning to the hydrogen peroxide example, I will demonstrate the process for balancing the chemical equation as a system of equations. Starting with $H_2O_2 \rightarrow H_2O + O_2$, we need to set one coefficient equal to one, and use it to solve for the others. In this example I will start with one molecule of H_2O_2 and add variable coefficients for all other molecules in the equation:

$$1H_2O_2 \rightarrow aH_2O + bO_2$$

The next step is to use the subscripts and coefficients to write an equation for each element in the reaction. For example, the subscript on hydrogen tells us there are 2 hydrogen atoms in one molecule of water, so we need to multiply the coefficient a by 2 on the right side of our equation. There is one oxygen atom per molecule in water plus two atoms of oxygen per molecule of the second product (oxygen gas).

$$\begin{aligned} &\text{H:}\ \ 2 = 2a \\ &\text{O:}\ \ 2 = a + 2b \end{aligned}$$

Examining this system of equations, we can easily solve the hydrogen equation to find that $a = 1$. We then substitute $a = 1$ into the oxygen equation to find the coefficient b:

$$\begin{aligned} &2 = 1 + 2b \\ &1 = 2b \\ &b = \frac{1}{2} \end{aligned}$$

This leads us to the balanced equation $H_2O_2 \rightarrow H_2O + \frac{1}{2}O_2$. Now, we don't really like to have fractions as coefficients, because half a molecule cannot exist, so we can double all of the coefficients, keeping the relative amounts of each molecule constant, to rewrite the balanced chemical equation as $2H_2O_2 \rightarrow 2H_2O + O_2$.

Although this system of equations was simple to solve using substitution, I still want to demonstrate how to set up and solve a matrix equation for this reaction. The first step is to write each (element) equation in standard form ($Ax + By = C$, where A, B and C are integers), adding zero as the coefficient for any variables not present in the equation. For example, in the peroxide equation, the variable b is the coefficient of O_2, which does not contain hydrogen.

H: $2 = 2a$ can be written as $2a = 2$

(1) H: $2a + 0b = 2$ (add b term with 0 as coefficient)

O: $2 = a + 2b$ can be written $a + 2b = 2$

(2) O: $1a + 2b = 2$ (coefficient of a is 1)

Using equations (1) and (2), the matrix equation becomes $\begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} * \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$. We solve for $\begin{bmatrix} a \\ b \end{bmatrix}$ by multiplying the inverse of the coefficient matrix with the "answer" matrix to get $\begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}^{-1} * \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$. Again, we got a fractional coefficient, so we would double all of the coefficients to make them integers, and the balanced chemical equation is $2H_2O_2 \rightarrow 2H_2O + O_2$.

Nuclear Chemistry/Physics

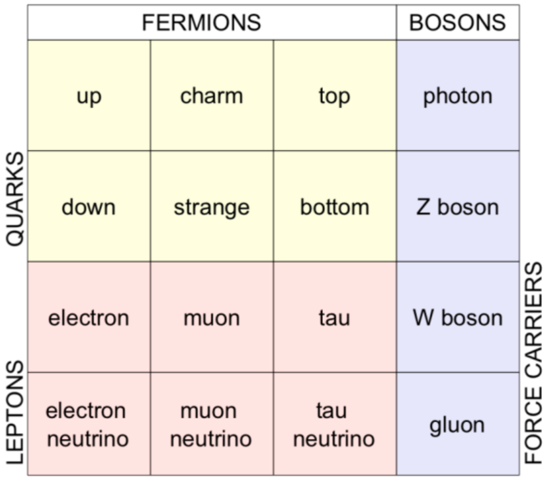
Chemistry and Physics tend to blend together when we study the nuclei of atoms. This was the focus of my research for this unit since so many discoveries were made since I was in school. The Laws of Conservation of Mass and Energy still apply in nuclear reactions, and the two are connected by Einstein's famous formula, $E = mc^2$. According to this formula, mass can be converted to energy (a lot of energy) in a nuclear reaction such that the total amount of mass and energy remains constant. The mass of atoms are measured in atomic mass units (amu) which is defined as 1/12 the mass of a carbon atom having 6 protons and 6 neutrons in its nucleus.[2] This atom is called Carbon-12 and can be written with the notation ${}^{12}_{6}C$. The subscript 6 refers to the six protons, and the superscript 12 is the atomic mass equal to the sum of the number of protons plus neutrons. One amu is equal to $1.66057 \text{x} 10^{-27}$ kilograms. That number is written in scientific notation, shorthand that reflects our Base 10 number system to avoid writing 26 zeroes between the decimal point and 166057! Applying Einstein's formula, where $c = 2.998 \text{x} 10^8$ m/s is the speed of light, we find that $E = (1.66057 \text{x} 10^{-27} \text{ kg})(2.998 \text{x} 10^8 \text{ m/s})^2 = 1.4925 \text{x} 10^{-10}$ Joules of energy per amu (remember that's from one proton or one neutron). In my readings, I learned that scientists usually convert Joules to electron volts (eV) when working at the atomic level. The conversion factor is 1 eV = $1.60219 \text{x} 10^{-19}$ Joules. Doing the conversion, we get:

$$E = (1.4925 \times 10^{-10}\, J) \times \frac{1eV}{1.60219 \times 10^{-19}\, J} = 9.315 \times 10^{8}\, eV$$

Therefore, the mass from one tiny little neutron can be converted into $9.315x10^8$ eV (or 931.5 MeV) of energy! The bottom line is that a nuclear reaction releases a lot more energy than a chemical reaction.

Quantum physics, also known as quantum mechanics, is the branch of physics that explains what is going on in nature at a very small scale – the scale of atoms and smaller subatomic particles. It is based on the idea that these small objects behave like both particles and waves. With the evolution of particle accelerators, physicists have been able to produce and identify smaller and smaller particles. Some well-known particle accelerators are SLAC at Stanford University, CERN near Geneva, Switzerland and Fermilab near Chicago. Physicists need particle accelerators because "most of the objects of interest to the elementary particle physicist today do not exist as free particles in Nature; they have to be created artificially in the laboratory. The famous $E = mc^2$ relationship governs the collision energy E required to produce a particle of mass m."[3] Particles are created in pairs during collisions. This must be true because nuclear reactions obey Laws of Conservation of total mass and energy and also of electric charge.[4] For example, when high-energy particles gamma ray photons penetrate an atom, they create two particles – an electron and a positron. Electrons and positrons are called antiparticles; they have the same (extremely small) mass, same internal symmetries, but opposite charges.[5] In a reverse collision, when antiparticles collide, the pair annihilates each other, converting mass into 2 photons of energy.

In his book *101 Quantum Questions*, Kenneth Ford states, "There is a confusing array of names [in quantum physics], some of them whimsical."[6] Professor Dave on YouTube[7] refers to these names as the Particle Zoo. Figure 2 below shows one version of the Standard Model of Particle Physics that organizes all of these particles.



	FERMIONS			BOSONS	
QUARKS	up	charm	top	photon	
	down	strange	bottom	Z boson	
LEPTONS	electron	muon	tau	W boson	FORCE CARRIERS
	electron neutrino	muon neutrino	tau neutrino	gluon	

Figure 2 – The Standard Model of Particle Physics

The current model shows that subatomic particles are divided into two categories – Fermions and Bosons. They are considered fundamental particles because they are not made of any smaller particles (that we know of). Bosons are force carriers (yes, they are considered particles in quantum physics) and have no mass. Photons carry the electromagnetic forces that hold protons and electrons together in atoms, among other things. The Z boson is neutral and along with the positive and negative W bosons carries weak nuclear forces that allow for radioactive decay. Gluons carry strong nuclear forces that hold quarks together. There is a theoretical graviton boson that carries gravitational force, but has not been found yet.[8] The Higgs boson is another theoretical force carrier that gives other particles mass.[9]

Fermions are the particles that make up matter. They are broken into two categories – unstable *quarks* and stable *leptons*. There are six leptons – electrons, muons and tau and their corresponding neutrinos.[10] Neutrinos are electrically neutral particles that are emitted in radioactive decay.[11] Quarks are the fundamental particles that experience strong nuclear forces and combine to form composite particles called hadrons. To complicate this model, all of these fundamental particles have *antiparticles* with the same mass but opposite electric charges, although some neutral particles are their own antiparticles.[12]

Mesons are a subset of hadrons made of one quark and one antiquark. Pions, etas, and kaons are types of mesons. Baryons are another subset of hadrons made of three quarks. Protons and neutrons (the nucleons) are examples of baryons. The protons made of 3 quarks and neutrons made of 3 quarks are the particles that make up atomic nuclei. They join with electrons from the group of leptons to make atoms that can bond together to form molecules and all matter.[13]

Radioactive Decay

We began our seminar with discussions about alchemists. Alchemists did some research and generated theories to explain things they observed. Their work was “unscientific… subjective and often mystical….”[14] The alchemists’ goal, at least in the Middle East, was to produce gold from other elements. To me, with my classic chemistry background, it sounded ridiculous since the number of protons in its nucleus defines an element, and the alchemists were not changing that. Then I learned more about nuclear reactions in which elements do transmute into other elements through radioactive decay, and my world has changed.

Most elements in nature are found in different forms called isotopes. Isotopes have the same number of protons (and electrons), but different numbers of neutrons. Isotopes have different atomic masses, but react identically in a chemical reaction. Sometimes one or more of an element’s isotopes are radioactive and will spontaneously decay. It is through radioactive decay that elements become different elements and particles. There are four types of radioactive decay – alpha particle emission (α), electron emission or beta decay (β^-), positron emission (β^+), and electron capture. The different types of decay depend on the size of the atomic nuclei and on the ratio of neutrons to protons in the nucleus.[15]

The strong nuclear forces that hold protons and neutrons together in the nucleus decrease as distance increases. When nuclei are too large, as in elements with atomic mass above 200 amu, the forces cannot always prevent positively charged protons from repelling each other. These nuclei can emit alpha particles which are stable $^{4}_{2}\mathrm{He}$ (helium) nuclei to reduce their size. Since the original element lost two protons and two neutrons it transmuted to a new element. An example of such *alpha decay* is $^{226}_{88}\mathrm{Ra} \rightarrow {}^{222}_{86}\mathrm{Rn} + {}^{4}_{2}\mathrm{He}$. Notice that the number of protons (subscript) and the atomic mass (superscript) remains constant from reactant to products in the equation, and since the products are neutral each product has the same number of electrons as protons.

Smaller atomic nuclei tend to be most stable when the number of protons and neutrons are equal. When the ratio of neutrons to protons in the nucleus is too high, a neutron can transform into a proton and emit an electron (β^-) and antineutrino in *beta decay*. Carbon-14 has six protons and eight neutrons and is a radioactive isotope of carbon. Because only living things continually absorb the radioactive isotope, scientists can perform mathematical calculations based on its half-life to determine the age of formerly living things. The beta decay equation $^{14}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N} + {}^{0}_{-1}e + antineutrino$ shows carbon transforming to nitrogen with a higher atomic number.[16] The atomic mass and charges remain constant by emitting an electron.

Positron emission (β^+) is a type of radioactive decay that occurs when a nucleus has too many protons relative to the number of neutrons. In this type of decay, a proton transforms into a neutron and emits a positron (an electron's antiparticle) and neutrino. An equation demonstrating positron emission is $^{18}_{9}\mathrm{F} \rightarrow {}^{18}_{8}\mathrm{O} + {}^{0}_{+1}e$ + *neutrino*. When fluorine transmutes to oxygen by the transformation of a proton to a neutron, a positron is emitted, along with excess energy in the neutrino, which balances the loss of a positively charged particle.[17]

The fourth type of radioactive decay is *electron capture*. Electron capture can also occur when there is an excess of protons in the nucleus. A proton combines with an electron from within the atom to form a neutron. The atomic number decreases by one; it is more stable with fewer protons to repel each other. There is a small amount of excess mass created by the transformation that is released as energy in a neutrino.[18] These nuclear equations provide another source of examples for writing and solving algebraic equations.

Teaching Strategies

This curriculum unit will require some direct instruction followed by guided and independent practice. The most challenging instructional piece will be the technology because students are typically at different places. It would be helpful to have students work in pairs and small groups as they grapple with using the technology.

KWL Chart

Some of the instruction will be done through videos rather than teacher lectures. Before beginning an activity, students can fill in a KWL chart with information they already know (K) and what they want to learn (W) in the first two columns, respectively. After instruction/video they return to the chart to add what they learned (L) to the third column; some students may record things in the (L) column as they listen.

K What I Know About Atoms	W What I Want to Learn	L What I Learned

Figure 3

Schoology

The Delaware Department of Education has adopted (purchased) a learning management system called *Schoology*. Teachers and students can access it from any computer, tablet or Smartphone; there is a downloadable app, also. My students use *Schoology* in multiple ways in different classes. All of them are familiar with how to login, access files, submit assignments, send messages to teachers, and write discussion posts. I upload class notes from my SMART Board files daily. I also add folders with extra practice problems or links to tutorials as needed. For this unit, I will have a folder that has links to useful videos and Answer Keys for assigned problems.

Classroom Activities

Activity #1: Chemical Equations as Systems of Equations

Objectives: Students will see a connection between math and chemistry. They will write and solve systems of equations in order to balance chemical equations. Students will apply the Law of Conservation of Mass/Matter from science and choose an appropriate solution method (Substitution, Elimination or Matrix Equation) to balance each chemical equation.

Warm-up: Hand out or project a copy of the Periodic Table of Elements. This activity assumes students have learned about the basic structure of an atom.

1. An element has 88 protons and an atomic mass of 236 amu. What is the element? How many neutrons are in the nucleus? [Answer: Radium; 236 – 88 = 148 neutrons]
2. An element has 86 protons and an atomic mass of 222 amu. What is the element? How many neutrons are in the nucleus? [Answer: Radon; 222 – 86 = 136 neutrons]
3. An element has 2 protons and an atomic mass of 4 amu. What is the element? How many neutrons are in the nucleus? [Answer: Helium; 4 – 2 = 2 neutrons]

Part 1: Chemical Equations

Review the Law of Conservation of Mass/Matter – in a chemical reaction, matter can be neither created nor destroyed. That means that all atoms in the reactants must be accounted for in the products.

Examples: Demonstrate as many examples as needed and then allow students to practice.

(a) Hydrogen peroxide breaks down into water and oxygen: $H_2O_2 \rightarrow H_2O + O_2$
(b) Fermentation (anaerobic respiration) reaction: $C_6H_{12}O_6 \rightarrow C_2H_5OH + CO_2$
(c) Burning propane fuel in a grill: $C_3H_8 + O_2 \rightarrow H_2O + CO_2$
(d) Formation of rust:[19] $Fe + H_2O + O_2 \rightarrow Fe(OH)_3$
(e) Ammonia production: $N_2 + H_2 \rightarrow NH_3$
(f) Gunpowder: $KNO_3 + S + C \rightarrow K_2CO_3 + K_2SO_4 + CO_2 + N_2$

Procedure for Balancing Chemical Equations:

1. Make a table to organize the number of each type of atom in the reaction.
   (a) $H_2O_2 \rightarrow H_2O + O_2$

Element	Reactants	Products
H	2	2
O	2	3

2. If the number of each type of atom in the reactants is not equal to the number in the products, then rewrite the equation adding coefficients. Designate the coefficient of the first reactant as one, and assign variables to all other reactants and products:
   (a) $1H_2O_2 \rightarrow aH_2O + bO_2$
3. Write an equation for each element, taking into account subscripts, such that the number of atoms in the reactants is equal to the number in the products:
   (a) H: $2 = 2a$
   O: $2 = 1a + 2b$
4. Solve the system of equations using any method.
   (a) H: $a = 1$; substitute in O: $2 = 1(1) + 2b \rightarrow 2 = 1 + 2b \rightarrow 1 = 2b \rightarrow b = \frac{1}{2}$

5. Replace variables in the chemical equation. If any coefficients are fractions, multiply ALL coefficients by the lowest common denominator to clear the fractions.
    (a) $H_2O_2 \rightarrow H_2O + \frac{1}{2}O_2$
    $2H_2O_2 \rightarrow 2H_2O + O_2$
6. Check that the number of each type of atom in the reactants is equal to the number in the products:
    (a)

Element	Reactants	Products
H	4	4
O	4	2 + 2 = 4

The Answer Key for Examples (b) through (f) is available in a separate document.

Activity #2: Big and Small Numbers

Objectives: Students will recognize very large and very small numbers generated by their calculators. They will recognize that the E## at the end of a number represents the exponent on base 10.

Warm-up: Which is greater 2^{60} or 3^{20}? Be prepared to support your answer.

On the calculator 2^{60} =1.15292...E18 and 3^{20} = 3,486,784,401. If past experience holds, several students will claim that 3^{20} is greater because the first digit is 3 or because the number on their screen is in the billions and 2^{60} is only 1.15292…they ignore the E18 at the end. This exemplifies the need to teach students about scientific notation and how it is denoted on the calculator.

Part 1: Review Base 10 number system

Demonstrate that the number 429.7 can be written as $400 + 20 + 9 + \frac{7}{10}$. Using place value, the number can be written as $4x100 + 2x10 + 9x1 + 7x\frac{1}{10}$and also as $4x10^2 + 2x10^1 + 9x10^0 + 7x10^{-1}$. Provide students with more examples to write numbers in expanded form based on place values and powers of 10. Towards the end, provide some very large numbers in the billions or higher so students can see the benefit of using a shorthand notation instead of writing all of the zeroes.

Part 2: Introduction to Particles

For an entertaining introduction to particle physics, show the *Big Bang Theory* video clip “Sheldon’s Rebus - Higgs Boson Particle” (link can be found in the **Bibliography for Teachers** section below). Next, have students enter what they know about atomic particles in the first column of a KWL chart and anything they want to learn in the second column (see Figure 3). Students should share what they have written with small groups first and then with the full class.

Show the Professor Dave Explains video “The Standard Model of Particle Physics” (link in **Bibliography for Teachers**). Students should fill in the third column of the KWL chart as they watch. Unless this lesson is for a Physics class, it is not necessary for students to record all of the information. Instruct students to record things they find interesting as they watch, even if it’s just some of the silly names of particles. Again, they should share what they recorded with small groups, and add anything else they find interesting.

Now that students are a little more familiar with the subatomic particles, the follow-up discussion will be about the size of those particles. Figure 4 below shows the (approximate) radius and/or mass of some subatomic particles. Teachers should choose a value from the table to demonstrate the meaning of numbers written in scientific notation.

Particle Name	Radius (m)	Mass (kg)	Mass (eV)
Electron	$2.818x10^{-15}$	$9.1094x10^{-31}$	$5.11x10^{5}$
Proton	8×10^{-16}	$1.673x10^{-27}$	938.3
Neutron	8×10^{-16}	$1.675x10^{-27}$	939.6
Electron Neutrino	$2x10^{-23}$	$3.6x10^{-42}$	$< 2x10^{-6}$
Muon	N/A	$1.884x10^{-28}$	$1.057x10^{8}$
Tau	N/A	$3.1673x10^{-27}$	$1.7768x10^{9}$
Up Quark	$1x10^{-18}$	$\sim3.6x10^{-42}$	$\sim2x10^{6}$
Down Quark	$1x10^{-18}$	$\sim8.9x10^{-30}$	$\sim5x10^{6}$
Strange Quark	$1x10^{-18}$	$\sim1.8x10^{-28}$	$\sim1x10^{8}$
Charm Quark	$1x10^{-18}$	$\sim2.3x10^{-27}$	$\sim1.3x10^{9}$
Bottom Quark	$1x10^{-18}$	$\sim7.7x10^{-27}$	$\sim4.3x10^{9}$
Top Quark	$1x10^{-18}$	$3.07x10^{-25}$	$1.72x10^{11}$

Figure 4[20, 21]

For example, the radius of an electron is $2.818\text{x}10^{-15}$ meters which is equivalent to 0.000 000 000 000 002 818; multiplying by 10^{-15} is the same as dividing by 10 fifteen times or moving the decimal point to the left 15 places. Have students enter the decimal number into a calculator and hit the Enter button. Most calculators will immediately convert to scientific notation. This demonstration will show students how very small numbers are denoted on their technology. On a Ti-84 graphing calculator, for example, the value is 2.818E-15. When particle mass is converted to energy according to Einstein's formula, $E = mc^2$, mass measured in electron volts is a very large number having a positive exponent in scientific notation. With more exposure to very large and very small numbers, students should be more prepared to look for and recognize numbers displayed in scientific notation on their calculators.

To reinforce how their technology denotes large numbers, refer back to the Warm-up question comparing 2^{60} to 3^{20}. Students can multiply 2 by itself repeatedly on a calculator (2x2, Enter, x2, Enter, Enter…) and watch for the change from a large integer to a number in scientific notation. It should make sense that a 10-digit number does not get smaller by multiplying by 2 one more time; in fact the number got too large to display on the screen and scientific notation is the shorthand notation used. If students need more instruction on scientific notation there are links to videos in the **Students Resources** section below, along with links for additional practice problems.

Activity #3 – Radioactive Decay, Half-Life and Exponential Function

Objectives: Students will understand the concept of half-life and how it applies to exponential functions. They will use technology to solve exponential equations that arise in radioactive decay applications.

Warm-up: Suppose you fold a piece of paper in half, and then in half again, and again and again (4 folds total). What fraction of the original size paper are you looking at if it remains folded? [*Answer:* $1 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}$]

Explain to students that the *Warm-up* is an example of exponential decay and can be written as the function $y = 1\left(\frac{1}{2}\right)^x$ where $x = 4$. Radioactive isotopes of elements decay exponentially and each one decays at a different rate. The recorded measurement of the rate of exponential decay is called the *half-life* of the isotope – the time it takes for one-half of the original amount to decay. The type of decay (alpha particle emission (α), electron emission, which is also known as beta decay (β^-), positron emission (β^+) or electron capture) is dependent on the element. The following examples show applications of radioactive decay and demonstrate how to use technology to solve related problems. (For writing and solving exponential equations (including different applications) using logarithms for more advanced math students, refer to the curriculum unit that I wrote for the 2013 Thinking and Reasoning seminar, titled *Why We Need Logarithms: From Early Navigation to Nuclear Meltdown.* It can be found under the Curriculum Units tab at dti.udel.edu.)

Example #1: Carbon-Dating

Carbon-14 is a radioactive isotope of carbon. As a carbon atom, it contains six protons and six electrons. It has an atomic mass of 14, so the nucleus contains eight neutrons in addition to the six protons. All living organisms constantly take-in carbon-14, either via photosynthesis or indirectly from food. All living organisms contain the same ratio of carbon-14 to carbon-12 (the most common, and stable, carbon isotope) and that ratio is the same as the ratio in the atmosphere. That ratio changes once the organism dies because it no longer takes up new carbon-14 and the existing carbon-14 decays with a half-life of 5730 years.[22] The radioactive decay of carbon-14 is an example of beta decay in which a neutron in the carbon atom's nucleus transforms into a proton and the atom emits an electron to maintain its neutral charge: ${}^{14}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N} + {}^{0}_{-1}e$.

Instruct students to choose any number as the starting amount of carbon-14. Ask them how much will remain after 5730 years. Then, how much will remain after another 5730 years (total of 11,460 years). Each student now has three data points that they can enter into a table on a graphing calculator (e.g. Ti-83/84 or Desmos online shown in Figure 5 below). The example below starts with 100 grams of Carbon-14. This is a great time to help students find an appropriate viewing window for data in the table. From the table, they can see that the x values need a large range, such as –100 ≤ x ≤ 15000, but the y values have a smaller range, such as 0 ≤ y ≤ 150.

L_1	L_2
0	100
5730	50
11460	25

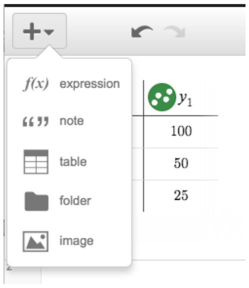


Figure 5

The next step is to find the exponential equation that best fits the data using Regression capabilities of the calculator. On the Ti-83/84, students push the STAT button, then the right arrow to CALC, and scroll down (or up) to 0: ExpReg for Exponential Regression and ENTER. If their data is in Lists 1 and 2, then they can push ENTER again. If their data is in any other list(s), they must enter the list name (use 2nd #

– list names are above the numbers on the keypad) after ExpReg before pushing ENTER. The calculator screen shows the values of a and b to be inserted into the general form of an exponential function, $y = a*b\text{^}x$. The procedure for finding a Regression equation using Desmos is to type the general form of an exponential equation as $y_1\sim a(b)^{x1}$. The calculator automatically makes numbers following the variables into subscripts. The symbol "~" is essentially the command for finding a Regression.

Once students have an exponential equation, they should compare with others to see that everyone got the equation $y = a(0.99879)^x$ where a was their starting amount of carbon-14; the base will be the same in all cases. (For more advanced math students, the teacher can show how to find the base without regressions by solving for the decay rate using the known half-life.) Now, students can answer questions such as "How much carbon-14 will remain after 1000 years if the original sample contained 250 ng?" simply by entering the value $x = 1000$ into the equation $y = 250(0.99879)^x$. To answer questions like "How long will it take a sample containing 5000 mg of carbon-14 to decay to a level of 1000 mg?" they need a way to find the exponent, x, that represents time. First, they should enter their regression equation as a y = equation. Then they can scroll through tables generated in the calculator or trace along the graph. To find a more exact value between data points (without adjusting the scale/steps in the table), help students understand that when solving the equation $1000 = 5000(0.99879)^x$, they are looking for a specific y value. If they enter the second equation $y = 1000$ into the calculator they will see a horizontal line at 1000. At the point of intersection (13,350, 1000) the x-value represents the number of years

As a final carbon-dating example, suppose a group of archaeologists uncovered a set of bones. They measure the ratio of carbon-14 to carbon-12 in their sample and find the ratio is $9.2\text{x}10^{-14}$. If a contemporary bone sample has a ratio of $1.2\text{x}10^{-12}$, determine the age of the bones that were uncovered. To solve, write the exponential equation as $y = (1.2\text{x}10^{-12})(0.99879)^x$ and a second equation for the value of y that we are trying to reach: $y = 9.2\text{x}10^{-14}$. It may take some Window adjustments to see the graph, but the intersection point is $(2121.3, 9.2\text{x}10^{-14})$. That tells us that the set of bones is approximately 2100 years old.

Example #2 – Giant Penguins and dating fossils more than 50,000 years old

I heard an article on NPR while I was putting together these activities. A team of scientists announced that they discovered an unknown species of a prehistoric giant penguin off the coast of New Zealand. The penguins were close to six feet tall and weighed about 220 pounds, about the size of an average-size man. The penguins lived a few million years after the extinction of dinosaurs and about 20 million years before the existance of whales.[23] The fossils of these giant penguins are between 55–60 million years old. Carbon dating does not work for fossils more than about 50,000–75,000 years old because its half-life is too short; the level of carbon-14 would be too low to be detected. There are other elements with radioactive isotopes that have longer half-lives that can be used to date older fossils. They can also be used for nonliving samples, such as rocks (including rock samples from the moon).[24] For example, Uranium-238 decays via multiple stages of both alpha and beta decay to Lead-206 with a half-life of 4.468 billion years. Uranium-235 decays to Lead-207 with a half-life of 704 million years and Rubidium-87 decays to Strontium-87 in $4.88\text{x}10^{10}$ years. In these cases, scientists measure the amount of the decay product (lead or strontium) relative to the original element (uranium or rubidium) because all of the product came from the original.

A sample problem about the giant penguins could be "How much of the original Uranium-238 remains in the giant penguin fossil if it is 60 million years old?" To solve the problem, we must first find the equation that models the decay of Uranium-238. Using technology and the two data points (0, 1) and ($4.468\text{x}10^9$, 0.5) – x represents *time* in years and y represents the *amount of uranium* remaining – an Exponential Regression gives us the equation $y = (0.999999999528)^x$. Since I used a starting value of one unit, we only need to substitute the age of the penguin fossil for x to determine the fraction of Uranium-238 that remains: $y = (0.999999999528)^{60{,}000{,}000} = 0.97207$, or about 97.2% of the original amount. This result could lead to a discussion about one of the concerns about nuclear waste – the half-life is so long that we need safe ways to store it for millions of years.

Additional Examples

Teachers can find additional examples of exponential decay based on students' interests. Some other radioactive decay applications include 1) Radon gas, a radioactive decay product of Uranium-238, often seeps into homeowner's basements from the soil, 2) patients are injected with radioactive isotopes for scans of the body, and 3) tracking movement of contaminants in the environment.[25]

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Student Resources

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This video defines the Particle Zoo of subatomic particles and organizes them into understandable categories.

Appendix: Common Core State Standards for Mathematics

CCSS.MATH.CONTENT.HSA.REI.C.5-6, 9: Solve systems of equations in two variables….

Activity #1 uses the context of balancing chemical equations to write and solve systems of equations in two or more variables. The activity presents multiple solution methods including graphing, substitution and matrix equations.

CCSS.MATH.CONTENT.8.EE.A.4: Perform operations with numbers expressed in scientific notation…. Interpret scientific notation that has been generated by technology.

Although it's an 8th grade standard, many of my high school students did not learn it. Activity #2 is devoted to ensuring that students recognize very large and small numbers generated by their calculators. They should recognize that the E## at the end of a number represents the exponent on base 10.

CCSS.MATH.CONTENT.HSF.IF.B.4: Interpret functions that arise in applications in terms of the context.

CCSS.MATH.CONTENT.HSF.BF.A.1: Build a function that models a relationship between two quantities.

Activity #3 presents real-life application problems for exponential decay. Examples include dating fossils using radioactive isotopes of carbon and uranium.

CCSS.MATH.PRACTICE.MP1: Make sense of problems and persevere in solving them.

Students are presented with application problems with varying entry points. They can write and solve algebraic equations using multiple solution methods, including technology.

CCSS.MATH.PRACTICE.MP4: Model with mathematics.

Students are modeling with mathematics as they set up systems of equations to balance chemical equations, and as they write exponential equations to describe radioactive decay.

CCSS.MATH.PRACTICE.MP5: Use appropriate tools strategically.

Students will gain experience in how to use the Ti-83/84 graphing calculator or Desmos online graphing calculator to solve matrix equations, solve systems of equations by graphing, and to find Regression equations to fit data.

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