Show Me How You Know - ***A Unit in Kindergarten Problem Solving***

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Introduction

I am currently a Kindergarten inclusion teacher and I serve approximately 20 to 22 general education and special education students per year. As of the 2014/2015 school year, my students were exposed to mathematical reasoning and problem solving opportunities embedded within the class mathematics curriculum. The curriculum, *Math Expressions Common Core*, by Houghton Mifflin Harcourt, was adopted by the school district, and is being formally implemented throughout district elementary schools for the 2015/2016 school year. The *Math Expressions* curriculum was chosen by my district in order to have a comprehensive math curriculum that encompasses the rigor of the mathematical practices and standards defined in the Common Core.

I was fortunate to be selected to trial the program last year, and my goal is to strengthen students' mathematical reasoning and problem solving skills this year. Using strategies of cognitively guided instruction, I plan to develop a stronger "Math Talk" community where students can share and discuss different strategies that may be used solve mathematical story problems. By exchanging ideas, and engaging in rich discussions, students will develop stronger math reasoning skills that will ultimately lead to deeper conceptual understandings. Such practices will ensure that my students are adequately prepared to handle the rigor of the Common Core Standards as well as the demands of the 21^{st} century.

Background/Unit Goal

Prior to the 2014/2015 school year, students in my classroom were taught the practice of mathematical reasoning as part of a more fragmented math curriculum that pieced together components of two separate curriculums, along with various teacher created supplemental materials. Each year, students enter Kindergarten with a range of pre-requisite skills. Some students may not know the difference between a letter and a number, while other students can rote count to 10, or 20, or higher. Typically only a few students are actually counting quantities using one to one correspondence with the last number counted representing "how many". Counting using one-to-one correspondence is the next learning progression and is practiced by pointing to each object as the number word is said, or by moving each object from one place to another as each number name is said. Counting one-to-one is learned from hands-on opportunities allowing students to connect mathematics to real world examples.

With respect to solving story problems, some students can listen to a simple story problem with up to five objects, people, or animals, and can count to join and tell how many objects. The Common Core State Standards requires that students conceptualize, represent and explain problems in a variety of ways. *Math Expressions Common core, written by Karen Fuson*, is our district's new math curriculum adopted for school year 2015/2016.[1] *Math Expressions* utilizes a research-based algebraic problem solving methodology that focuses on grade level problem types. In Kindergarten, the problem types include: result unknown (add to and take from), total unknown (put together/take apart), and both addends unknown (put together/take apart).

My research will focus on how I can use the district curriculum, along with Cognitively-Guided Instruction (CGI) strategies, to strengthen the reasoning skills of my kindergarten students. With stronger reasoning skills, my students will be able to solve the aforementioned problem types by creating math drawings, and using "Math talk" to discuss the drawings and explain their solutions. Given the high population of ELL students in my class, I will also focus on specific language needs and accommodations that will be needed in order to help them make connections and elicit mathematical thinking.

School Demographics

Forest Oak Elementary school is part of the Red Clay Consolidated School District. It is located in the suburb of Newark, Delaware. In 2014/2015, there were 529 students enrolled, 89 of which were in Kindergarten. Kindergarten inclusion classes are made up of approximately 18 to 22 students. Students with IEPs are serviced in general education classrooms led by dual certified inclusion teachers. According to student data, in 2014/2015, enrollment by race/ethnicity was as follows: 10% African American, 2.3% Asian, 24% Hispanic/Latino, 59.9% White, and 3.8% Multi-Racial. Other student characteristics include: 9.8% ELL, 37.2% low income, and 5.9% special education. This year I have 21 students enrolled in my class, one of which has an Individualized Education Plan (IEP). In addition, 57% of my students are ELLs, and I am experiencing significant communication challenges due to language barriers with students and families.

Mathematical Reasoning -Past Practices

In the past, students had limited opportunities to reason within the parameters of district- selected units from the Math Trailblazers curriculum, as well as specific online units identified within the Engage NY curriculum. Problem solving/reasoning was not embedded within the curriculum, but was taught for one session each week. Many teachers would also integrate supplemental materials found online, or purchased from the Teachers Pay Teachers website in order to give students more problem solving/reasoning opportunities. My Kindergarten team used a supplemental problem-solving journal so that students could practice solving a "problem of the day". Most of the problems in the journal were basic part/part/whole, result unknown, *joining all* addition problems. There were also some basic subtraction problems that would allow students to *separate from* to determine an unknown result. Examples of these problem types include:

Joining All (Result Unknown): Leticia has 5 red balloons. Joshua has 3 blue balloons. How many balloons do Leticia and Joshua have altogether?

Separating From (Result Unknown): Estephany has 8 cookies. She gives 3 cookies to Taylor. How many cookies does Estephany have left?

While the journals gave our students practice with the strategies for *joining all* and *separating from*, they learned how to solve these types of problems based on the familiarity of the problem types. The educational concern was that the supplemental framework did not allow enough class time to explore more complex problems. As a result, students did have sufficient opportunities to engage in rich collaborative discussions that foster deeper conceptual understandings that are necessary for solving more complex problems. Our first grade teachers also commonly reported that incoming first graders lacked in their problem solving and reasoning abilities.

Mathematical Reasoning – Adoption of New Common Core Curriculum

Last year, I trialed the *Math Expressions* curriculum. I found the lessons easy to follow, and I noticed that many of the standards were incorporated into all of the lessons. As a result, students were able to dig deeper into the material, because activities were comprehensive, and involved investigation that covered the various domains. I did find that it was challenging to develop the "Math Talk" community that is necessary to place students on an "Inquiry Learning Path" as Karen Fuson, author of Math Expressions, states. Fuson refers to Math Talk as a discussion tool that makes “students’ thinking visible”[2]. A “Math Talk” is an active discussion about the different strategies that were used to solve a problem with an emphasis on why a particular strategy was used[3]. She states, “Teachers need to use carefully designed visual, linguistic, and situational conceptual supports to help students connect their experiences to formal mathematical words, notations, and methods[4]. In Volume 1 of the Kindergarten Teacher Edition of *Math Expressions Common Core*, in her introduction to teachers, Fuson identifies four features essential in Math Talk Communities, and she correlates them with CCSS Mathematical Practices[5]:

Math Talk Feature	CCSS Mathematical Practice Correlation
1. Math Sense	Make sense of problems and persevere in solving them (MP.1). Attend to precision (MP.6)
2. Math Structure	Look for and make use of structure (MP.7). Look for and express regularity in repeated reasoning (MP.8)
3. Math Drawings	Model with mathematics (MP.4) Use appropriate tools strategically (MP.5)
4. Math Explaining	Reason abstractly and quantitatively (MP.2)

Construct viable arguments and critique the reasoning of others.

Mathematical Reasoning within the Common Core Standards/Kindergarten

Mathematical reasoning is required to create the math drawings explicitly mentioned in the CCSS in the following Kindergarten domains: Operations and Algebraic Thinking, Number and Base Ten, Measurement and Geometry. Students must also reason mathematically as they engage in the eight mathematical practices identified in the Common Core. In 2000, the NCTM stated, "The ability to reason systematically and carefully develops when students are encouraged to make conjectures, are given time to search for evidence to prove or disprove them, and are expected to explain and justify their ideas."[6] Students must be able to reason mathematically in order to develop the deep conceptual understanding required by the Common Core.

According to the Common Core State Standard for Operations and Algebraic Thinking in Kindergarten, students must understand addition as putting together and adding to, and understand subtraction as taking apart and taking from. More specifically, CC.K.OA.1 requires Kindergarten students to represent addition and subtraction with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions or equations. CC.K.OA.2 requires Kindergarten students to solve addition and subtraction word problems, and add and subtract within 10.

Reasoning is also mentioned explicitly in CCSS Mathematical Practices 2,3, and 8, and is implicitly encompassed within mathematical practice 1, and 4-7 as follows:
CC.K-12.MP.1: Make sense of problems and persevere in solving them.
CC.K-12.MP.2: Reason abstractly and quantitatively.
CC.K-12.MP.3: Construct viable arguments and critique the reasoning of others.
CC.K-12.MP.4: Model with mathematics.
CC.K-12.MP.5: Use appropriate tools strategically.
CC.K-12.MP.6: Attend to precision.
CC.K-12.MP.7: Look for and make use of structure.
CC.K-12.MP.8: Look for and express regularity in repeated reasoning.

Unit Goals/Essential Questions:

My research will focus on how I can use the district curriculum, along with Cognitively-Guided Instruction (CGI) strategies, to strengthen the reasoning skills of my kindergarten students. With stronger reasoning skills, my students will be able to solve the aforementioned problem types by using a variety of direct modeling, counting, and number fact strategies. Students will create math drawings, and models, and they will also engage in "Math talk" to discuss their drawings and explain their reasoning and solutions. Since I have such a large population of ELL students, my unit will incorporate language development strategies to extend student understandings. My unit will also provide opportunities for flexible groupings, student collaboration, and student leadership.

The following essential questions will be addressed within my unit:

1. In what ways can students discuss number relationships, and tell/model story problems through counting, visualizing, and drawing 1-10 objects.
2. How can students represent and solve addition and subtraction stories from family experiences?
3. 11. How can students use drawings and write expressions to solve addition and subtraction story problems?

Research

In 1993, Carpenter, Ansell, Franke, Fennema, and Weisbeck conducted a study of 70 Kindergarten children from six different classrooms, and two schools, with diverse populations[7]. Observers came in on at least four separate occasions to spend time in each of the classrooms. In all of the classes, students had engaged in problem solving throughout the year. Each of the teachers distributed counters to students in order to model and solve story problems throughout the year. Most of the teachers assigned single step story problems, and used numbers to 20 within the problems. In May, the 70 students were given a post-test of 9 story problems to solve. The researchers found that 46% of the students utilized a valid mathematical strategy to solve all of the problems[8]. Sixty-three percent of the students used a valid strategy and came up with the correct solution for at least seven of the nine problems[9]. Out of the 70 students, only 5 students received a 0 for not solving any problems correctly[10]. More specific data regarding the range of problems the students used a valid strategy for solving is summarized below[11]:

- Separate (Result Unknown): 62 children used a valid strategy
- Join (Change Unknown): 56 children used a valid strategy
- Compare: 50 children used a valid strategy
- Multiplication: 60 children used a valid strategy
- Measurement Division: 51 children used a valid strategy
- Partitive Division: 49 children used a valid strategy
- Division with Remainder: 45 children used a valid strategy
- Multistep: 47 children used a valid strategy
- Nonroutine: 41 children used a valid strategy

In 1999, Carpenter, Fennema, Franke, Levi and Empson later wrote *Children's Mathematics: Cognitively Guided Instruction*. A second edition of this book was released in 2015[12]. In their book, the researchers define and discuss Cognively Guided Instruction (CGI). Cognitively guided instruction is a research based professional development program that was developed by Thomas Carpenter, Elizabeth Fennema, Penelope Peterson, Megan Loef Franke, and Linda Levi. [13]Through their studies and work over the years, the researchers learned that if teachers can understand students' mathematical thinking, then they can make changes in their own practices and lesson delivery that will support students to unpack their math problems, and elicit student thinking about how they solved the problems. This collaborative process of unpacking the problem, and discussing and reflecting about various solutions will deepen students' conceptual understanding, hence improving student achievement.

In a Cognitively Guided Instruction classroom, teachers understand the development of students' mathematical thinking, and they are able to enrich the learning process by providing many constructivist opportunities for students to question and challenge each other's thinking. Within the various problem types, most students solve problems using three strategies[14]:

1. Direct modeling using manipulatives or drawings
2. Counting
3. Number Facts

Direct Modeling Strategies

The age that students use each of the problem solving strategies above varies significantly. However, the strategies are typically learned in progression therefore young students begin with direct modeling, and progress to counting and using number facts. At times, the strategies are used simultaneously for a period of time, as students are developing their mathematical thinking. For example, a student may problem solve using direct modeling and counting strategies until he or she feels comfortable with using the counting strategies exclusively. According to Carpenter et. al, Kindergarten students can use direct modeling to solve some story problems even if they have not had any formal instruction[15]. However initially, some younger children may only be able to handle one-step problems, such as joining all or separating from. Joining to strategies require more steps and planning ahead, which some students may not be ready to execute as yet. Consider the following example:

Carlos has earned 4 tokens. He needs to earn 10 tokens to get a prize. How many more tokens does Carlos need to earn to get a prize?

Carlos sets up his model by making a set of 4 cubes/tokens. He then adds 6 more cubes/tokens and counts "5, 6, 7, 8, 9, 10". Carlos doesn't know to keep the initial group and the additional group separate, so he is not able to indicate the number of additional tokens he needed to get the prize. He is puzzled, and answers the question by counting all of the cubes and stating "10" as the answer. Carlos needed to know ahead of time how he would obtain the answer from the model he constructed. Both the question and the action had to be considered prior to modeling the problem.

According to Carpenter et al., students can students require many experiences to practice solving simple problems in order to learn how to think about the question, and the action required to solve it correctly[16].

Comparing problems are more difficult to model than joining to problems. If the language in the problem provides matching cues, or the information is presented in a bar graph, then direct modelers may be able to solve the problem without much difficulty. However, joining and separating problems where the start is unknown, are very difficult for direct modelers to solve because they cannot represent the starting set. Trial and error is the only way for direct modelers to solve this type of problem.

Counting Strategies

Over time, as students gain rich experiences with direct modeling, they begin to move on to more efficient counting strategies[17]. Students who are able to use such counting strategies have more developed number sense, and can think of numbers in a more abstract way. As previously indicated, both direct modeling and counting strategies may be utilized simultaneously as students' mathematical thinking is developing. Below is an example of a counting on strategy:

Lilliana has 15 cookies. Olivia gives her 4 more cookies. How many cookies does Lilliana have now?

Since Lilliana is comfortable with counting on, she can solve the problem by saying 15, and then holding up 4 fingers one at a time as she counts "16, 17, 18, 19". Lilliana is then able to state that she has 19 cookies now.

As students gain more experience with counting strategies, they gain a deeper understanding about part-whole relationships, as well as an understanding that actions are reversible. Students who develop these deeper understandings are able to apply flexible strategies to solve problems with different structures.

Number Fact Strategies

According to Carpenter et al., teachers should not assume that students have number fact fluency because they are able to solve problems rapidly[18]. On the contrary, students who are able to apply counting strategies become very efficient, and they are able to count very quickly to arrive at solutions to problems. The efficient application of counting strategies does not necessarily lead to number fact fluency. Number fact fluency is developed from a student's ability to identify relationships between numbers and operations and memorization. Students require significant problem solving experiences and reflection/discussion opportunities to start developing number fact fluency. Students that gain number fact fluency from their experiences with counting strategies, and their rich discussions with peers, have a better understanding of addition and subtraction than their peers who simply memorize number facts.

The CGI Classroom

How do you implement CGI practices in the classroom? Carpenter et al. report that in CGI classrooms, teachers pose appropriate problems and elicit student thinking[19]. But first, teachers must carefully consider the level of their students mathematical thinking in order to identify appropriate problem types. Once the problem types are selected, teachers can anticipate possible solution strategies that students will pursue. In order to ensure students' success, teachers should select examples where the direct modeling strategy can be easily utilized/implemented. After specific problems are selected, depending upon the grade level of the students, teachers should read the problem, the teacher and students should read the problem together, or the students should read the problem independently. Teachers then proceed to support students as they "unpack" the problem context through rich discussion[20]. Unpacking the problem will ensure that students understand the actions that are occurring in the problem so that they will be able to use a modeling strategy to solve the problem. CGI teachers also provide story scenarios that allow students to make connections with their own experiences so that they can make sense of the problems. For example, teachers may choose to pose story problems about breakfast or lunch items served in the cafeteria. Since all of the students eat in the cafeteria, students will be able comprehend the problem context and make sense of the problem. Carpenter et al provide five principles for unpacking problems[21]:

- make sure that all students understand the problem
- align the story and the math
- support students to unpack the problem independently
- do not solve the problem for students
- remove barriers so that all students are able to participate in the discussions

A common feature in all CGI classrooms is the explicit discussion of students' thinking and reasoning[22]. Questions cannot be preplanned, because it is difficult to anticipate what ideas students will share. CGI teachers must carefully listen to their students' explanations so that they can offer support as necessary to complete or correct students' chosen strategies. Without listening to student explanations, it is difficult to determine the level of students' mathematical thinking and development. For example, on the surface, it may appear that a student is using a direct modeling to solve a problem, however once they explain how they arrived at the answer, it is possible that they used manipulatives, but counted on to arrive at a solution. See the example below:

There are 8 kittens in one litter. There are 4 kittens in another litter. How many kittens are their altogether?

In this problem a student may make a set of 8 and a set of 4 and tell you that there were 12 kittens altogether. If the teacher does not elicit a more detailed explanation from the student, the teacher may assume that this student used a direct modeling strategy to solve the problem. With further discussion however, the student may explain that there were 8 kittens in the first litter, and the second litter has “9, 10, 11, 12” kittens, thus there are 12 kittens altogether. Once the student represented the 8 kittens, he/she was able to count on to determine how many more kittens there would be if there were 4 kittens in the second litter. So the student who appeared to be using a direct modeling strategy, was actually transitioning to using a counting strategy to solve the problem.

Eliciting student thinking aloud can be helpful to explore both correct and incorrect solutions to problems. In the event that a student has solved the problem incorrectly, this type of a discussion allows everyone to see where exactly the error occurred so that the student can make sense of the problem by identifying their own error, and persevere in solving the problem correctly.

CGI and English Language Learners

Since more than 50% of my students are ELLs, I was determined to research the impact of CGI on English Language Learners. In their new book, *Children's Mathematics: Cognitively Guided Instruction Second Edition* , Carpenter et al. referenced a number of articles about CGI and ELLs. Below is a summary of the findings:

In their article, “Mathematical Problem Solving Among Latina/o Kindergartners: An Analysis of Opportunities to Learn”, authors Turner and Celedon-Pattichis, studied 7 students from three different Kindergarten classes in the southwest[23]. A total of 21 students were followed in the study. The 7 students selected from each class represented balanced achievement levels. Although all of the students showed measurable growth from the comparison of pre and post assessment data, the students in Ms. Arena's class performed significantly better than the students in the other two classes[24]. The research from the study indicated that Ms. Arena spent more time engaging her students in complex problem solving that required more than just a single step to solve. She also used consistent accommodations for testing and the students were given all pre and post assessments in their native language of Spanish.

A review of the pre-assessment data indicated that rote counting past 10 was a skill that 75% of the students in all three classes had mastered[25]. Approximatly 63% of the students could identify some numbers from 1 to10[26]. Seventy-five percent of the students in the other two classrooms (not Ms. Arena's) could count with sets greater than 10 with one to one correspondence[27]. In contrast, more than 50% of the students in Ms. Arena's class were unable to count a set higher than 10 using one to one correspondence[28]. Consequently, Ms. Arena's class had lower scores in these basic number skills foundations. In terms of problem solving, 43% of the students could solve a simple addition word problem, and 48% could solve a simple subtraction word problem without any prior instruction[29]. This finding confirmed to the authors that children have an intuitive sense for problem solving, even prior to having any formal education[30]. All in all, the basic skills demonstrated by the 21 students from the three classes were typical for the beginning of the year in Kindergarten.

A teacher practice analysis was conducted. The researchers learned that all three teachers were utilizing similar strategies to teach problem solving to their students. Open-ended questions were posed to the students to elicit mathematical thinking and reasoning. If students were not able to answer the open-ended question, closed questioning was utilized so that students could clarify their ideas by providing briefer responses in the form of a yes/no response, or by filling in a blank statement with a word or short phrase[31]. Originally cited by Franke et al. in 2007, this strategic scaffold made learning accessible to struggling learners and thus enabled all students to participate in the problem solving discussions[32]. Peer discussions took place regularly in all three classrooms, and all three teachers were in the habit of celebrating students for their contributions to the problem solving process.

A problem solving posttest was conducted at the end of the year. According to NCES, 2000, students in the study scored higher than predicted by national assessment results[33]. The kindergarteners studied also performed equally to the kindergarteners studied and published by Carpenter et al. in 1993, and those students were from white middle and upper middle class families[34]. Ms. Arena's students scored significantly higher than the students in the other two classrooms. One reason for the higher outcomes is suspected to be due to students in Ms. Arena's class having more opportunities to solve more types of problems. For example, on Ms. Arena's test, 41% of the problems were join/separate result unknown, whereas the other two teachers had 63% of these problem types on their assessments[35]. Ms. Arena also posed 5.4 practice problems per lesson on average, whereas Ms. Perales and Ms. Field pose 3.8 and 3.2 problems per lesson on average, respectively[36]. It is also significant to note that Ms. Arena's students receive all mathematics instruction in their native language of Spanish, whereas the students in the other two classes only counted and did peer sharing in Spanish. The last difference between Ms. Arena's instruction and her two counterparts is in her delivery of story problems. Ms. Arena uses an informal story telling approach and makes context connections to students' experiences both in and out of the classroom. In contrast, the other two teachers pose story problems in a straightforward and simple way, as is recommended for ELs and supported in research conducted by Echevarria, Vogt, & Short in 2007[37].

Classroom Teaching Strategies

In the future, I plan to incorporate what I learned about cognitively guided instruction in my classroom. Below are some of the teaching strategies that I plan to utilize:

Direct Instruction - Whole Group Problem Posing:

I will make sure to choose problems that have objects and/or context that the students can understand. Students must be familiar with the contexts of the problems. For example, most of the students buy lunch in the cafeteria every day, and all of the students eat lunch in the cafeteria every day. As a result, most of the students will be able to relate the problems to their daily lunchroom experiences which will improve their understanding of problems such as the following:

Vanessa has 6 goldfish, and Antonio has 4 goldfish. How many more goldfish does Vanessa have?

I will first read the problem aloud and have the students repeat the problem. I will have a variety of manipulatives available and each student will receive enough manipulatives to model the problem posed. I will tell students that using manipulatives is optional, but I will remind them that they need to make sense of the problems in order to solve them.

Role Playing - Acting out the Problem:

Students will take turns acting out some of the problems that are posed. Role playing will be modeled by the teacher and student volunteers. Role playing will help students to make context connections and will ensure story comprehension, particularly for ELLs who are working on English language development.

Music –Children's Songs:

Students will problem solve as they sing popular nursery rhymes and children's songs The repetition and familiar structure will be a form of scaffolding lessons to increase story comprehension.

Wordless Pictures and/or Picture Books:

Students will analyze pictures and engage in Math Talk to create their own mathematical story problems.

"Math Talk" - Problem Unpacking and Questioning:

We will discuss what is happening in the problem so that everyone understands the context. What is the story about? I will elicit student thinking by asking questions such as:

- Can you tell me how you solved the problem?
- What did you do first?
- What did you do next?
- Can you explain the strategy you used?
- Can you share your thinking?
- Tell me about your drawing.
- How did the drawing help you to solve the problem?

I will choose different students to come up and model “Math Talk” with me so that the other students can observe what “Math Talk” looks and sounds like.

Collaborative Learning – Think/Pair Shair, Small Groups:

Students will have many opportunities to work together and engage in Math Talk as they unpack problems and solve them. Afterwards, students will share their experiences with the class as they answer group work questions such as:

- Did you and your partner solve the problem in the same way or in different ways?
- Can you explain your partner’s strategy? Tell me one thing.

Differentiated Instruction:

Prior to beginning my unit, I will first give students a brief pre-assessment so that I can determine students’ levels of mathematical thinking. I will determine the problem solving strategy or strategies that my students are using (direct modeling, counting strategies, or number fact strategies) so that I can differentiate student groupings and pose appropriate problems for all learners. Given that it is early in the Kindergarten year, it may be that students are on similar levels, however I do not want to make that assumption without evidence.

Scaffolding:

I will scaffold our Math Talk discussions as necessary to ensure story comprehension and connection to story context. I will use a combination of open and close questions as necessary, particularly for ELL learners who may require questions that they can answer with a yes/no response, or a short phrase.

Math Journals:

Students will keep math journals, and will use them throughout the unit. The math journals will provide opportunities for engagement in story content for those students who enjoy drawing. Students will be able to record their problem solving steps and strategies and will be able to refer to their ideas and reflect upon their mathematical thinking throughout the unit. I will use the math journals as a written formative assessment tool, so that I know how students are progressing from time to time. I will keep track of how often students are using ***valid strategies*** to solve problems. It is important to remember that students do not have to have the correct answer all of the time to prove that their mathematical thinking and reason skills are developing.

Unit Activities

I will also utilize the activities below in conjunction with the district adopted Math Expressions curriculum which has problem solving opportunities embedded within all of the lessons.

Activity 1: Pre-Test/Interview:

Objective: To the best of their ability, students will solve story problems using direct modeling strategies. I will use an observation sheet to record information for each student. The purpose of this activity is to establish each student's ability to think mathematically and to problem solve. I will use the information that I collect in order to develop appropriate problems to pose to my students both in whole group and in small group settings.

Materials: Hiders (1/student), manipulatives (10 counters/student), Pre-test paper and pencil (per student)

Procedure: Some or all of the questions below will be posed to the students so that the teacher can determine various levels of mathematical thinking. The questions will be stated in English and in Spanish so that I am able to determine the knowledge level of my ELL students. Students will be given the option of modeling and/or drawing their responses A copy of the pre-test questions has also been included in Appendix C.

1. Juan has 4 cookies, and Cynthia has 2 cookies. How many cookies do Juan and Cynthia have altogether? (Join – Result Unknown)

2. Carlos has 3 little pancakes. Carlos wants to eat 7 little pancakes. How many more pancakes will Carlos need to get so that he can eat 7 pancakes? (Join – Change Unknown)

3 Olivia and Hailey have 8 pieces of gum. They gave 4 pieces of gum to Daniela. How many pieces of gum do Olivia and Hailey have left? (Separate – Result Unknown)

4. Josh has 10 crayons. He lost some of his crayons, and now he only has 6 crayons. How many crayons did Josh lose? (Separate – Change Unknown)

5. Eduardo has 3 bags of candy. Each bag has 2 pieces of candy in it. How many pieces of candy does Eduardo have altogether? (Multiplication)

Activity 2: Problem Solve While Singing a Children's Song:

Objective: To give the students an opportunity to solve a subtraction story using the direct modeling strategy for the following problem type: Subtraction/Separate – Result Unknown

Materials: 5 counters/student, teacher computer, audible sound system, and internet access to the "Five Green Speckled Frogs" you tube link (see Teacher Resources section at the end of this unit).

Procedure: As a class we will first listen to a song and view the animated You Tube video for the children's song "Five Green Speckled Frogs". Next, students will be given 5 counters each, and they will independently model the story problem as we slowly sing the song aloud together. I will then give students some time to engage in Math Talk with a shoulder buddy so that they can discuss the strategies that they used during the activity.

Activity 3: Problem Solve – Partners Model for the Group Using Real Objects that Facilitate Context Connections:

Objective: To give students an opportunity to actively connect with the context of the story as they observe partners modeling the story using real objects. This activity will help students develop their story comprehension skills.

Materials: 6 animal crackers for the scenario, 5 crayons for the scenario, bag of animal crackers for practice activity

Procedure: I will call pairs of students up to directly model 2 scenarios 2 different problem types.

Partner Scenario #1: Aidan gets 4 cookies out of the bag. Then Aidan gets 2 more cookies out of the bag. How many cookies does Aidan have altogether? (Addition/Join – Result Unknown, Direct Modeling of Joining All, or Counting On)

Partner Scenario #2: Stephanie gets 5 crayons. She gives some of her crayons to Carter. Now Stephanie has [student determines] crayons left. How many crayons did Stephanie give to Carter? (Subtraction/Separate – Change Unknown, Direct Modeling of Separating To, and/or Counting Down To)

Then students will return to their seats for snack. Each student will receive 7 animal crackers and (2/student will be placed in the center of each group table) students will be told not to eat their crackers until they hear the story problem that they will have to model using their crackers:

Individual Seatwork Practice Problem: You have 7 animal crackers. Ms. Gallagher gives you some extra animal crackers. Now you have 9 animal crackers. How many extra animal crackers did Ms. Gallagher give to you? (Addition/Join – Change Unknown, Direct Modeling of Joining To, and/or Counting On To)

Activity 4: Role Playing (Using Student Volunteers):

Objectives: Students will have an opportunity to actively connect with the context of the story as they observe classmates act out an animal story problem using body motions and sounds to represent the actions of the animals in the story. This activity will also help to develop students' story comprehension skills.

Materials: Math Expressions Warm Up Quick Practice Routine from Unit 1, Lesson 1, Teacher's Edition: Creative Movement and Sounds (Fuson, 2013, p. 5)

Procedure: As a warm up, I will first do the Creative Movement and Sounds Quick Practice Routine from Math Expressions that the students are already familiar with doing. Next I will call up student volunteers to role play different animals:

Scenario #1: Call up 6 students to act like chickens. Call up 4 students to act like horses. Pose the following scenario: There were 6 chickens on the farm (6 students will cluck and flap their wings 6 times like they do during the quick routines warm up), there were also 4 horses on the farm (4 students will neigh and gallop in place with one foot in front of the other 4 times), how many animals were there on the farm altogether (all chickens and horses will make their respective animal sounds and movements simultaneously)? (Addition/Join – Part-Part-Whole with Whole Unknown, Direct Modeling of Joining All and/or Counting On).

Activity 5: Collaborative Learning/Small Groups:

Objectives: Students will work collaboratively in small groups to solve two types of story problems. Students will be given individual jobs to do within their groups. Groups will share their work with the class.

<u>Problem Types:</u>

- Multiplication /Join – Result Unknown, Direct Modeling of Joining to
- Compare – Difference Unknown, Direct Modeling of Matching

Materials: story problem card/group (7 total), and paper and pencil per group (7 total), set of manipulatives (per group)

Story Problem Card #1: Maya has 2 bags of chips. Each bag has 4 chips inside. How many chips does Maya have altogether?

Story Problem Card #2: Johnny has 4 cubes. Leila has 2 cubes. How many more cubes does Johnny have?

Procedure: Students will be divided into groups of 3. Each group will be given a story problem card that is typed on paper with picture symbols wherever possible to remove language barriers and facilitate story comprehension. First the students will be directed to engage in Math Talk together so that they can develop their mathematical thinking and story comprehension skills as they discuss what is happening in the problem. Next, each student in the group will be assigned a job. One student will model the problem using manipulatives, one student will draw the problem, and one student will check the work in both problems, and elicit group thinking about the two representations of the problem. Each group will then have to share how they solved the problem. Four groups will be assigned the same problem, and three groups will be assigned a different problem. So other groups will be familiar with the same problems and can interject and compare strategies during the sharing discussions.

Activity 6: Creating and Solving Story Problems from a Wordless Picture Book:

Objectives: Students will connect math to the real world by counting objects and creating words and story problems to go with the story, *Anno's Journey,* written by Mitsumasa Anno. This lesson is an extension to the early lessons in Unit 1 of Karen Fuson's Math Expressions curriculum (2013, p. 1EE). In the previous book, children counted objects in different scenes that corresponded with numbers1-10, and they created their own scenes to represent various numbers 1 through 10. In this activity, using Anno's Journey, students will discuss the pictures and they will create story problems to represent the scenes that they see in some of the pictures.

Materials: 1 class copy of Anno's Journey, by Mitsumasa Anno, 1 page of the book assigned to each pair of students.

Procedures: Will take a picture walk through the book and students will generate words and vocabulary to go with each page. Together we will create some story problems to go with a couple of the pictures. Then, students will return to their seats and they will work with their math partner to analyze a page that they are given from the book. Each pair will discuss their picture and describe what they see in the scene. Using mathematical thinking, partners will collaborate to develop a story problem to go with their picture. Each pair of students will present their picture to the class and share their story problem. Student in the audience will be asked whether they can come up with any other story problem to go with the target picture.

Conclusion

I plan to utilize the classroom activities and strategies that I developed for this unit throughout the school year. At the end of the school year, I will administer two post-tests to my students. One of the post-tests will be exactly the same as the one that I administered at the beginning of the unit, so that I can compare the results directly, and measure the growth of each individual student. The second post-test will contain the same number of problems, but they will have greater complexity. This will allow me to gain valuable insights about the growth of my students and will help me to plan future lessons that will extend my students' thinking and mathematical reasoning skills.

Teacher Resources

You Tube Video: 5 Green Speckled Frogs Song/Animation:

(http://r.search.yahoo.com/_ylt=A2KLqIIX7SxWDGIAZwX8w8QF;_ylu=X3oDMTByZnJkbGl0BHNlYwNjZC10aHVtYgRzbGsDcGxheQR2dGlkAw--/RV=2/RE=1445813655/RO=10/RU=https%3a%2f%2fwww.youtube.com%2fwatch%3fv=WSC-gHBU_d0/RK=0/RS=tX.wKF.BC_GEUkeEXaQewlKca7U-

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Notes

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[2] Karen Fuson, "Math Talk and Helping Community in Math Expressions," Www.d25.k12.id.us/pdf/curric/math_expressions-math_talk.pdf, Accessed August 1, 2015.

[3] Ibid.

[4] Ibid.

[5] Karen Fuson, *Math Expressions Common Core*, Teacher Edition, Volume 1, Orlando, Florida: Houghton Mifflin Harcourt Publishing Company, 2013. xviii-xix.

[6] "Reasoning and Proof Standard for Grades Pre-K-2," In *Principles and Standards for School Mathematics*, 121-125, Chapter 4 ed., National Council of Teachers of Mathematics (NCTM), 2000.

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[8] Ibid, 435.

[9] Ibid.

[10] Ibid.

[11] Ibid, 435-438.

[12] Thomas P. Carpenter, Elizabeth Fennema, Megan Loef Franke, Linda Levi, and Susan B. Empson, *Children's Mathematics: Cognitively Guided Instruction*, Second Edition ed., Portsmouth, NH: Heinemann, 2015.

[13] Ibid, 200.

[14] Ibid, 34.

[15] Ibid, 35.

[16] Ibid, 36.

[17] Ibid, 37.

[18] Ibid, 40.

[19] Ibid, 134.

[20] Ibid, 135.

[21] Ibid, 140.

[22] Ibid, 141.

[23] Erin E. Turner, and Sylvia Cledon-Pattichis, "Mathematical Problem Solving Among Latina/o Kindergartners: An Analysis of Opportunities to Learn," *Journal of Latinos and Education* 10, no. 2 (2011): 146-49, Accessed October 23, 2015, doi:10.1080/15348431.2011.556524.

[24] Ibid, 146.

[25] Ibid, 154.

[26] Ibid.

[27] Ibid, 155.

[28] Ibid.

[29] Ibid.

[30] Ibid.

[31] Ibid, 156.

[32] Ibid.

[33] Ibid, 157.

[34] Ibid.

[35] Ibid, 160.

[36] Ibid, 162.
[37] Ibid, 163.

Curriculum Unit Title: Show Me How You Know – A Unit in Kindergarten Problem Solving

Author: Jacqueline K. Gallagher

KEY LEARNING, ENDURING UNDERSTANDING, ETC.

The Common Core State Standards requires Kindergarten students to represent addition and subtraction with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explantions, expressions or equations (CC.K.OA1). Students are also required to solve addition and subtraction word problems, and add and subtract within 10 (CC.K.OA.2). In a Cognitively Guided Instruction classroom, teachers provide constructivist opportunities for students to solve problems. As they unpack complex problems, students engage in deep discussions about their thinking and reasoning. Students solve problems using direct modeling with manipulatives or drawings, counting, and/or number facts (Carpenter, Et al.)

ESSENTIAL QUESTION(S) for the UNIT

1. In what ways can students discuss number relationships, and tell/model story problems through counting, visualizing, and drawing 1-10 objects?
2. How can students represent and solve addition and subtraction stories from family experiences?
3. How can students use drawings and write expressions to solve addition and subtraction story problems?

CONCEPT A

Provide opportunities for explicit discussions of student thinking and reasoning (Carpenter Et al.)

CONCEPT B

Remove barriers so all students can participate in discussions. Present open-ended problems in an informal story-telling format, make context connections to students' home/school experiences

CONCEPT C

Utilize wordless pictures and/or picture books to facilitate discussions focused on unpacking the problem, and proving conjectures based on illustrated evidence.

ESSENTIAL QUESTIONS A

In what ways can students discuss number relationships (1-10), and tell/model story problems?

ESSENTIAL QUESTIONS B

How can students represent and solve addition and subtraction stories from family experiences?

ESSENTIAL QUESTIONS C

How can students use drawings and write expressions to solve addition and subtraction story problems? What other strategies can students use to solve complex story problems?

VOCABULARY A

Direct Modeling, counting, joining, separating, comparing, number facts, part, whole

VOCABULARY B

"Math Talk" (Fuson), prove, joining, separating, math word wall

math terms in students' native language

VOCABULARY C

Expression, number sentence, add, subtract, plus, minus, equals, sum, does not equal, more than, less than, partners, 5-groups

ADDITIONAL INFORMATION/MATERIAL/TEXT/FILM/RESOURCES

Math Expressions Materials: *Math Expressions Common Core*. Teacher Edition ed., by Karen Fuson

CGI Reference: *Children's Mathematics: Cognitively Guided Instruction*, by Carpenter, Fennema, Franke, Levi, and Empson

You Tube Video: 5 Green Speckled Frogs Song/Animation**:** https://youtu.be/WSC-gHBU_d0

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