Teaching What "Equals" Means to First Graders

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Introduction

How many of you were shocked or disturbed when you first became aware that you would now be required to teach algebra to your kindergarten or first grade students? The Common Core State Standards mandate that we make algebra real for even the youngest students. There is an entire domain dedicated to "Operations and Algebraic Thinking." I know my fellow colleagues and I have been discussing this "travesty" at length with as much contempt as it deserves. Do they expect our five and six year olds to be headed to Harvard tomorrow? First, they take away play and now we need to teach them higher level math? How absurd!

The fact is, however, that we have always been teaching algebraic thinking to our little ones, just not thoroughly enough. Also, we think of arithmetic as being a separate subject from algebra when we should consider that arithmetic falls into the realm of algebra.[1] Additionally, our language has caused misconceptions with regard to equality.[2] We consistently focus on finding ***a*** solution to ***a*** problem[3]; we use words that have double meanings when teaching arithmetic[4]; and, we omit language that would aid teachers in later grades[5]. Our practice has been to teach problem solving in one direction (left to right, if you will). And our instruction has been in short cuts and finding the most efficient method of getting an answer. After quite a bit of research and discussion, I have come to understand that we are actually setting our little people up for a lifetime of misconceptions and most of it has to do with how we use and talk about the equal sign.

Rationale

As we hear so often these days, "research has shown" that at the elementary levels, students are taught to have an *operational view of equality*. We teach (and our students use) procedures that enable them to get correct answers in order to pass tests, when we should be teaching concepts which will ultimately make finding solutions to problems easier. We teach students that there is one, and only one, solution to an "equation". And why do we do this? So they can pass timed math facts fluency tests. 1 + 3 = 4, 0 + 7 = 7, etc. Don't get me wrong, I understand the need for math fact fluency, but what we are doing is limiting our students' understanding of what it means to be "equal."

You might make the argument that we give our young math students a start on understanding what equality is and that it is up to the middle school teachers to *add on* *to* that understanding. This gets to the heart of the problem. We are establishing an operational view of equality, i.e. that given two addends (or even three), if we join their quantities they should add up to a new, greater quantity, which is true. But we never go beyond this simple addition. Our students then arrive in a middle school algebra class and they see an equation like 2x = y and they expect that x is going to be equal to only one number when in reality it will have a set of solutions. That set of solutions is found because of the relation of the two sides of the equation. This means that the middle school teacher has to undo the understanding we have developed in elementary school. I read the results of a study done in 2006 by a group of professors of math and psychology[6] in which the team set out to disprove that "understanding the equal sign does matter" to the middle school student when learning to use variables. They did their research with sixth, seventh and eighth grade math students. Once their research was completed they discovered that their hypothesis was wrong. It really **does** matter that students understand that quantities are equal because of their relation to each other and that students have to understand that from the beginning. Misconceptions are not easily undone as students move from grade to grade.

I read another study in which the research was designed to find the level of understanding that pre-service teachers have of "equivalence and relational thinking"[7]. The study concluded that while pre-service teachers can see the importance of teaching relational thinking (of the equal sign), they probably would not encourage students to use that type of thinking. But accordingly, these strategies help students to reason at an abstract level[8]. As teachers of young children, I would think that encouraging this type of thinking would be beneficial to the cognitive development of our students. We know that children who are under the age of seven are not typically abstract thinkers, so as we are trying to grow that skill in our students, what better way than through algebra?

Overview

I am hopeful that this curriculum unit will be used to teach young students to have a *relational view of equality*. I hope to help teachers find a developmentally appropriate way to teach that two things being equal is determined by their relation to each other, and that students will be able to determine if equations are true because of this understanding. Additionally, I hope to get teachers who use this unit to use more appropriate mathematical language when teaching addition and subtraction to young children in order to develop a clear and correct understanding of what they are doing. This should help them to be more successful when they reach the middle school grades and begin learning about inequalities.

This curriculum unit will be designed for teachers of first grade, however it could be simplified for kindergarten students as well (and may even benefit struggling second graders). I will be teaching first grade students in a middle class, suburban community. The 2014 demographics of my class of nineteen are: 47% female, 42% African

American, 16% bi-racial, and 84% benchmark in DIBELs (a measure of literacy).
The 2013 demographics of my school with an enrollment over 600 are 35% African American, 35% White, 36% Low Income, and 80% or more meet expectations in both math and reading. Students at this school are expected to enter first grade being able to solve simple addition problems with sums to 10, being fluent with sums to five. By the end of the year, these same students are expected to master addition and subtraction of equations with sums to twenty. They are required to achieve 80% accuracy on a timed test (10 minutes) of 50 math facts (25 addition/25 subtraction) to be considered benchmark by year's end. We use the Houghton Mifflin Harcourt Math Connects curriculum which is aligned with the Common Core State Standards. Some teachers have begun implementing Number Talks strategies to develop number sense. The children have math workbooks which are quite colorful, use pictures of manipulatives, but the program writers also have the students working rather quickly with abstract symbolism versus hands-on problem solving. Teachers at my school are expected to provide whole group instruction to introduce concepts, small-group instruction focused on individual needs, one-on-one check-ins (for RTI), and leveled, independent work within a 70 minute period. Additionally, students who need more focused intervention meet with an interventionist for half hour periods three times per week.

This curriculum unit will address one Common Core State Standard: 1.OA.7 – Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. Additionally, I feel that this unit can specifically address the second mathematical practice, "reason abstractly and quantitatively" and, the seventh, "look for and make use of structure."

Content Knowledge

As the authors of the book Putting the Practices into Action remind us, in everyday life we are met with problems to solve whether analyzing data or calculating how much is left in our checkbook[9]. But, they say "it would be impossible to memorize how to solve every math problem." Therefore, it is an important life skill to learn how to reason abstractly. Understanding that arithmetic is just a subset of algebra should guide our practice when teaching our students addition and subtraction. In order to teach algebra to children in a meaningful way and to eliminate misconceptions that can develop from inaccurate language, we need to begin by defining what a "unit" means. Next we must determine the subset of the set of numbers that our little guys work with, namely the Whole numbers, call it S={0, 1, 2, 3, …, 30}. Then we will determine the mathematical operations we will perform with that set (in our case addition and subtraction). As the content experts that we are supposed to be, we should know (and teach) the properties associated with those operations, as well as, understand the algebraic structure of that set of numbers when using those operations.

Concept of Unit

Often, early elementary math curriculum begins with counting. Teachers feel that students should master a one-to-one correspondence using the counting numbers correctly before they can master the rest of the curricula. There are some studies that consider that this progression may not be developmentally correct. Researcher, Catherine Sophian has completed a lifetime of research on finding a way to make formal math education mesh with the development of the young child. She has stated that often research is done to discover how pre school-aged children acquire math knowledge but that once they reach school the formal learning replaces development, “so that once schooling has begun there is nothing more to learn about development.” Some of her research has found that mathematical thinking begins not with counting but with comparisons between quantities, in particular the identification of equality and inequality relationships. In her examination of how preschoolers acquire an understanding of concept of unit, she noted the importance of using collections of similar (but not the same) objects. For example, a teacher might use pompoms of varying sizes when asking students to find sets that have the same number of items. This may aid them in understanding that two sets of objects being equal does not have to do with the amount of space that a group takes up, but how many discrete objects are in each group.

Misconceptions in later grades might also be avoided if we differentiate the mathematical concept of unit from physical objects. This will help children when they are learning how “variation in unit size affects the numerical outcome.” Think of centimeters and inches. These are units with which we measure and an object that is six centimeters long is smaller than an object that is six inches long. Both are six but they are not equal. If an assignment is to compare two sets of shoes to see if they are equal, it is important for the teacher to state that she wants the student to count each shoe as one, or to count each pair. Defining the unit prior to comparing is paramount to being able to accurately compare one set to another. We would be very wrong if we considered six centimeters to be the same as six inches. Consider the cliché “comparing apples to oranges.” It should be our mission to make that phrase archaic because we’ve done such a fantastic job developing a concept of unit!

The Set of Whole Numbers

Sophian has examined the work done in the 1970s by V. V. Davydov (1975). He suggested that a set of objects is not defined by a number because they are a physical collection, but rather that the collection becomes a set of objects when we consider its relation to other physical collections using the concepts of equivalence, greater than and less than. Again, there is that term “relation.” So it is easy to understand that a set having three objects is less than a set having 5 objects. Students can see that even before we start assigning a numerical value to each set.

This is important for the students to understand as we begin to teach them operations like addition because they can see that a set resulting from joining two smaller sets becomes larger than either of the sets that were joined. This knowledge will help them once they begin using the abstract symbols associated with addition. It is even more important to establish this knowledge before teaching subtraction with the abstract symbols. How many times have students come up with an answer when subtracting that is greater than the sets they were subtracting (because they added instead of subtracted)? Sophian makes an extremely valid point based on research done in Brazil[10]. "Symbolic problem solving becomes error prone and illogic only when children… adopt symbol manipulation procedures without understanding the mathematical logic behind them."

Fortunately, in first grade we spend almost all of our time working with the set of whole numbers, that is the numbers on the number line beginning at zero and moving to the right (the positive numbers and zero). This is important to consider as there are many situations that we can talk about with our students that could result in a solution that is a negative number (for example when we discuss temperature or owing money). Because we are using the whole numbers we are limited in the results we can get when working with these numbers.

Properties of Equality

The *relation of equality* on the set of whole numbers is defined by three properties:

1) transitivity: for any numbers a, b, and c in our set, if a = b and b = c then a = c,
2) reflexivity: a = a, any number equals itself,
3) symmetry: if a = b then b = a, for any numbers a and b in S.

These properties are important to understand before we begin teaching equality to our students because if we truly examine what these properties are telling us, then we shouldn't be limited by programs or curricula that have us teaching our students only one way. So many times the curricula that we use have students work with join problems where the result is unknown. If we are to teach these properties effectively, then we need to be sure to use all types of problems (join, separate, part-part-whole, and compare) as well as moving the unknown to a different place in each problem on which we work.

Properties of Equality Related to Operations

In first grade we work with the operations of addition and subtraction. The properties of equalities hold true when working within these operations. Take for instance, the transitive property: for any numbers a, b and c in the set of whole numbers (0-30), if a =b, then a + c = b + c, and a - c = b - c.

If you recall, arithmetic is a subfield of algebra. Therefore, the operations of addition and subtraction of whole numbers give algebraic structure to the set of whole numbers. The operation of addition of whole numbers has the following properties:

1) closure: if a, b are whole numbers then a+b is a whole number. If a and b are numbers in S, then a+b could be a number bigger than 30 (so S is not closed under addition, but W is closed under addition).
2) associativity: (a+b)+c=a+(b+c) for any a, b, c in S
3) identity element: There is 0 in S with the property that 0+a=a+0=a for any a in S
4) commutativity: for any a and b in S, we have that a+b=b+a

The operation of subtraction does not have these properties, i.e. only some numbers make these properties true for subtraction - not all numbers of S.

While working within these parameters of unit, set, properties of equality and algebraic structure, it is important to establish for our students an understanding of when an expression or equation is true or false. So often, our students will work through a problem until they get an answer and never determine if the answer they came up with is correct. This goes back to the necessity of understanding equality as relational and not operational. Students who have a solid understanding of equality can just "see" that an answer is not reasonable if they have separated a set into two subsets, and yet they record that one of those subsets is larger than the initial set.

Strategies

Guided discovery, cooperative learning, Think-Pair-Share are all well-known, best-practices for facilitating student learning and these are excellent methods to employ when teaching mathematics. It is important to have first grade students using math language regularly so that language becomes part of their natural vocabulary. To that end, the following lessons are designed to have students talking about what equals means. Differentiating instruction is necessary so that each student has the best environment in which to learn. Questioning using Bloom's Taxonomy is important to encourage higher-order thinking. We not only want our students to know the meaning of equality, but to be able to apply this property when manipulating sets and to use the property when creating their own reasoned responses. None of the following activities are wholly original because of the breadth and depth of math curricula. However, I have extended or embellished activities that I have used in the past, because I felt that either the concept was not developed thoroughly enough, or the language used was misleading or would guide students to misconceptions. When appropriate I will state from where the idea came.

Classroom Activities

Lesson #1 - Comparing Quantities

Enduring Understanding

The definition of a set: A set is a collection of objects (called elements) with one or more common properties. The size of a set is the number of elements in the set. Mathematically, two sets are equal if the elements of the first set are included in the second set and the elements of the second set are included in the first set. When we compare two (or more) sets, they are considered equal if their elements have the same defining properties and the sets have the same number of elements.

Essential Question

How do I know when sets are equal?

Objective

This activity can be used as an activating strategy to begin the discussion of equality.

Procedure

A beginning of the year activity, this activity can be done once at the beginning of a unit as with Math Connects Lesson 2 or it can be used repeatedly throughout the unit always increasing the size of the group of students standing. Begin with students in a whole group setting (whether at a carpet or in their seats). Choose students randomly (I use popsicle sticks with their names on them) to stand in front of the whole group where those still seated can see all the students chosen.

Begin with a small group, say four or five students. Have those seated decide attributes that the students fit, say gender, hair/skin color, sneaker/shoes, or color/type of clothing. Record these attributes on chart paper. Then choose one attribute on which the students should focus. It is easiest to begin with boy/girl because at the beginning of the school year that is what first graders will key in on. [If those chosen randomly are all boys or all girls use a different attribute.]

Ask those seated to decide if there are more boys or girls standing. Ask how they know. Explain that what they are doing is what mathematicians do and it is called comparing. We compare groups of objects to determine the relationship between them, or to see how they are alike. We use vocabulary like greater than, less than and equal to talk about this relationship.

Have the students standing take their seats and randomly choose four different students. Tell the class that they will use the next attribute from the chart to find the relationship between the students standing.

For example, there are four children standing, one has blond hair, three have dark hair. Two are boys and two are girls. All are wearing shorts and zero have long pants. Three are wearing shoes with velcro and one has shoes with laces. While these four children are standing have the class decide the relationships of these same students with regard to each of the attributes. In fact, have individual students direct the four to stand in groups depending on the attributes. So if talking about hair color, the one student with blond hair should be separated from the three with dark hair. Then have a different students direct the students to separate according to gender, then a different student direct for clothes, then shoes, etc.

All the while use the vocabulary of compare, relationship, greater than, less than and equal. Have all students sit.

Closure/Assessment

Direct students to look at the chart with the attributes listed. Ask them to talk with an elbow buddy to remember how many students (from the last group) had light colored hair and how many had dark colored hair. Record these quantities on the chart paper with numerals (next to the appropriate attribute). Repeat this procedure for the other three attributes. Once the numerals are recorded, have them talk with an elbow buddy one last time to determine if they see a pattern with the numerals. Someone will most likely notice that even though all of the pairings of numerals are different (1, 3); (2,2); (4, 0); (3,1) there was always a set of four children standing. Listen to the students for vocabulary: compare, relationship, greater than, less than, and equal.

Lesson #2 - Comparing Quantities

Enduring Understanding

When we compare two (or more) sets, they are considered equal if their elements have the same defining properties and the sets have the same number of elements.

Essential Question

How do I know when sets are equal?

Objective

This lesson will help to extend the students' grasp of finding a relationship between sets. Students should begin to develop the concepts of reflexivity and symmetry.

Materials and Setup

For each group of three or four students, you will need an 12X18 sheet of construction paper, and call it a mat. [Ideally, you could use a 12X18 piece of felt so that the pompoms won't move, but the paper will work so long as students do not touch it, so you will need to establish this rule before giving out materials.] You will also need a zippered sandwich sized bag, a number of pompoms of varied size (equal to the sum with which you want your students to work) [Earlier in the year, you might work with sums to three to six, and later in the year with larger sums.] Prepare plastic bags with different quantities of pompoms (you could write the number of pompoms on the bag).

Procedure

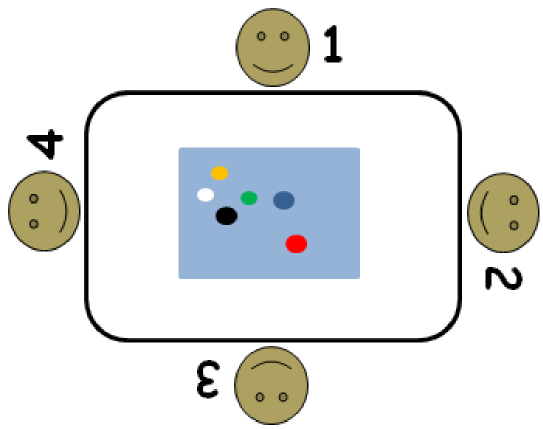
This activity came to me as a variation of a Number Talks activity in which the students look at cards with random collections of dots and talk about what patterns they may see. I felt that it could be adapted to help students see the properties of equality: reflexivity and symmetry. Demonstrate the activity for the whole class or in small groups before expecting independence.

Initially you can use the Numbered Heads strategy for grouping students (count off 1, 2, 3, 4 and all ones work in a group, all twos work in a group, etc.). Later, you can group students according to ability so that students with a firm understanding of comparing are working with larger sets and students who still are struggling to understand are working with smaller sets.

Review the vocabulary that you have introduced in the last activity: set, compare, relationship, number, value, equal. Introduce new vocabulary, numeral and quantity, while doing the activity.

The first time you do this activity, you want every group to use the same number of pompoms. Place the mat in the middle of the group. Choose one student to be the first and then they take turns in a clockwise rotation. The first student dumps the pompoms onto the mat allowing them to fall randomly.

Figure 1:



Each partner reports what they see from his/her side of the mat. For example:
#1 may say "I see four and two."
#2 may say "I see two and two and two."
#3 may say "I see one, three, and two."
#4 may say "I see three and three."
Accept all correct responses. For those incorrect, ask the child to point (without touching) to the pompoms to show what they saw. (Usually, they will realize their error, however, in my experience, students will sometimes count the same pompom twice. Assuming that the child has a concept of one-to-one correspondence, this is why having different colors and sizes helps.) There will often be two partners who see the image in the same way, and that is a good moment to talk about how the quantity does not change even though they are looking at it from a different perspective. Once all partners have had a turn they all say, "We see six."

The four (or three) partners stand up and rotate one place around the mat. The next partner collects the pompoms (in the bag or not) and drops them on the mat, again they fall randomly. They repeat the same procedure talking about what they see. Once they have all had a turn they say "We see six." Repeat these steps until all partners have had a turn to dump the pompoms.

Have the students clean up the materials and return to the whole group to discuss their findings. Ask open-ended questions. "What did you notice?" "Did you always have the same number of pompoms, the same quantity?" "Did you all see the quantity the same way?" "What do you think about that?" Guide the students' responses to the mathematical vocabulary: joining sets, same value. Record some of their expressions on chart paper for all to see.

Figure 2:

2 + 2 + 2

3 + 3

4 + 2

1 + 3 + 2

2 + 4

6

Be sure to use the language same value and equal during the discussion. By writing the students expressions on the chart paper, they will begin to connect the symbols with the sets with which they are working. So often, first grade students have practiced math facts at home, committing the symbols to memory, not really grasping what the symbols mean.

You can vary this activity for different ability groups by giving them different sets of pompoms. Students can play this as a center activity. They can record each others expressions on a white board as each partner says what they see. Then they compare the expressions on their white boards with one another. This is similar to an activity I remember from Math Trailblazers, “How to Make 5” (or any other value). The students used a chart and recorded the numerals as they used two-sided counters to “make 5.” So the first numerals would be 5 (red) and 0 (yellow), then they would flip one counter and now they would record 4 (red) and 1 (yellow). Flip another counter and then record 3 (red) and 2 (yellow), etc. They would then talk about the pattern how as the numerals on one side of the table decreased, the other side increased. And those were “the ways” to make 5. I found this limiting, in that the students would only ever work with two addends and while they were counting and comparing the number of red and yellow counters, and they could notice the pattern of one addend decreasing as it’s “partner” increased, they did not transfer this information to their problem solving.

Assessment

Are the students using the vocabulary correctly? Do they begin to connect the relationship to the different expressions. For example, can they see that 2 + 2 + 2 and 4 + 2 are similar (equal)?

Lesson #3 - True/False

When I was a child, I watched Sesame Street as much as today's children watch Spongebob, Square Pants. There was a song and game called "One of These Things"

"One of these things is not like the others.
One of these things doesn't belong.
Can you tell which thing is not like the others by the time I finish my song?"

Our district places resources on Sharepoint and one of those resources is called Elimination. It is a similar activity, where the students have to figure out which expression of four given does not make a true statement. The following activity takes both of these further as it begins with pictures and simple values (like those used in the Sesame Street game) and builds using the dot patterns found in Number Talks and finishes with expressions from the Elimination activity.

Enduring Understanding

When comparing sets, students need to be able to decide if a statement is true or false based on the properties of equality and the sets with which they are working.

Essential Question

How do I know if a mathematical statement is true or false?

Objective

To develop the idea that some statements are true and some are false. [We want to get students away from the idea of right/wrong answers as that way of thinking contributes to an operational view of equality.]

Materials and Setup

You will need a set of cards with pictures, dot patterns or expressions and a collection of 1" X 1" squares of construction paper for markers (or other material to place over the set that makes the statement false, and counter as needed.

I have created a collection of picture, dot pattern and expression cards. You can print them, cut them and I organized mine into envelopes for each number, then plastic zipper bags for each type (picture, dot pattern, or mathematical expressions). You can download these cards for free from Teachers Pay Teachers.

Procedure

This can be an individual center activity, a partner activity or even used as a formative assessment. The student takes an envelope from one of the set of cards and a square marker. They place one card in front of them, looking at all four sets (which are enclosed in circles) on the card they must determine which of the sets does not belong with the others. There is a numeral on the card that shows the value that each set should equal. There is one set on each card that clearly does not belong. Once the student finds the set that does not equal the numeral they place the paper marker over the set. The student takes another card from the same envelope and repeats the procedure until all six or eight cards have been used.

If working independently, the cards have the answers on the back, so the student can then look at each card to determine if they were correct. If working in pairs, the students can take turns with each card or work together on each card and tell each other why they think a set doesn't belong. They should check their work once all six or eight cards have been used. If using this as a formative assessment, have the student defend why they chose a particular set as not having the same value as the numeral on the card.

Closure/Assessment

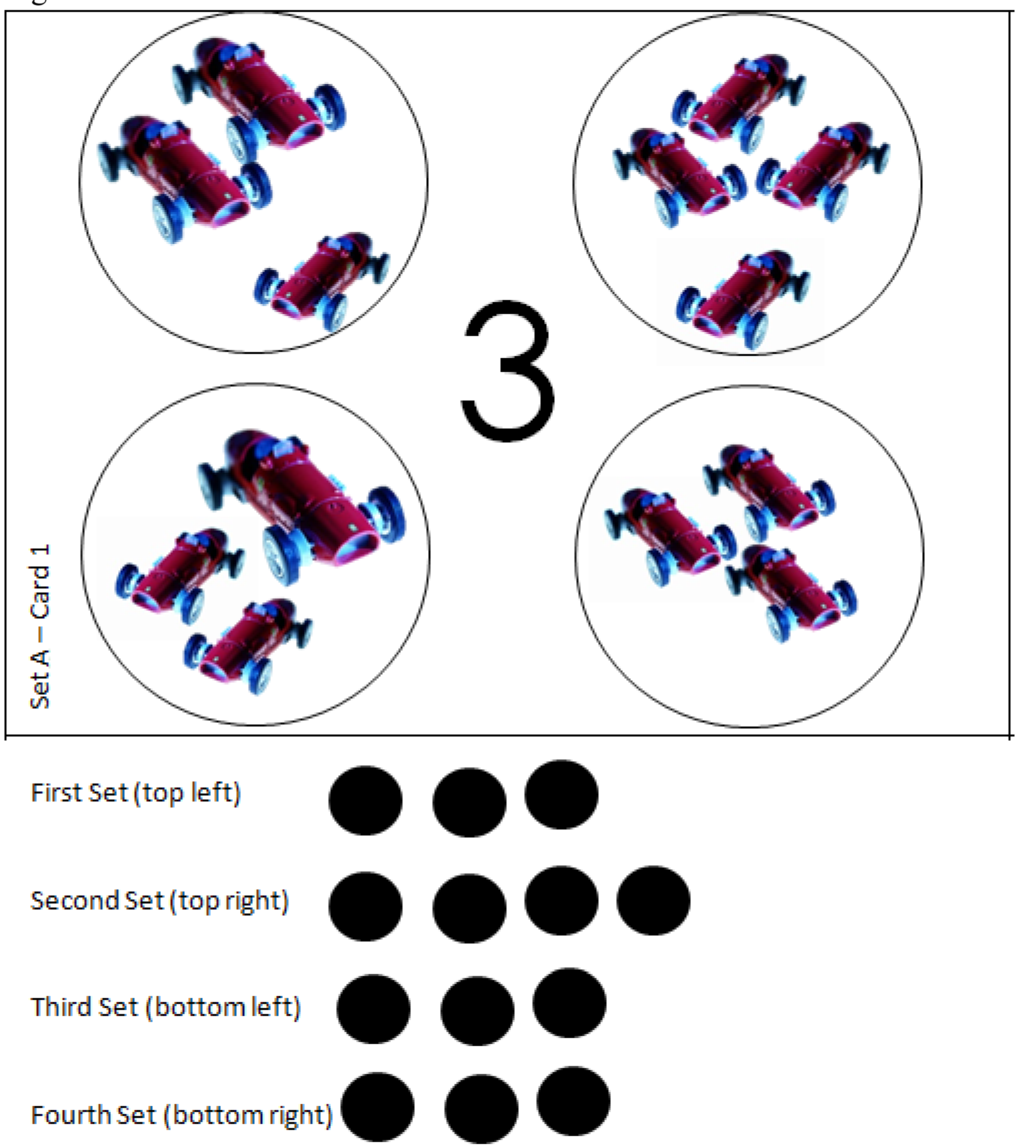
Students will begin to defend their understanding of what makes a statement true or false. When working with students who are using this activity, it will be purposeful for the teacher to use the language "when working with this set" and "makes this statement true (or false)."

Variation

You can use this as a game similar to the 24 Math game. Students can work as partners, placing the card between them and they tap the card once they see the set or expression that makes a false statement. Before they can claim the card, they must defend their choice stating why.

If a student has difficulty finding a set or expression that makes a false statement, they should be encouraged to act out each with counters (even if using pictures). A teacher can help a student develop one-to-one correspondence by placing a counter either over each picture, or on a table near each picture while counting. But the teacher can also begin very basically by having the student line up counters for each set (without counting, but rather matching) and then the student can see which set has more or less than the others.

Figure 3:



Lesson #4 - Using Mathematical Notation to Show Relationship

At the beginning of each first grade year, the students learn to compare quantities and they learn how to use the mathematical symbols for greater than and less than (>, <). To make this more "hands on," we make the symbols with popsicle sticks. It occurred to me that we could get more "bang for our buck" if we used these symbols in the following game.

Enduring Understanding

Relationships between sets can be illustrated using mathematical notation (symbols) and numerals.

Essential Question

How do I use mathematical symbols to show a relationship between two quantities?

Objective

This activity has students using mathematical symbols as they begin to make sense of how to show relationships between quantities.

Materials and Setup

After introducing the idea that numbers are used to represent sets of items and that sets can be compared to each other to find a relationship, we now introduce symbols to illustrate those relationships. I had the students make the symbols during a whole group lesson, but you could certainly have this part done as a center activity (with cards showing the steps) or you could have a volunteer work with small groups. See Appendix B for photographs of the signs.

You will need a 12 X 18 piece of manila construction paper, six craft sticks (color ones are best but the students can use markers to color them), a bottle of white glue for each group or pair, a pencil, a gallon sized plastic zipper bag, 15 multi-link or Unifix cubes to use as counters, number cubes (I have red and blue cubes from the Math Connects materials, but you could use a marker to color 1" wooden cubes red and blue and program numbers on them. The red have numbers 0 to 5, the blue have numbers 6 to 10), an 8 X 11.5 piece of paper to use as a workspace (mat), and two sets of number cards 0 to 15 (optional).

The student places the folded symbol papers, fifteen counting cubes, the "mat," and number cards in a zippered bag. Each time they play they can get a red number cube and a blue number cube from the math tools you have at hand.

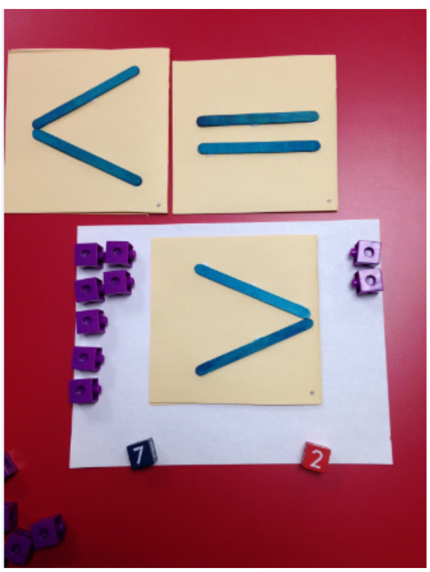
Procedure

Students can play this game independently or in pairs. They take the materials out of the bag and place the mat in front of them. They line up the three symbols (folded so they cannot see the words) above the mat. They should check the dots to be sure they have laid them out correctly. They should be sure that the cubes are separated but they can remain in the bag. They should have picked up the number cubes from the math tools center.

They roll each number cube and place each on the bottom corners of the mat (one in the left corner and the other in the right). It does not matter which number cube is place in which corner. They count out the number of cubes from the left number cube and they count out the number of cubes on the right number cube. Then they determine which symbol will make the inequality (or equality) true. They "read" the statement then check the words on the back of the symbol to see if they chose the correct symbol.

If playing as a pair, their partner can check the words to see if the statement was set up correctly. The students should be using the mathematical language as they play this game.

Figure 4:



Closure/Assessment

Do the students use the vocabulary correctly? Can they use the symbols correctly to explain the relationship between the sets? Do the students use the symbols correctly for smaller sets, but not for larger sets? Have the students connect the counting cubes to make "towers" and then compare them. Reteach the language if necessary.

Variations

Students can use two red number cubes or two blue number cubes depending on their ability/understanding. Students can use the number cards to set up the mathematical statement using numerals and symbols. Students can be given a worksheet to record the statements that they create. They can hand this in for assessment/accountability.

Lesson #5 - Problem Solving

Enduring Understanding

The numerals and symbols in mathematical statements are representations of real quantities and their relationships.

Essential Questions

How do I use mathematical symbols to show a relationship between two quantities? How do I know if a mathematical statement is true or false?

Objective

Students will be able to correctly represent real life situations using mathematical symbols.

Materials and Setup

Collect a list of several real life situations in which students are grouped or collections of things are grouped so that students must talk with each other about how to represent the joining, separating or comparing of these groups. It is important to use the names of the students in the problems to give them a sense of belonging (they are not just solving random problems but these are problems that they may need to solve for real reasons).

Procedure

This activity can be used for a whole group lesson on problem solving. The teacher can help students to figure out what information is given and what needs to be worked out. The teacher can guide students through finding a solution using counters or drawings, and ultimately toward using mathematical notation.

This can be a whole class activity where the class is divided into groups of four and given a piece of paper with the problem written (be sure there is a strong reader with each team, or parent volunteers/fifth grade buddies can help). The team has to solve the problem on a piece of chart paper and then once all the teams are given a set amount of time, they come together to share what they have discovered. This can generate discussion about problem solving. The students have to defend their solutions to others. They can see how their solutions compare with others. They can be guided to see different solutions, yet equal results.

This activity can be used in small groups, where the teacher tailors the problems to the ability of the group with which s/he is working. These problems can be used as an assessment to see where students' thinking is with regards to equality and the properties of equality.

Closure/Assessment

The teacher will hear the students vocabulary. The teacher is able to assess each student's level of understanding through the discussions. The students are able to show what they understand about mathematical notation as they use the symbols to illustrate the problems.

Appendix A

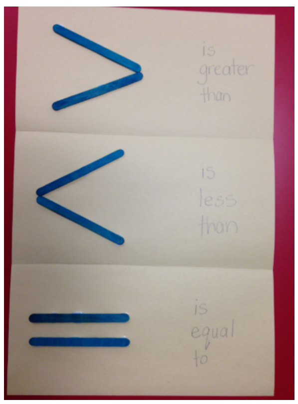
Vocabulary to be defined throughout lessons:

compare: to see how sets are alike
numeral: the symbol we use to show how many
relationship: how sets compare to each other
equal: having the same value
not equal: having different values
greater than: having more objects in a set than another set
less than: having fewer objects in a set than another set
set: a collection of objects
quantity: the number of objects in a set
number: how many objects in a set
value: the number assigned to a set after counting
mathematical expression: showing the sets of numbers being joined or compared using numerals and mathematical notation (i.e. plus sign, minus sign, greater than sign, less than sign)
mathematical statement: showing an equality between two mathematical expressions (using an equal sign).
equality: two sets having the same value
inequality: two sets having different values
true: an equal sign can be placed between two expressions after they have been compared and found to have the same value.
false: an equal sign cannot be placed between two expressions after they have been compared because they do not have the same value.

Appendix B

Show the students how to fold the paper in thirds on the long side. Show the students how to make a greater than sign using two craft sticks. They place dots of glue on each end of each stick and one dot in the middle of each. Then they place the sticks appropriately on the paper in the top third section. Repeat for the less than sign in the middle third of the paper and the equal sign in the bottom third section. All three symbols should be placed on the left side of the paper. Across from each sign, the students write the words to match the symbol "is greater than," "is less than," and "is equal to." Once these dry (the next day), the students cut on the fold lines and fold each piece in half so the words are behind the symbol (they cannot see them). Before folding though, the students should be guided to draw a small dot on the top left hand corner of each piece so they know which way the symbol should go as they use them.

Figure 5:



Appendix C

[Teacher's Name] has sharpened new pencils for your table group. If there are four students in the group, how many pencils does the teacher need to sharpen so that each student can have two? What if she sharpened seven pencils? ten? twelve?

[Child 1's Name] has two sisters. [Child 2's Name] has one brother and one sister. They say they have the same number of children in their families. Is that true?

[Teacher's Name] gave one student seventeen Goldfish for snack and then took five back. He gave a different student fifteen Goldfish and then took three back. Did he give both students the same number of Goldfish?

Our class was having "Bring a Toy to School Day." Two students brought toy cars. [Child 1's name] brought four red cars and six green cars. [Child 2's name] brought six green cars and four red cars. One of them thinks they have the same number of cars and the other says they don't. What do you think? Why do you think that?

It is [Child's Name] birthday and she brought 24 cupcakes to share. If she gave the boys 11 cupcakes, how many cupcakes will she have left? If there are eight girls, will she have enough? Will there be any leftover?

The Phys. Ed. teacher was preparing for Field Day. She has to have enough hula hoops for nine students to participate at the same time. If she has two hula hoops how will she group the nine students? What if she has three, four, or five hula hoops? How many hula hoops will she need so that all of the groups have an equal number of students?

Two children brought apple slices for the class party. [Name] brought seven slices and [Name] brought five slices. Which of the children brought more/less? If they combine the apple slices will they have enough for eight, nine, or ten students to have one slice? What is the largest group of students so that everyone can get one slice?

Our school has a safety patrol in which fifth grade students can participate. On Mondays and Tuesdays, there are eight boys and three girls. On Wednesdays there are six girls and five boys. On Thursdays and Fridays there are seven girls and four boys. Are there enough students each day for them to patrol fourteen stations? If not, how many more students are needed each day?

During reading group, [Teacher's Name] meets with two girls and four boys. During math group, [Teacher's name] meets with four girls and three boys. Does she have enough chairs at her table for all the students of each group?

[Teacher's Name] has fourteen dictionaries. S/he has twenty one students. How many students can work by themselves? How many students will have to share a dictionary? How many more dictionaries will [Teacher's Name] need to get so that everyone can have their own?

Resources

Math Connects (curriculum)
Math Trailblazers (curriculum)
Number Talks: Helping Children Build Mental Math and Computation Strategies, Grades K 5, Updated with Common Core Connections, by Sherry Parrish

Ten Black Dots by Donald Crews
Domino Addition by Lynette Long
More, Fewer, Less by Tana Hoban
Pair of Socks by Stuart J. Murphy
Quack and Count by Keith Baker
Subtraction Action by Loreen Leedy

Annotated Bibliography

Hattikudur, Shanta, and Martha W. Alibali. "Learning about the equal sign: Does comparing with inequality symbols help?" *Journal of Experimental Child Psychology* 107, no. 1 (2010): 15-30.

Karp, Karen S., Bush, Sarah B., and Dougherty, Barbara J.. "13 Rules That Expire" *Teaching children mathematics* 21, no. 1 (2014): 19-25.

Knuth, Eric J., Ana C. Stephens, Nicole M. McNeil, and Martha W. Alibali. "Does Understanding the Equal Sign Matter? Evidence from Solving Equations." *Journal for Research in Mathematics Education* 37, no. 4 (2006): 297-312. http://delcat.worldcat.org/search?q=Does+Understanding+the+Equal+Sign+Matter%3F++Evidence+from+Solving+Equations&fq= (accessed July 4, 2014).
This article was very difficult to find without the help of the resource librarian at the University of Delaware. The article is very interesting as the hypothesis posed was disproved.

Molina, Concepcion. *The problem with math is English a language-focused approach to helping all students develop a deeper understanding of mathematics.* San Francisco: Jossey-Bass, 2012. This book touched on quite a few of the misconceptions that teachers inadvertently create through the use of "simpler" language when teaching mathematical concepts to young children.

O'Connell, Susan, and John SanGiovanni. *Putting the practices into action: implementing the common core standards for mathematical practice, K-8*. Portsmouth: Heinemann, 2013.
We read Chapter 3: Exploring Standard 2: Reason Abstractly and Quantitatively during a school professional development meeting. The team of teachers leading the professional development brought it back from a workshop they had attended in order to learn more about implementing the Common Core Math standards. It gives MANY ready-to-use ideas and activities while providing the teacher with the research to know why they are useful ideas and activities.

Schliemann, AnaluÌ cia Dias, and David William Carraher. *Bringing out the algebraic character of arithmetic: from children's ideas to classroom practice*. Mahwah, N.J.: Lawrence Erlbaum, 2007.

Sophian, Catherine. *The origins of mathematical knowledge in childhood*. New York: Lawrence Erlbaum Associates, 2007.

Stephens, Ana C.. "Equivalence and relational thinking: preservice elementary teachers' awareness of opportunities and misconceptions." *Journal of Mathematics Teacher Education* 9, no. 3 (2006): 249-278.

Notes

[1]Schliemann and Carraher, *Bringing out the algebraic character of arithmetic: from children's ideas to classroom practice*. (Mahway, NJ, 2007), .

[2]Molina, *The problem with math is English a language-focused approach to helping all students develop a deeper understanding of mathematics.* (San Francisco, 2012), .

[3]Stephens, *Journal of Mathematics Teacher Education* 9, no. 3 (2006): 249-278.

[4]Molina, 2012.

[5]Stephens, 2006.

[6]Knuth, Stephens, McNeil, and Alibali. "Does Understanding the Equal Sign Matter? Evidence from Solving Equations." *Journal for Research in Mathematics Education* 37, no. 4 (2006): 297-312. http://delcat.worldcat.org/search?q=Does+Understanding+the+Equal+Sign+Matter%3F++Evidence+from+Solving+Equations&fq= (accessed July 4, 2014).

[7]Stephens, 2006.

[8]Stephens, 2006.

[9]O'Connell, Susan, and John SanGiovanni. *Putting the practices into action: implementing the common core standards for mathematical practice, K-8*. Portsmouth: Heinemann, 2013.

[10]Schliemann and Carraher, 2007.

Curriculum Unit Title: Teaching What "Equals" Means to First Graders

Author: Janet B. Zegna

KEY LEARNING, ENDURING UNDERSTANDING, ETC.

When we compare two (or more) quantities, they are considered equal if their values are the same. Additionally, the comparison is considered an inequality if the values are different. We show equality mathematically when we have two expressions on either side of an equals sign. It is important for the students to understand that equals DOES NOT mean the same, and that the expressions on either side of the equal sign are usually different.

ESSENTIAL QUESTION(S) for the UNIT

Do you understand the meaning of the equal sign?
Can you determine if statements involving addition and subtraction are true or false?

CONCEPT A	CONCEPT B	CONCEPT C
Comparing Quantitites	Which Statements are True/False?	Using Mathematical Symbols
ESSENTIAL QUESTIONS A	**ESSENTIAL QUESTIONS B**	**ESSENTIAL QUESTIONS C**
How do I know when quantities have the same value?	How do I know if a mathematical statement is true or false?	How do I use mathematical symbols to show a relationship between two quantities, whether equal or not equal.
VOCABULARY A	**VOCABULARY B**	**VOCABULARY C**
compare, set, number value, numeral, quantity less than, greater than, equal to not equal to	mathematical statement true false	symbol, < +/-, > =

ADDITIONAL INFORMATION/MATERIAL/TEXT/FILM/RESOURCES