Addressing Misconceptions when Solving Multi-Step Linear Equations

Kathleen Fagan

I have taught how to solve multi-step linear equations to 9th graders several times as well as reinforced the concept in Academic Support Math to students who typically struggle with mathematics. During my experience teaching, I have noticed that while some students have a natural inclination to solving equations, many students struggle with certain mechanics of the process and make similar mistakes. Most students are able to grasp two step equations in the form of $ax \pm b = c$, where a, b, and c are real numbers; however, when more complex forms of multi-step equations are encountered, many of my students have gotten overwhelmed, confused, and frustrated when trying to isolate the variable in question. The unit I am creating for Delaware Teacher's Institute has a focus of solving multi-step linear equations and addressing the mistakes that students typically make when developing the skills needed to successfully solve multi-step linear equations. I would like to combat the typical errors made when solving equations so that students may be successful in their math classes, with the assumption that the conceptual understanding for solving multi-step equations is essential to most high school math courses and lends itself to many other topics within mathematics.

School and Classroom Background

I am a special education mathematics teacher at a public high school in Wilmington, Delaware. Approximately two thirds of the population at my school qualifies for free and reduced lunch. Also, there is a greater percentage of students with disabilities attending the school where I teach compared to other public schools in Delaware. One challenge that I face with my 9th grade students, aside from dealing with the transition from middle school to high school, is that my students come from many different middle schools and thus have varying levels of mathematical confidence and mathematical backgrounds. I will have to carefully assess each student's individual prior knowledge of solving equations and make ensure that every student has the appropriate mathematical background knowledge in order to successfully solve multi-step equations while offering extension activities to those who already have a solid prior knowledge.

The course I am teaching is Interactive Mathematics Program Year 1 to classes that consist exclusively of students with disabilities. This textbook can be described as an integrated mathematics program consisting of mainly Algebra 1 and Geometry topics as well as a brief introduction to Statistics. This course is assigned to the majority of 9th graders as their first mathematics course. The unit that presents multi-step equations is third unit in Year 1 and is entitled “Overland Trail”. This unit looks at mid-19th century Western migration in terms of the many linear relationships involved[1]. In this unit, students are introduced to the concept of using variables to represent unknown values and work towards creating equations for math story problems which represent linear equations. Students are assigned approximately four activities that provide an introduction to solving equations. These activities use manipulatives and the balance scale method to help students visualize the conceptual theory behind solving equations. The unit does not provide direct instruction on solving equations, and I feel the unit does not give the students enough opportunities to practice solving equations. I feel these instructional techniques are important to include when teaching students with disabilities; therefore, I am planning on teaching the unit I have created after the activities from the textbook that introduce solving equations in order to scaffold and solidify the concept of equation. As well, I typically make graphic organizers for each activity so that students may benefit from prioritizing the information and chunking the material in such a way that my students have easier access to the material.

Students should feel comfortable solving two step equations prior to my unit. Students also should understand the concept of like terms and have a basic knowledge of how to combine like terms (also called similar terms). Students also need to have an understanding of how a coefficient relates to the variable it is attached to. For example, students must understand that 3x represents “3 multiplied with x” or “x + x + x”.

Objectives

I will have three main objectives for the unit. First off, I want students to become familiar with solving equations that require more than two steps. When students are presented with an equation that requires more than two steps within the Overland Trail unit, often they do not have any idea what the initial step for solving this equation would be, nor do they have any idea the appropriate steps to take in order to solve the equation or what order to perform these steps. I will provide students with some direct instruction for solving multi-step equations using the balance scale method and model several examples for them. Together, we will identify a rough outline of basic steps to take for solving equations. The next step I will take in order for students to become familiar with the process of solving multi-step equations is to be able to identify the different types of multi-step equations. I plan on providing students with many examples of multi-step equations and we will categorize the type of equation such as including: distribution, variables on both sides, variables on the same side that need to be combined, etc. We will discuss strategies for solving each type of multi-step equation and the steps to take for solving each type. I also feel that it will be important to address that it is not necessary to follow these steps in exact order and that different approaches will be allowed instead of the methods presented, as long as we are ensuring fidelity to the properties of equality. By taking the time to deconstruct the multi-step equation by becoming familiar with the different steps to take for solving equations as well as identifying the type of multistep equation that is being presented in order to identify strategies for solving specific types of equations, students will become more comfortable and confident when presented with a complex multi-step equation.

My second objective will be that students will be able to recognize that each term within the equation has either a positive or a negative coefficient which will impact how we solve equations. I have found that one of the main mistakes students make when solving equations is that they don't realize when working with the equation that a negative number or a number being subtracted from a term affects the overall outcome when manipulating equations. For example, how does a negative number in front of a set of parentheses change the values of the terms inside when distributing compared to a positive number? As well, when combining two terms, it is important to identify whether each term has a positive or a negative coefficient so that an error is not made when combining the terms. Also, I would like my students to become comfortable with the difference between solving an equation that involves a variable term with a negative coefficient such as $-3x = 12$. I have noticed many of my students tend to have no issues solving an equation $3x = 12$ by dividing both sides by three, but many students will try to add three to both sides when solving $-3x = 12$. Also, be emphasizing that (-3) is multiplied with x to give us 12, and writing the negative coefficient using parentheses might reinforce the operations involved in the equation $(-3)*x = 12$. I will have to do some different activities to address this issue. By having students work with problems that include negative terms within the equation, taking care to monitor individual student's progress toward solving the equation, and viewing the differences in how to solve the equation when the term has a negative coefficient compared to when the term has a positive I will be able to avoid these common errors students make when it comes to working with terms with negative coefficients within equations.

My last objective is to ensure that students understand the difference between combining like terms on the same side of an equation compared to combining terms on different sides of the equation. There have been countless times when I have observed a student working on an equation that has variables on the same side and seen him/her attempt to combine the variable terms incorrectly. For example, when presented with the equation $3x – 6x + 14 = 32$, many students attempt to add 6x underneath the -6x as well as the 3x. To assist students with this issue, I will encourage students to check their solution with the equation. I also plan on presenting several examples of student work that includes mistakes and having students explain the error and give the correct solution. Students will be able to construct a viable argument to justify a solution method in this way.

Connection to the Common Core

In the state of Delaware, we utilize the Common Core State Standards, which reinforce the concepts that I will use in my unit. Many of the standards directly connect to my objectives. My focus throughout the unit are that standards that state students should "solve linear equations and inequalities in one variable, including equations with coefficients represented by letters[2]", as well as "explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution.[3]" Lastly, students should be able to "construct a viable argument to justify a solution method.[4]" I will make sure to keep these standards in my mind when designing the activities of my unit.

Content

My unit covers the key concepts for a basic understanding solving multi-step equations while addressing some common misconceptions students have when learning how to solve multi-step equations. The unit will cover the process for solving equations that require more than two steps, with a focus on identifying whether each term has a positive or negative coefficient and how that affects the process of solving equations, as well as addressing how the location of the variables within the equation affects the process for solving the equation. The unit starts with an introduction to the steps for solving multi-step equations. When manipulating equations, there are a few important properties regarding real numbers and the equality relations that students need to understand.

Equality

When discussing equations, it is important to define what an equation is. An equation is assertion that two different expressions represent the same value. For example, if I was to state that 60 seconds = 1 minute, I would be implying that these two expressions have the same overall meaning. We use an equality sign to represent this relationship. In other words, equations are based on the properties of equality as an equivalence relation. When we state that two expressions are equivalent by using an equality symbol, we assume that the properties of reflexive, symmetric, and transitive property of the equality relation hold true.

Reflexive Property

The reflexive property states that for every real number x, x = x. This may already be obvious to most students; however, it is good to keep this concept in mind when solving equations and when checking the solutions. For example if I know that 17x + 27 = 45, I know that 17x + 27 represents the value of 45, which allows me to begin figuring out what x must be if 17x + 27 is 45. Also, if x = 18/17 is the solution, then 17*(18/17) + 27 = 45 is a true statement.

Symmetric Property

The symmetric property states that for any real numbers x and y, if x = y, then y = x. The symmetric property is important for students to understand so that equations can be reorganized in a way that makes the most sense to the student. For example, some students are uncomfortable solving the equation 35 = 5x + 10, but using the symmetric property to reorganize the equation so that it looks like 5x + 10 = 35 may be a simple way for students to visualize an equation in a way they recognize the steps to take to solve the equation.

Transitive Property

The transitive property states that for all real numbers x, y, and z if x = y, and y = z, then x = z. Again, if students understand the transitive property, it will be easier for them to justify their solutions. For example, if a student knows that 3x + 7 = 10, and they also know that 10 = 3 + 7, then using the transitive property students also know that 3x + 7 = 3 + 7, which may help them understand that x must be equal to one. The transitive property allows students to make connections between two or more logical statements that have the same relation and to simplify equations.

The reflexive, symmetric, and transitive properties are important to understand when solving equations as well as justifying the steps that students will take when solving equations. If a relation is missing one of these properties, then the relation is not an equivalence relation. In other words, if a student has an equivalence statement (equation) then the student can also assume that the reflexive, symmetric, and transitive properties apply when simplifying equations, justifying the steps, finding solutions, and drawing conclusions.

Substitution

For the real numbers A and B that represent the two sides of an equation, if A = B, then A and B have the same value that can be obtained by substituting the solution into the expressions given by the two sides of the given equation. This notion is important in solving equations for students in order to check their solution and justify their answers. For example, when presented with the problem $4(2a + 3) = -3(a - 1) + 31$ a student isolated the variable and found out that a = 2. Substituting the value back into the equation, $4[2(2) + 3] = -3[(2) - 1] + 31$, and simplifying the equation would give the value of 28 for each side. Because one can substitute the solution 2 into the equation in place of *a* and get the same value for each side, the student can justify his/her answer by checking the truth value of the statement he/she obtains.

Properties of Equality which Allow Balancing and Solving Equations

In order to be able to manipulate equations so we can isolate the variable, we must first understand properties of equality. The properties of the addition, subtraction, multiplication, and division operations with real numbers, together with the properties of the equality relation allow us to operate on both sides of the equation without changing the equivalence relation between the two sides of the equation. In other words, if I add, subtract, multiply, or divide (with the exception of dividing by zero) the same number or term to both sides of an equation, I still have a statement that both sides of the equation still have the same meaning or value. For example, when altering an equation in one of the ways listed below, if one side of the equation has an overall value of 25, the other side of the equation will also have an overall value of 25. These properties are important because they allow us to simplify equations in such a way that we can solve for unknown values.

The addition property of equality states that if you add the same number to both sides of an equation, the sides remain equal and the equality property of the equation remains true. In other words, for any numbers a, b, and c if a = b, then a + c = b + c. So if I add the same value to both sides of an equation, I am not changing the fact that both sides represent the same value. This is easy enough to explore with students by looking at equalities without variables. If I had an equation that stated 2 + 3 = 4 + 1, the overall value represented by each side of the equation is 5. Now if I add 10 to each side of the equation, I would then get 10 + 2 + 3 = 10 + 4 + 1. Now both sides of the equation still represent the same value, but it this case the value the equation represents is now 15.

Similar to the addition property of equality, there is subtraction property of equality. The subtraction property of equality states that if we subtract the same number from both sides of the equation, the sides remain equal and the equality property of the equation remains true. Again for any real number a, b, and c if a = b, then a – c = b – c. If I show a few examples of equations using actual numbers, students can more easily understand the subtraction property of equation. For example, if I had an equation that stated 10 = 6 + 4 and then subtracted 3 from each side, my new equation would look like this: 10 – 3 = 6 + 4 – 3. In this case, both sides of the equation still represent the same value, but in the new equation the value each side of the equation is 7 now.

The multiplication property of equality is similar to the above properties of equality. If two expressions are equal to each other and multiply the exact same, then the two new sides will remain equal. If I choose any real numbers a, b, and c and a = b, then a*c = b*c. Using the real numbers 2*3 = 6, if I multiply both sides by 4, my new equation would then look like 4*2*3 = 4*6. Now both sides are still equal to each other, but the new value is 24.

The division property of equality is similar. The division property of equality states that if you divide both sides of an equation by the same nonzero number, then the sides remain equal. In other words, for real numbers x, y, and z (such that z is not zero), if x = y, then x/z = y/z. I can show this to students easily enough by again using real values. If we look at the equation 12 = 12, if I divide both sides of the equation by 6, my new equation looks like 12 ÷ 6 = 12 ÷ 6. Now each side of the equation represents 2, so the equality property still holds true. One way to understand that the division property does not hold when the division is by zero, is to think of the multiplicative inverse. For example, I know that 24 ÷ 8 = 3 because 3 * 8 = 24. In the same sense, when trying to figure out what the value of 24 ÷ 0 is, I can ask myself zero multiplied by what number equals twenty four. We know that zero multiplied by any number is zero; therefore, there does not exist a number such that when I multiply it by zero the answer would be twenty four. Thus, the division property of equality does not hold when dividing by zero.

By utilizing the addition, subtraction, multiplication, and division properties of equality, students make simplify equations in such a way that they can solve for an unknown value. These properties will also assist students with constructing a viable argument to justify a solution method.

Properties that Allow for Simplifying Equations

The associative, commutative, and distributive properties can also be helpful when solving equations. These properties allow us to see equations in a different light that may allow us to simplify and therefore understand the value of the variable in question.

Associative Property

The associative property refers to the way an expression is grouped. The addition or multiplication of a set of numbers is the same regardless of how the numbers are grouped. This property does not hold for subtraction or division. Basically, the associative property holds when moving parentheses around in an expression does not affect the value of the expression. The addition property states that for any real numbers a, b, and c (a + b) + c = a + (b + c). For example, (2 + 9) + 12 = 2 + (9 + 12). In this equation, both sides of the equation represent a value of 23, no matter what order you add the numbers together. Similarly, the associative property works for multiplication. For example, with the equation (3 * 4) * 5 = 3 * (4 * 5), both sides represent 60. By understanding the associative property, students may find it easier to deal with adding (or multiplying) certain terms first before other terms; thus, students may find it easier to work with certain equations by utilizing the associative property.

Commutative Property

The commutative property refers to the way expressions are reorganized or moved around without changing the value of the expression. The addition or multiplication of a set of numbers is the same regardless of how the numbers are ordered. This property also does not hold for subtraction or addition. The commutative property for addition can be stated as such: for any real numbers a and b, a + b = b + a. Again, showing this to students may be simplest by using actual numbers. For example, 1 + 3 = 3 + 1. This property may be beneficial to students when working on multi-step equations. If a student is unsure of how to solve the equation 18 + 4x = 76, by using the commutative property to rearrange the equation, a student may be more comfortable with the equation if it was visually represented as 4x + 18 = 76 and understand the steps for solving an equation when it is viewed in this way.

The commutative property referring to multiplication states for any real numbers a and b, a * b = b * a. If a student comes across a problem that requires multiplying 4x * 5, using the commutative property, the student may be more comfortable having the expression represented as 5 * 4x. Understanding that values can be arranged in this way without changing the value of the expression will assist students in solving equations with fluency.

The Distributive Property of Multiplication over Addition

The distribution property of a factor states that $a(b + c) = a * b + a * c$ where a, b, and c are any real numbers. Using actual numbers as an example for students, one could state that $7(2 + 10) = 7 * 2 + 7 * 10$. If I simplified this equation, I could say that $7*(2 + 10)$ is really equivalent to $7*12$, or 84. In other words, each side of the equation represents the number 84 and the expressions are equivalent.

Distribution of a factor is a concept students will need to utilize when solving multi-step equations, as some of the equations students will encounter will have distribution within the problem. I will spend some time going over distribution in regards to positive and negative terms. Students will need to spend additional time practicing distribution when the various terms have negative coefficients, as I have noticed students have had difficulty with this. One activity may include having students color-code these problems, possibly coloring or highlighting the negative terms red and the positive terms blue. By having students practice distribution problems with this color coding method, it may make the sign of the term more salient to students so that they remember to keep the sign of the term included when performing operations with the terms. Another method to make the sign of the terms more conspicuous is to have students circle each term with its sign in order to encourage students to consider the sign when manipulating equations.

All of these properties are essential for manipulating equations in order to isolate a variable. By isolating the variable in an equation, we can identify the value of the variable and thus find the solution to the equation of the value of the variable which makes the equation valid or true. Students who have a grasp on these properties should have an easier time solving equations that involve more than two steps. This information should be communicated to students before this unit is presented and reinforced throughout the unit with different activities and examples.

Inverse Operations

Inverse operations are opposite operations that undo each other. Addition and subtraction are inverse operations. Multiplication and division are inverse operations. Using inverse operations is an effective strategy to solve equations. Overall, the goal of solving an equation is to isolate a variable. In order to do that, we must reverse the operations acting on the variable. We do this by applying the inverse of each operation on both sides of the equation.

For example, when presented with the problem $6x – 7 = 41$. The operations that acting on the variable are that the x is being multiplied by six and then having 7 subtracted from in. Because we are working backwards and doing the opposite operation to isolate the variable, we must also work backwards in terms of the order of operations. Normally we would perform multiplication and division before addition and subtraction according to the order of operations, but because we are working backwards, the first thing we need to use the inverse operation on is the subtraction. To undo or "fix" the variable term being subtracted by 7, we want to add 7 to both sides of the equation. Now the equation becomes 6x = 48. Lastly, we address the six being multiplied with the x. Because division is the inverse of multiplication, we shall divide both sides of the equation by 6. Our equation now becomes x = 8. We can substitute to check and justify our answer, 6(8) – 7 does in fact equal 41 so we know we performed the inverse operations correctly.

Set Theory

Set theory is a branch of mathematical logic that involves sets. Sets are a collection of objects. For the purpose of this unit, students will be working with the set of real numbers. Real numbers can be thought of as numbers that can be found on the number line. Real numbers include rational as well as irrational numbers. A rational number is a number that can be expressed as a fraction or ratio. Some examples of rational numbers are $\frac{1}{2}$, $0.\overline{6}$, or 7. An irrational number cannot be expressed as a fraction or ratio. I like to think of irrational numbers as numbers that cannot be thought of as terminating or repeating decimals. Some examples of irrational numbers are π , $\sqrt{3}$, or e.

Students will first begin to work with natural numbers. Natural numbers can be described as positive whole numbers. Students typically feel most comfortable with these numbers, so while learning the process of solving multi-step equations, we will begin working with natural numbers only.

As students become more comfortable with each step in the process of solving equations, we will begin to work with the set of integers. Integers are the set of positive and negative whole numbers including zero. While we transition from natural numbers to integers, we will discuss the differences in strategies for solving equations that only use natural numbers compared to working with equations that use integers. During this point in the unit, we will spend time working with identifying whether each term is positive or negative to make sure that students are correctly operating on these terms according to the signs of each term.

Once students are finally comfortable with working with positive and negative whole numbers, we will move into the set of rational numbers. This includes addressing multi-step equations that include fractions and terminating decimals. This is typically an area where students feel uncomfortable and tend to avoid when possible. Having students work with all types of rational numbers will force them to really comprehend the process of solving multi-step equations.

Ideally, students will eventually be able to solve multi-step equations that include irrational numbers as well. While I would like my students to be competent in solving multi-step equations involving irrational numbers, I do not believe they will get to that level this current school year. Since I am teaching solely students with disabilities, I will focus my efforts on making sure students are comfortable working with natural numbers and integers when solving equations.

Strategies

The strategies that will be utilized in this unit will include: think pair share, web-based games that involve solving equations with a balance scale, direct instruction, scaffolding, guided practice, independent practice, games that reinforce the process of solving equations, graphic organizers, "I have, who has" technique, physically solving linear equations[5], and methods such as pictorial representations, abstract or symbolic representation[6].

Activities

I have listed a few activities as a resource to solving complicated linear equations below. I have also added thoughts for modifications for each activity to accommodate diverse learners and ideas for adjusting the activities in order to best serve the population being taught. Please feel free to adjust the activities so they work best for you and your population of students.

Kinesthetic Solving Multi-step Equations

Students each need a whiteboard and marker. There needs to be a pile of flashcards with a single multi-step equation written on each card. The student who is the leader will choose a multi-step equation from the flashcards, assign the other students a term to write on his/her whiteboard, and arrange the other students to form the multi-step equation. The leader student will then go through the process of solving the equation by some process of reasoning (subtraction property of equality, inverse operation to addition, etc.). The lead student can tell each "term" what to do in order to isolate the variable and thereby find the solution to the equation. The lead student can record the steps taken on his/her own whiteboard to track the process of solving the equation. Students can take turns being the leader in the activity and you can encourage movement or specific kinesthetic moves to represent different types of operations performed to the equations.

This activity can be modified so that students use a balance scale model to represent the equation instead of assigning each student a literal term (for example, if a student had the term assigned "2x" the student could write "2x", or "x + x". If the equation had a positive three, the student could write "+3", or "1 + 1 + 1" or draw 3 lines in a color that is designated as positive.

Sorting Equations and Dissecting Strategies

Students work in small groups and will be given a set of flash cards that have a single multi-step equation written on each card. Students will go through the flashcards and sort them according to different patterns or similarities they find. While the teacher may want to allow time for open-ended sorting, students may eventually need to be prompted to think of similarities in how the equations would be solved during the sorting process. Students will then discuss a specific strategy to be used for solving each category and make a poster for each strategy that explains the technique that is helpful for solving the equation with a solved example. Students may want to consider using some type of graphic organizer on their poster, such as a flow chart. Some possible category options could be equations that require students to: combine like terms on the same side of the equation, combine like terms that are on different sides of the equation, an equation that contains distribution without like terms, or an equation that requires distribution and combining like terms.

This activity can be modified to be taught in a jig-saw method where the whole class works together to sort the equations and then the class breaks off into small groups where each group is assigned a specific strategy to make one poster. Students can display the posters around the room and at the end of the activity students may participate in a gallery walk to look at other classmates' strategies. A possible option for an exit ticket would be to assign students an equation from the flashcards to solve and use the posters as a guide to do so. Groups could also be assigned a strategy to present in front of the class as a method of share-out. It is also possible to have one person nominated as the "teacher" from each group who explains the poster to other groups while the rest of the group rotates around the room to be taught by the other "teachers" from each group.

Recognizing Terms as Having Positive or Negative Coefficients

Students will need two different colored pencils. Students will use a set of flash cards that have a single multi-step equation written on each card. Students will pick an equation to color-code and identify which terms have a negative coefficient by outlining, shading, or writing the term in the color that is designated as "negative" and identify which terms have a positive coefficient by coloring these terms with the positive color. Students will go through the process of solving the equation, color-coding the terms as they work through the activity.

This activity can be performed individually or in small groups. If this activity is performed in pairs, assign one student to be in charge of terms with positive coefficients and one student to be in charge of terms with negative coefficients. Students must work together in order to solve the equation.

Flash Cards Resources

I have included in my bibliography two resources for generating equations for flashcards, from worksheetworks.com and www.math.com/students/worksheet.

Some examples of what these flashcards may look like are shown below. The equation is listed on the left hand side, while the solution is listed on the right hand side of the table created. When utilizing flashcards like these, simply cut horizontally across each equation and then fold the flashcard in half so that the answer is listed on the back. Alternatively, one could use the answer part of the card separately from the equation itself, allowing for an "I have", "who has" activity during which students must match their equation flashcard to the correct answer flashcard and increases interaction between students.

$3x + 2 - 7x = 2$	$x = 0$
$8 + 4x = -6x - 42$	$x = 11$
$x - 1 = -2x + 8$	$x = 3$
$33 = -3x + 2(-2x - 15)$	$x = -9$
$6x + 2(3x + 16) = 128$	$x = 8$
$-2x + 7(2x - 8) = 52$	$x = 9$
$6x - 5(3x + 5) = 47$	$x = -8$
$7x + 5(-x - 8) = -28$	$x = -6$
$-3x + 3(3x + 5) = 75$	$x = 10$
$7x + 7(x + 14) = 196$	$x = 7$

Annotated Bibliography

"A Balancing Act: Solving Multi-Step Equations." nsa.gov/academia. January 1, 2009. Accessed December 19, 2014.

This resource contains concepts of the distributive property and combining like terms and uses manipulatives as well as cooperative learning strategies in order to enhance students understanding of solving multi-step equations. The last activity has students create and solve equations with a solution by using a cross-word template.

Austin, Joe Dan, and H.-J. Vollrath. "Representing, Solving, and Using Algebraic Equations." *The Mathematics Teacher* 82, no. 8 (1989): 608-12. Accessed September 9, 2014. http://www.jstor.org/stable/27966416.

This text describes the pan balance method, pictorial representations, and discusses abstract versus symbolic representations. The text concludes with applications of equations so students understand how fundamental equations are to problem solving.

Fendel, Daniel M., and Diane Resek. *Interactive mathematics program: integrated high school mathematics, year 1*. 2nd ed. Emeryville, Calif.: Key Curriculum Press, 2009. Print.

This text is the curriculum currently used in my school. It contains problem based learning activities that accentuate group interaction in order to explore concepts and discover skills without being directly instructed on the topics addressed.

Griffin, Ms. "Multi-Step Equations." Quizlet. October 1, 2009. Accessed November 1, 2014. http://quizlet.com/1416480/multi-step-equations-flash-cards/

This website is a resource that has 20 multi-step equations for students to quiz themselves on multi-step equations.

Kinach, Barbara. "Solving Linear Equations Physically." *The Mathematics Teacher* 78.6 (1985): 427 - 430, 445 - 447. *JSTOR*. Web. 29 Sept. 2014.

This text describes in great detail the balance method for teaching how to solve equations and goes through several examples with the use of a physical and pictorial method which will assist students with solving equations at a symbolic level.

"Math Worksheets 4 Kids." Free Algebra Worksheets. January 1, 2014. Accessed November 1, 2014. http://www.mathworksheets4kids.com/algebra.html.

This website has practice questions with different difficulty levels.

"Math.com Algebra Worksheet Generator." Algebra Worksheet Generator. January 1, 2005. Accessed December 19, 2014. http://www.math.com/students/worksheet /algebra_sp.htm.

This resource can be used to generate multi-step equations and number of equations by category can be chosen as well as choosing to include negatives within the equations.

"Mathematics Standards." Mathematics Standards. March 1, 2010. Accessed January 11, 2015. http://www.corestandards.org/Math/HSA/REI.

This resource lists the common core standards for high school mathematics that are the focus on this unit.

"Mixed Problem Types: Solving Multi-step Equations." Solving Multi-Step Equations. January 1, 2002. Accessed December 19, 2014. http://www.worksheetworks.com/math /pre-algebra/equations-mixed.html

This website is great for creating mathematical worksheets tailored to a teacher's needs.

"3.7: Solve Equations with the Distributive Property and Combining Like Terms." Solve Equations with the Distributive Property and Combining Like Terms. January 1, 2014. Accessed December 19, 2014. http://www.ck12.org/book/CK-12-Middle-School-Math-Concepts-Grade-8/r14/section/3.7/.

This resource has direct instruction, solved problems as examples, a tutorial video, as well as practice problems for students to work on.

Notes

[1] (Fendel and Resek, 2009) p. 175 - 211
[2] "Mathematics Standards"
[3] "Mathematics Standards"
[4] "Mathematics Standards"
[5] (Kinach 1985) p. 427 – 430, 445 – 447
[6] (Austin and Vollrath) p. 608 - 612

Curriculum Unit Title

Addressing Misconceptions

when Solving Multi-Step Linear Equations

Author Kathleen Fagan

KEY LEARNING, ENDURING UNDERSTANDING, ETC.

Students will solve complex multi-step linear equations with various methods (balance scale, pictorial, kinesthetic, etc.) by working carefully to recognize common errors and avoid misconceptions.

ESSENTIAL QUESTION(S) for the UNIT

How do I solve multi-step linear equations that involve more than two steps?

CONCEPT A

Solving Equations Requiring More than 2 Steps

CONCEPT B

Identifying the coefficient of each term as positive or negative in the equation

CONCEPT C

Solving Equations with Like Terms on the Same Side vs. Different Side of the Equation

ESSENTIAL QUESTIONS A

What are the strategies used for solving equations with more than two steps involved? How do the properties of equalities and operations with real numbers help us to solve equations?

ESSENTIAL QUESTIONS B

How does the coefficient of a term affect how I solve an equation?

ESSENTIAL QUESTIONS C

What is the difference in strategy when solving the equation where there are like terms on one side of the equation in comparison to like terms on opposite sides of the equation?

VOCABULARY A

equality, properties of equality, reflexive property, distributive property, transitive property, substitution, inverse operations, set theory, commutative, associative, and transitive properties

VOCABULARY B

coefficient, combine, positive, term, negative, like terms, expression, combine like terms

VOCABULARY C

opposite, transfer, sides of an equality, like terms

ADDITIONAL INFORMATION/MATERIAL/TEXT/FILM/RESOURCES

Flashcards with multi-step equations,

student whiteboards, markers, manipulatives,

and possibly use of websites listed in annotated bibliography section