Moving Students from Conjecture to Proof

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Overview

This unit presents a series of activities that are intended to promote mathematical habits of mind, encouraging students to autonomously search for the deeper relationships that arise from their inductive reasoning and experimentation. The activities involve numeric and algebraic reasoning as well as geometric reasoning situations.

The students who will be taught this unit are generally in the 11th or 12th grade and have been instructed using Core Plus integrated math materials.[i] The activities in the unit will draw on their prior algebra and geometry knowledge but will engage students at a deeper level. As students come to realization of what constitutes mathematical proof, they will also become more adept at communicating and justifying mathematical arguments. While they have some basic prior knowledge of similarity and congruence, they have much less experience with writing or communicating a mathematical proof involving them.

During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs.

Rationale

Research shows that "many students do not feel the need to ask why or justify (prove) the validity of their conjectures once they "see" the truth of the geometric relationships." [ii] They often fail to distinguish between inductive and deductive reasoning strategies, therefore also failing to recognize the strengths and weaknesses of each type of argument.

Anecdotal Evidence of Students' Weakness in this Area

It has been my experience that high school students struggle with the notion of proo. In general, a mathematical proof requires students to use only the information that they are given to prove other relationships or facts that are not given. However, if some fact seems visually obvious to them, students often see no need to go beyond their observations in proving it to be true. For example, when asked to prove that a triangle is isosceles, students may simply state that they can clearly see that two side lengths are congruent and they view this as a satisfactory proof. As another example, consider students who, when asked to prove if two triangles are congruent, they may simply mark the corresponding sides that they "see" as congruent as part of their proof approach, even if they were not given this information. They may then state the appropriate reason, such as SAS, but they arrived at this conclusion by mere observation.

In my classroom, I have witnessed that students have a basic lack of understanding of the concept of proof: what it is, what it requires, why it is done. Additionally, a vast majority is unable to distinguish whether or not they are proving a general claim or something specific to their problem. For example, when asked to prove that a perpendicular bisector of an isosceles triangle is also an angle bisector, students do not always see this as being always true for any isosceles triangle, not just for the one they have drawn before them. Conversely, if a student has proven something to be true for a particular case, they often erroneously conclude that it is true always. I have seen so many students who will falsely assume that an angle that is cut into two pieces is bisected, with no evidence to lead them to this conclusion. Anything broken in two, to them, is broken in half.

I hope to enhance the students' ability to develop a logical thought process, using only the information that is "given" in the problem to systematically determine other truths that serve as building blocks to arriving at their conclusion. Additionally, and perhaps most importantly, the students must satisfactorily communicate their thought process, whether it be in paragraph form or as a two-column proof, in manner that can be followed by the reader. They must be able to provide evidence for their assertions.

Roles of Proof in a Mathematics Classroom:

Studies indicate that students' experience with constructing proofs has been mainly limited to a high school geometry classroom. To these students, proof is a "stand alone" topic, with a very rigid structure, and has limited application to other aspects of math. To help combat this, the 2000 NCTM Standards recommended "reasoning and proof should be a consistent part of students' mathematical experience in pre-kindergarten through grade 12." The standards further argued that proof should not be seen as a distinct topic, but rather as an approach to thinking about any mathematical topic. [iii] The definition of proof and the role that it plays in teaching mathematics continues to evolve.

Perhaps one of the reasons that proof is such a notoriously difficult concept for students to learn and for teachers to teach is that there are "several notions of what mathematical proof is, or might be."[iv] While there are purists who hold a very formal view of what a proof should look like, there are those with a broader view of proof and accept proofs as logical progression of thought which are intended to convince a reader. They accept proofs laid out in paragraph format, or any form in which the progression of thought can be followed.

In his research, Keith Weber proposed the following list as the purposes that proof can have.[v]

- *Explanation:* Readers can understand a mathematical relationship as a result of having read a proof.

- *Communication:* Students and mathematicians can communicate and debate their ideas about relationships by documenting their thinking in the form of a proof.

- *Discovery of New Results:* New models or ideas or developed through exploration of old ideas.
- *Justification of a definition:* One can show that the definition adequately captures all of the essential properties of the concept being defined.

- *Providing autonomy:* Teaching students how to prove can allow them to independently construct and validate new mathematical knowledge.

Similarly, Michael de Villiers believes that proofs serve many functions beyond convincing and validating mathematical arguments. In numerous publications, he highlights the following as a model for the functions of proof:[vi]

- Verification: concerned with the truth of a statement

- Explanation: providing insights into why it is true

- Systematisation: the organization of various results into a deductive system of axioms, major concepts, and theorems

- Discovery: the discovery or invention of new results

- Communication: the transmission of mathematical knowledge

- Intellectual Challenge: the self-realization/fulfillment derived from constructing a proof

Often these functions may become interwoven within one problem. De Villiers promotes the use of interactive software for students to investigate and conjecture about geometric relationships. Verification can be achieved, he suggests, through infinite inductive trials using dynamic models. So the role of proof in this instance is not one of verification that a relationship is true, but more one of explanation of why it is true.[vii]

Why Students Struggle with Proof

Because students are exposed to such diverse notions of proof through their elementary grade mathematics classes, they "acquire a wide variety of beliefs about the criteria that make proofs valid and what constitutes mathematical proof."[viii] Some students lack the cognitive development and are prevented from understanding the level of the content being taught. Other students believe that non-deductive arguments can be used as the sole basis for a valid proof. Because they lack a clear vision of what a proof requires, most students are not fully aware of how inadequate their proof are.

Levels of Geometric Thought

Dina and Pierre van Hiele, two Dutch researchers, have proposed a learning cycle that students must progress through in order to learn geometry (Figure 1). Their research indicates that students cannot skip a level in the progression, and furthermore, instruction that is aimed at a student who is not cognitively ready for it will be incomprehensible.[ix] Students at the high school level should be in or very near level 3 of the progression, but prior mathematical experiences coupled with any other factors that may have stifled their cognitive development may have a student at a lower level.

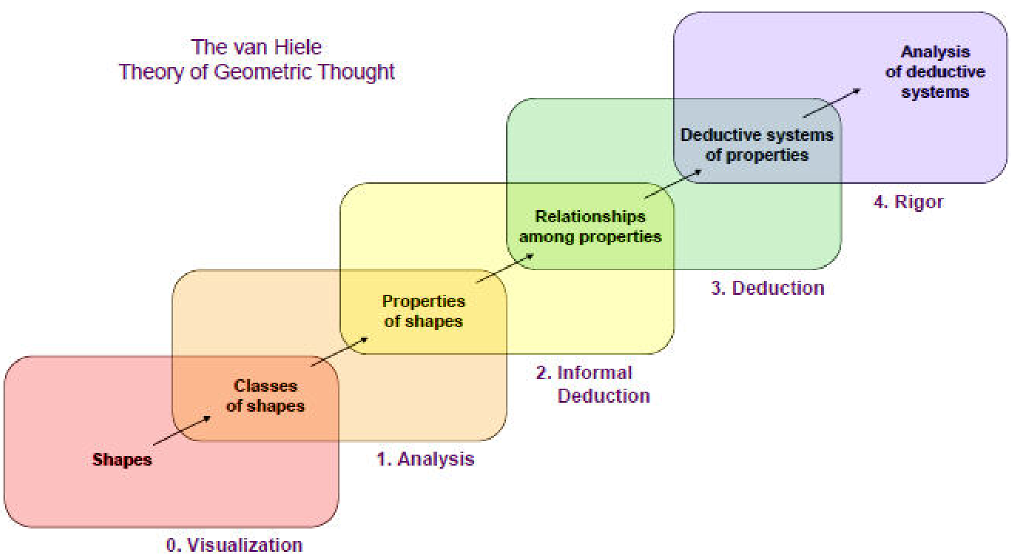


Figure 1[x]

Inductive Proof Approaches with Younger Students

Many students believe that proof can be achieved through inductive means alone. In fact, the inductive methods that students routinely employ and accept as justifiable proof can be a major roadblock to their development of deductive reasoning skills. In one study of middle school math students, researchers concluded that students primarily produced inductive arguments to prove their point. Students believe that more examples are better than fewer, and that the wider the variety of their examples, the more convincing their argument became.[xi] Once convinced that their argument is true, students generally feel no need to proceed further with the proof process.

Invalid Proof Paradigms

Through their research, Guershon Harel and Larry Sowder have defined a detailed taxonomy of "cognitive characteristics of the act of proving held by students, which they call *proof schemes*."[xii]

- Ritual: Proof is viewed as a very formal activity, with two-column format. Anything not written using a prescribed mathematical convention cannot qualify as a proof. Even more disturbing, some students will erroneously agree with any proof (even an invalid one) simply on the basis that it is written in the correct format.

- Authoritative: Students accept all proofs presented by an established authority, such as a teacher or their math book. They do not question or exhibit any curiosity about *why*, because they believe the authorities know what they are doing. However, this can be problematic in that they do not see themselves as having any mathematical authority. As a result, they do not believe they have the ability, or right, to prove a conjecture.

- Perceptual: Students are convinced of the veracity of a statement based on a rudimentary diagram or visual representation. While the diagram may indeed be appropriate to the proof, the student's thought process is geared more toward that one specific example and not sophisticated enough to consider transformations of the image.

Issues with Communicating Logical Arguments

Despite the recommendations of the National Research Council (2001), and more recently the U.S. Common Core State Standards Document (2010) to make proof more pervasive throughout all strands of mathematical instruction, in the United States, most opportunities for communicating formal reasoning and deductive arguments are found in the high school geometry classroom.[xiii] This is a natural source of rich opportunities for students to prove conjectures about shapes and structures found in geometry.

Two-Column Proof

Although NCTM listed two-column proof as a geometry topic that should receive decreased attention in 1989[xiv], many math teachers continue to teach these tremendously important geometric relationships using the same methods that they were taught. Since the two-column proof format was so prevalent in earlier texts, it continues to dominate instruction today. The rigidity of this approach is difficult for students to read, yet alone write. As mentioned above, students who are influenced by a Ritual Proof Scheme may be so focused on the form of their proof, that they will convince themselves that it is true because structurally it appears to be so.

Paragraph Proof

More recent textbooks have begun promoting paragraph proofs. Here students will document their progression of thoughts in sentence form. The form is more conversational and therefore, more accessible to students. However, by easing up on the formality, students may become too relaxed in their justifications and leave out key details that are necessary to supporting their arguments. Even though their logic may be correct, their communication of their logic is inadequate to qualify as a valid proof.

Teacher's Role in Moving Students along the Proof Continuum

The proof process can help students develop their abilities to question and justify mathematics. But questioning, conjecturing, and critiquing processes should not be limited to the math classroom, yet alone only the geometry classroom. Teachers should promote this as a habit of mind that students should engage every time they encounter something new to them. If teachers begin to incorporate proof and deductive reasoning outside of the geometry domain, the notion of proof will be far more pervasive and far less elusive to students.

One way to move in this direction is to explicitly teach inductive and deductive reasoning skills. Engage students in activities that will allow them compare and contrast the two approaches to solving a problem. Teachers should allow students to engage a problem inductively, form a conjecture about the situation, and then use deduction to prove or disprove their conjecture. The induction phase serves to convince the student that an assertion holds true. The deduction phase is means for the student to explain and/or communicate to others why the assertion is always true.

The nature of the tasks that students are asked to participate in is of paramount importance when trying to promote deductive habits of mind. Teachers must have tasks that are engaging enough that the students wish to investigate and experiment as well as validate and communicate their findings. Without appropriate motivation, students are less willing to assert the mental effort necessary to participate in a meaningful proof.

Additionally, the classroom environment is key. It is critical for teachers to create an environment that encourages risk-taking, where students are not afraid to make a false assertion but instead they revel in the learning that comes from proving an assertion is false. Collaboration with peers, where students share their ideas as well as their struggles should all be part of this risk-taking environment.

To aid in the investigation process, teachers should make certain that their classrooms are equipped with the necessary tools. For example, many geometry tasks will require construction of shapes using such instruments as straightedges, compasses, rulers, and protractors. By creating and constructing, students often gain a deeper insight into the relationships that exist in geometric figures. However, the availability of technology such as graphing calculators or dynamic software programs allow for greater access to a multitude of examples without the time that is required to make multiple constructions. While using these programs, proof takes on the discovery function that will serve as the springboard for the conjecturing and deducing phases of the proof process.

When mathematical proof is used for the function of explanation and communications, teachers must make their expectations clear. For a proof to be convincing, it must clearly convey a logical progression of thought and give insight as to why a proposition is true. The focus must be on the content of the proof, not its format. As research suggests, teachers must not concern themselves with the rigidity of the two-column proof, which is likely the way they were taught. However, they cannot let the relaxed format of a paragraph proof lead to a relaxed standard of the communication process.

Carefully designed tasks, in a collaborative and risk-tolerant environment, will ultimately result in higher engagement level and deeper understanding of the math content. This unit is designed to promote they cycle of induction, conjecture, and deduction through a series of investigations. In the end, students will not only see deduction as a necessary part of the proof process, but they will also autonomously seek to use deduction to satisfy their own need for intellectual fulfillment.

Classroom Activities

The following activities assume that students have prior knowledge of triangle similarity and congruence as well as fundamental algebra skills, including factoring difference of squares.

Activity One

This problem is taken from Core Plus Mathematics, Course 3, Unit 1. Students are asked to make a conjecture about what happens when they choose any four consecutive integers, add the middle two, and then subtract the smallest of the four from that sum. [xv] This activity serves to introduce some key vocabulary. First, students need to understand what a conjecture is. A conjecture is an inference or conclusion based on incomplete evidence. A discussion should ensue about why the evidence in this problem is deemed to be incomplete. Students should realize that there are infinite sets of numbers that can be tested, and therefore, it not even possible for them to have complete evidence.

Second, this activity is a nice way to introduce the concept of inductive reasoning. The majority of the students will try two or more sets of integers to test their conjectures. The process of forming a generalized conclusion based on isolated facts is inductive reasoning. Inductive reasoning is often very compelling and results in a great amount of conviction about the validity of a student's conjecture. Since students within the classroom have likely tried a wide variety of sets of integers, they may see no need to move beyond these tests to verify their conclusion. In this case, proof is not necessary to serve the verification function.

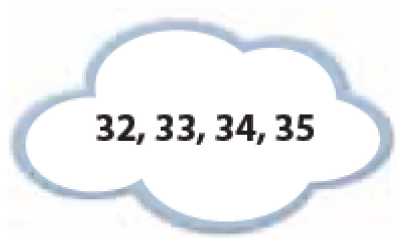
Check Your Understanding

Make a conjecture about what happens when you choose any four consecutive whole numbers, add the middle two, and then subtract the smallest of the four from that sum.

a. Describe the procedure you used to create your conjecture.

b. Write your conjecture in if-then form.

c. If n represents the smallest of four consecutive whole numbers, how would you represent each of the next three numbers?



d. Use your representations in Part c to write an argument that proves your conjecture is always true.

Although it is possible to achieve a high level of confidence regarding a conjecture's validity through inductive reasoning, this generally does not provide satisfactory evidence about *why* the conjecture is true. The next step is to define for the students the notion of deductive reasoning as a means of providing this explanation. "Deductive reasoning involves reasoning from facts, definition, and accepted properties to conclusions using principles of logic. Under correct deductive reasoning, the conclusions reached are certain, not just plausible."[xvi]

If this notion is new to students, it may take much prompting from the teacher to get the students started in the right direction. For example, the text suggests in part c of the problem, "Let the first integer be *n*. How would you define the next three integers in the set?" As students define the other integers, then perform the same procedure that they had previously using actual integers, they should arrive at a compelling argument as to why their conjecture is undeniably always true. At this point, a class discussion should take place comparing/contrasting the roles that inductive and deductive reasoning played in solving this problem. While inductive reasoning led to a conjecture, deductive reasoning was used as a means to prove the validity of the conjecture. Deductive reasoning is a critical component in this process.

Activity Two

Now that the process of *Inductive Reasoning* → *Conjecture* →*Deductive Reasoning* has been established, the next activity should follow the same process.

Prior to this next activity, a discussion should take place in the class about the definition of odd and even numbers. The teacher should ask the student to define the terms *odd* and *even* in their own words. The teacher should facilitate the discussion so that the students come to the realization that all even numbers are divisible by two without a remainder, and can therefore be written symbolically using the notation $2n$. Furthermore, every odd integer has a remainder of one when divided by two, so it can be represented by the notation of $2n + 1$. Students may be further convinced of these representations by generating tables of values, or looking at these tables in their graphing calculators when $y_1 = 2x$ and $y_2 = 2x + 1$.

In this activity, students are asked to make a conjecture about what type of numbers result from squaring an odd number. Students should use a number of examples during the inductive reasoning phase to conjecture that the result is always an odd number. Next, the teacher needs to encourage them to explain *why* this is true. Again, the ultimate goal is for students to ask this of themselves without prompting from the teacher. They should realize that the deductive reasoning phase is required in order to fully explain and prove their conjecture. Now that they have a symbolic representation of odd numbers ($2n$+1), they should be inclined to use it to see that tive reasoning approach, it is likely that students will emerge with two cases where they can visually ascertain that shaded area is one fourth of the total area (Figure 6).

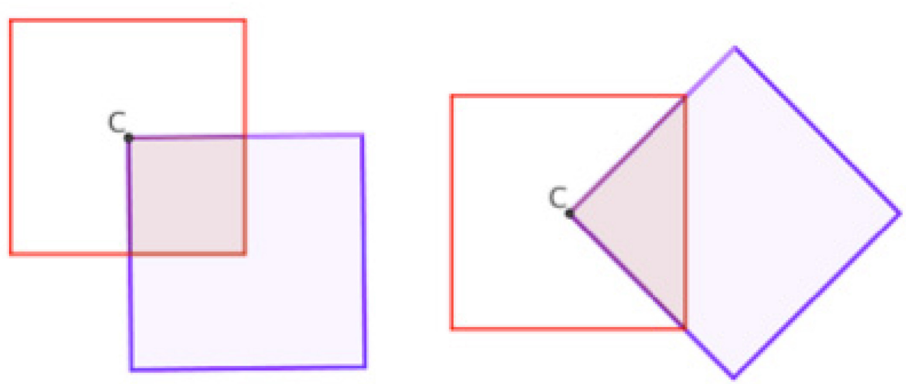
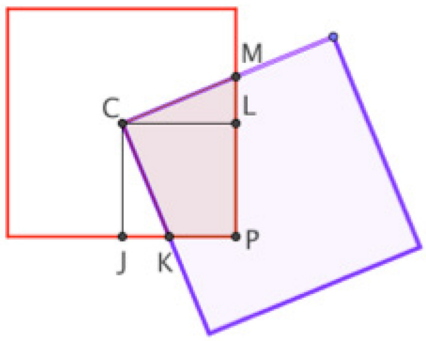


Figure 6

The most challenging part of the problem is determining the area of the shaded region in other cases, as is seen in Figure 5. The addition of auxiliary lines, as shown in Figure 7 may help.



$$m\angle JCK + m\angle KCL = 90^o$$
$$m\angle LCM + m\angle KCL = 90^o$$
$$\therefore m\angle JCK = m\angle LCM$$

Figure 7

Students should be able to use deductive reasoning to prove that triangle JCK is congruent to triangle LCM, therefore explaining why the area of the overlap is always one fourth of the total area of the square. For a dynamic exploration of this problem, visit http://geogebratube.com/student/m62464. This should be done after the students have explored with paper and pencil.

Conclusion

It is important that the notion of proof not be limited to the geometry classroom. The pattern of *Inductive Reasoning* → *Conjecture* → *Deductive Reasoning* should be encouraged whenever possible. Students should become accustomed to the expectation of explaining their reasoning and justifying that it holds true for more than just a few isolated cases. While these activities described in this unit will help students gain experience in communicating their mathematical proof of their conjectures, they should not be the only opportunities provided for students to experience this idea. Writing proofs should not be taught as an isolated skill, but rather, it should be a skill that is embedded throughout the curriculum whenever the opportunity arises.

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Appendix:

The unit is aligned with the Common Core State Standards as described in these excerpts:

- CCSS.Math.Practice.MP1 Make sense of problems and persevere in solving them.
- CCSS.Math.Practice.MP3 Construct viable arguments and critique the reasoning of others.

- CCSS.Math.Practice.MP5 Use appropriate tools strategically.

- CCSS.Math.Practice.MP7 Look for and make use of structure.

- CCSS.Math.Practice.MP8 Look for and express regularity in repeated reasoning.

- CCSS.Math.Content.HSA-SSE.A.2 Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$.

- CCSS.Math.Content.HSG-CO.B.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

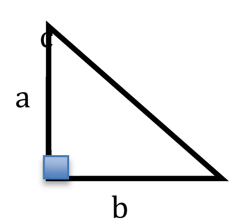
- CCSS.Math.Content.HSG-CO.B.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

- CCSS.Math.Content.HSG-SRT.B.4 Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.

- CCSS.Math.Content.HSG-SRT.B.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.[xvii]

Prerequisite Knowledge for Teachers

Triangle Angle Sum Theorem: The sum of the interior angles in all triangles is 180°.Pythagorean Theorem and its Converse: In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. Conversely, if the sum of the squares of the other two sides is equal to the square of the hypotenuse, then the triangle is a right triangle.



$a^2 + b^2 = c^2$

Figure 8

Properties of Parallel Lines Cut by a Transversal

When parallel lines are cut by a transversal, corresponding angles are congruent. Conversely, when cut by a transversal, if corresponding angles are congruent, then the lines are parallel.

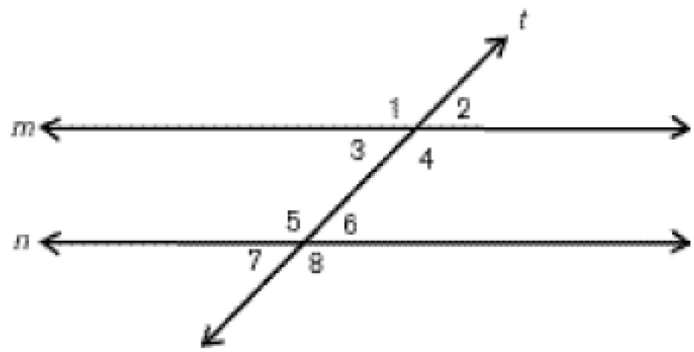


Figure 9

Vocabulary accompanying Figure 9 above

Corresponding angles – angles at the same location at each intersection
For example, angle 1 corresponds with angle 5. These angles are congruent.

Alternate interior angles – angles that are located between the parallel lines but on opposite sides of the transversal
For example, angles 4 and 5 are alternate interior angles. Alternate interior angles are supplementary.

Triangle Similarity Theorems

AA Theorem – When two angles of one triangle are congruent to the corresponding angles of another triangle, the triangles must be similar.

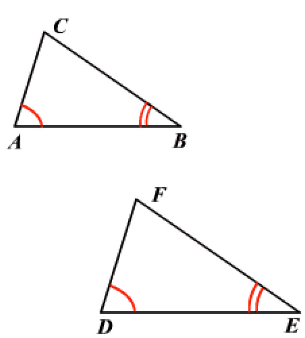


Figure 10

SSS Similarity Theorem – If three pairs of corresponding sides are proportional, then the triangles must be similar.

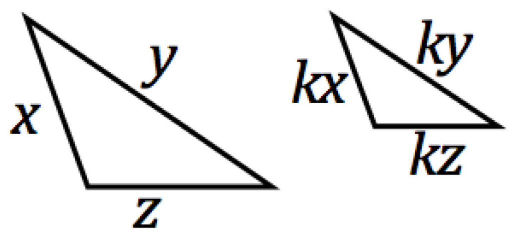


Figure 11

SAS Similarity Theorem – If two pairs of corresponding sides are proportional and the angles included between them are congruent, then the triangles are similar.

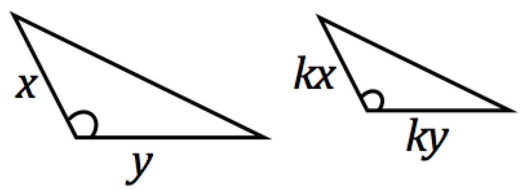


Figure 12

Triangle Congruence Theorems

SSS Congruence Theorem – If three sides one triangle are congruent to the three sides of another triangle, then the triangles are congruent.

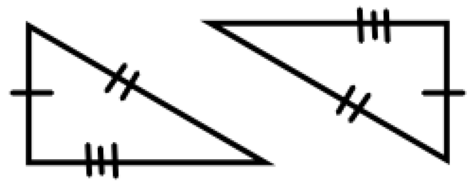


Figure 13

SAS Congruence Theorem – If two sides and the included angle are congruent to two corresponding two sides and included angle, then the triangles are congruent.

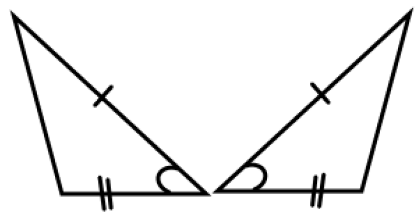


Figure 14

ASA Congruence Theorem – If two angles and the side between them are congruent to the two angles and side of another triangle, then the triangles are congruent.

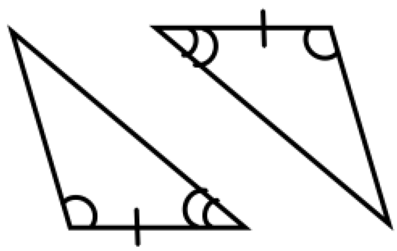


Figure 15

AAS Congruence Theorem – If two angles and the non-included side of one triangle are congruent to two angles and the non-included side of another triangle, then the triangles are congruent.

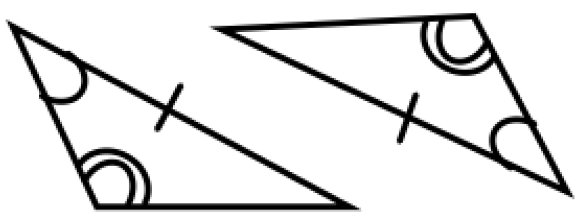


Figure 16

Hypotenuse Leg Congruence Theorem – If the hypotenuse and one leg of a right triangle are congruent to the hypotenuse and leg of another triangle, then the triangles are congruent.

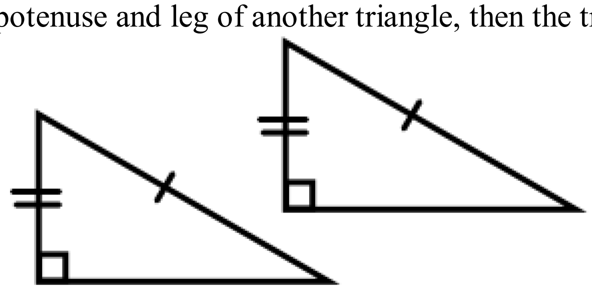


Figure 17

Basic Properties of Parallelograms
Parallelogram – a quadrilateral characterized by opposite sides that are congruent and parallel

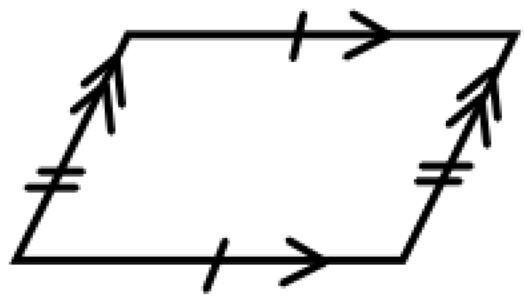


Figure 18

Notes

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[i] Core-Plus Mathematics Course 1, 2, and 3 2nd Edition, (New York: Glencoe, 2009)
[ii] Sharon M. McCrone et al., *Focus In High School Mathematics: Reasoning and Sense Making* (NCTM: 2010), 3
[iii]Keith Weber, "Research Sampler 8: Students' Difficulties with Proof", Mathematical Association of America, accessed November 2014, http://www.maa.org/programs/faculty-and-departments/curriculum-department-guidelines-recommendations/teaching-and-learning/research-sampler-8-students-difficulties-with-proof
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[v] Weber, "Research Sampler 8"
[vi] Michael de Villiers, "The Role and Function of Proof in Mathematics", *Pythagorus* (1990), 17-24
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[xi] Caroline Williams et al., "Understanding Students' Similarity and Typicality Judgments In and Out of Mathematics", (2011), http://idiom.wceruw.org/documents/PMENA_2011_paper.pdf

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[xiii] Michelle Cirillo and Patricio G. Herbst, “Moving Toward More Authentic Proof Practices in Geometry”, *The Mathematics Educator* (2011/2012), 13
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[xvi] Core-Plus Mathematics, 14

Curriculum Unit Title: **Moving Students from Conjecture to Proof**

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KEY LEARNING, ENDURING UNDERSTANDING, ETC.

Conjectures can be tested for validity with inductive reasoning strategies.

Conjectures can be proven true or false with deductive reasoning strategies.

ESSENTIAL QUESTION(S) for the UNIT

What evidence is sufficient to prove or disprove a conjecture?

What are the shortfalls of inductive reasoning? How can these shortfalls be overcome?

CONCEPT A	CONCEPT B
Inductive Reasoning	Deductive Reasoning
ESSENTIAL QUESTIONS A	**ESSENTIAL QUESTIONS B**
How can inductive reasoning strategies be used to test a conjecture? Why is inductive reasoning an insufficient proof strategy?	What is deductive reasoning and how does it compare with inductive reasoning? Why is deductive reasoning a more convincing proof strategy?
VOCABULARY A	**VOCABULARY B**
Conjecture Inductive reasoning	Deductive reasoning Proof

ADDITIONAL INFORMATION/MATERIAL/TEXT/FILM/RESOURCES

Geogebratube applets:

http://geogebratube.com/student/m60787 http://geogebratube.com/student/m62464

http://geogebratube.com/student/m62355