Making Sense of Fractions: Beyond Equal Parts

Beth Vanaman

Introduction and Rationale

In March of every year, I begin to get anxiety over teaching an essential unit on fractions. Fractions represent at least half of the questions on our end of the year testing (approximately 30 of 60 questions). Some students enter the unit having a notion of what a half is (i.e. half of a sandwich, half of your homework problems), but a pretest (Addendum 1) shows that most of them cannot translate their understanding to numbers (writing a half) or splitting an object into more than 2 equal pieces. Using the results of the pretest, I then have 4 weeks (or less) to teach my students equal parts, benchmark fractions, and comparing and contrasting size of the whole. From scaffolding to independent problem solving, the pressure is on!

Common Core State Standards for Mathematics (CCSSM)[1] say that by the end of the third grade, students need to have a deep understanding of fractions, despite only partitioning circles and rectangles into two and four pieces in first grade and partitioning circles and rectangles into two, three, or four equal pieces in second grade. In second grade, they also are introduced to the terms halves, thirds, half of, a third of, etc. The conceptual leap to third grade fractions is great. The standards state that students will-

> "1.) Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a/b* as the quantity formed by parts of a size 1/*b*. CC.3.NF.1,
> 2.)Understand a fraction as a number on a number line; represent fractions on a number line diagram. CC.3.NF.2,
> 3.) Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. CC.3.NF.3"[2]

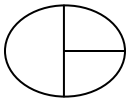
In addition to overcoming conceptual leaps between second and third grade, my students have also exhibited trouble in previous units (not fractions, but number operations books) with Math Practice Standard 1 CCSS.Math.Practice.MP1 "Make sense of problems and persevere in solving them."[3] Too often I find my students saying "I can't do this!" without even attempting the problem, saying "I don't know what to do!" without reading the directions or the problem, and "What do I do now?" despite being one step from an answer. In this unit, I hope to begin by scaffolding, but continually provide students with opportunities to work in cooperative groups and independently so that they can solve problems without teacher help despite problems being complex or multi-step.

Demographics

Wilmington Manor Elementary School, of the Colonial School District, has approximately 500 students in grades K-5. Most of the students have little to no exposure to the world beyond their own neighborhood and a small stretch of highway known as Route 13 (fast food restaurants, schools, and drug stores). In this small school, there is a high population of Low Socio-Economic Status (LES) students and English Language Learners (ELL). 68.4% of students in the building receive free and reduced lunch. With so many students below poverty level and so many parents working multiple jobs, my students often report that they have no help after school with homework or assistance with reading. They are also unable to attend after school events or curriculum nights which offer guidance with math strategies, reading focus skills, and so forth. 37.1% of students at Wilmington Manor Elementary School are classified as ELL. Many students are their family's only English speaker. Many parents cannot read my instructions or notes in English, and they are unable to help their students with any work sent home in English.

With little to no help available to students after school, my unit needs to address all of the third grade standards, correct any misconceptions, and teach test taking strategies during a one hour daily math block and a thirty minute math intervention block. The core lessons (whole group) will be taught to all of my students during the regular math block. I will reserve the intervention time to pull small groups of students who are struggling with a particular concept or misconception, while others work on extension activities. In particular, I will work closely with any special education students in my classroom to ensure that they are receiving the needed support to be successful with fractions in regards to their IEP goals.

A pretest given and work from previous years shows that students typically struggle with dividing a shape into equal pieces (i.e. cutting a circle with straight vertical lines rather than pie pieces) and incorrectly identifying the number of equal parts.



A student lacking proficiency in identifying equal parts would say that this drawing represents thirds. They count just the pieces but are unable to initially recognize that one piece is larger than the others. This shape actually shows fourths.

The pretest shows that students also have difficulty comparing the size of and ordering fractions. Often students have the misconception that the bigger the denominator, the bigger the fraction. A student who is unable to compare fractions with accuracy would explain that *¼ is bigger than ½ , because 4 is bigger than 2.* Rather ½ is the larger fraction. This misconception is resolved in the unit by drawing the fractions or using fraction bars.

Problem Solving in Mathematics

Many factors contribute to a student's success in mathematical problem solving. Addendum 2 shows a web featuring those factors[4]. Aside from my lesson planning, unit writing, and teacher implementation, a lot hinders on a student's attitude towards learning and their investment towards the task and goal.

CCSS describes mathematically proficient students as ones who can "start by explaining to themselves the meaning of a problem and looking for entry points to its solution."[5] Students' procedural fluency and strategic competence are steps towards proficiency that allow them to attack a problem and formulate an answer. However, I have found that for most of my students, it's not that easy. After selecting meaningful problems and creating worthwhile instructional tasks that measure the selected goals, I have to work on my class's conceptual reasoning. In order to problem solve, my students need to be able to identify the math operation needed and how the problem relates to what I have modeled in class. In my unit on fractions, I want students to be able to recognize the part-to-whole relationship and whether or not they are dividing one whole into parts or whether they are finding what is part of the whole. Students should be able to devise and attempt a plan after having read the problem. This unit aims at eliminating the "I don't know" comments and replacing them with confident students who are not afraid to try. According to Empson[6], a math book writer, students should devise their own methods that make sense to them. This will lead to the concepts making sense.

I also want my students to show proficiency through adaptive reasoning. My unit will not only ask them to solve fraction problems, but they should also be able to justify and defend their answers (i.e. which is bigger? How do you know?) and explain others' reasoning and thoughts. Teaching multiple strategies to my students will allow them to become so comfortable with their work that they can teach it to others.

While all of these steps to proficiency will prove challenging, I think that my biggest challenge in mathematical problem solving will be for my students to have productive disposition. By the end of my unit, I want my students to see the understanding of fractions as useful and worthwhile. Students should recognize real world applications. In Polya's "How to Solve It," he discusses that students need to have buy in to the problems that they are solving. Without a clear understanding as to why the math is necessary, students do not see a need to go through all of the work to solve the problem.[7] To exemplify real word applications, I will highlight fractions found in every day life such as shopping fliers, timed events, and careers using measurement.

Learning Objectives

- Students will be able to divide a whole into equal parts.
- Students will understand the meaning of numerator and denominator.
- Students will be able to represent fractions on a number line.
- Students will identify fractions larger than one by their place on a number line.
- Students will be able to compare and contrast the size of fractions.
- Students will recognize equivalent fractions.
- Students will be able to rationalize their thinking and justify their answers.

Essential Questions

How can I divide the whole into equal parts?
What is the whole in this problem?
How can the number line be divided to represent different fractions?
Which fraction is bigger? How can I prove it?
Are these equal fractions? How can I show that?

Strategies

To teach my unit on fractions, I will rely on several teaching methods including direct instruction, collaborative pairs, cooperative groups, small group instruction, videos, Smart Board activities, etc.

Whole group, direct instruction will occur daily as I start lessons with activating strategies, introduce the concept to be learned, and walk students through problems by scaffolding. During this part of my lessons, I will rely on the use of my Smart Board.(

The website Smart Board Exchange[8] provides many great fraction lessons for a range of abilities. For example, students just starting out can come up to the Smart Board and draw lines to show equal pieces. They can manipulate drawn lines by dragging them to correct unequal pieces. As students become more proficient with fractions, the Smart Board is a great tool for representing equal fractions by overlying shapes and quickly creating number lines and fraction bars that can be used an indefinite amount of times.

In small group instruction, during the Math block or during our intervention time, I can work with students to review concepts with which they are struggling and provide enrichment through videos on Discovery Education[9] and using self-created fraction bars from construction paper. An example video is "Fractions and All Their Parts,"[10]

which includes a lot of visualization of fractions while using the vocabulary part, whole, numerator, and denominator. With small dry erase boards, I can recreate problems students had missed on homework or in their math workbooks.

Background Knowledge

In order to be successful in this unit, students must be able to partition squares and rectangles into equal parts. They must know and use terminology such as part, whole, numerator, denominator, halves, thirds, fourths, etc. Addendum 3 lists suggested vocabulary for the unit.

Students should also have familiarity with the number line. In earlier units on addition and subtraction, my students used the number line as a tool for problem solving. They also used it to create a line plot in different unit on data. The exposure line plots helped them to recall that fractions are found between whole numbers. I can best introduce and explain this by discussing their age. Many kids can relate to being 8, 8 1/2, and then 9 years old. We also talk about halves by using shoe size as an example.

Knowledge of the concepts greater than, less than and equivalent is necessary for student proficiency in the unit. Students will use the terms to describe the size of fractions.

Instruction

As a teacher of this unit, it is my job to foster an environment where students feel comfortable learning, making mistakes, revising their work, and engaging in discussion. Students will always be aware of directions and I will ensure that all students understand their roles, my expectations, and the desired outcome. As a facilitator of the classroom, I will make sure that students are participating and demonstrating their understanding informally through discussion and in group work, as well as formally with learning checkpoints and work samples.

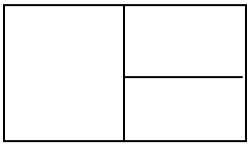
Activity 1

In the introductory lesson, my goal is for students to understand equal parts and sharing. I will hook students with a simple fraction problem like sharing pizza slices or cookies found in real world examples (i.e. fliers, menus, magazines). Students will be able to tackle the problem using fractions or by drawing pictures. Their solutions will be a lead-in to the term “fair shares” and “equal pieces.”

In my first activity, I want to work solely with fractions with a numerator of 1. Once students are comfortable with dividing shapes into equal pieces using the Smart

Board and colored paper fraction bars, I will have them color in one piece of an object and write the fraction represented. We will continually practice using the terms numerator and denominator as we work. Students will also use the terms numerator, denominator, half, thirds, fourths, etc. when describing and labeling their work.

As students become more familiar with dividing objects into fair shares using pictures provided in their math textbooks, I will begin to challenge them with more difficult problems where they are dividing groups of objects into fair shares. A sample problem is "Hannah and Kate's mom brought home 5 donuts on Saturday morning. She told the girls to split the donuts fairly (implying equal shares). So Hannah and Kate started picking out the donuts that they wanted. How many donuts did each girl get?" I will also have students use error analysis to approach common misconceptions in fractions. They will look at a sampling of pictures and student work where students divided common shapes and justify whether or not the shape is cut into equal and fair parts. They will argue their point by showing how they would correct the picture, if they said the picture showed unequal groups. An example would be showing students the picture below.



Using the picture, you can show sample student work such as *"This shows thirds, because the rectangle has 3 pieces."* Students can argue whether or not they think that it is thirds and why or why not.

Activity 2

In my second lesson, I want students to recognize fractions as part of a collection, not just as a single picture. I also want them to be able to show how to share a remainder. Students will work with a problem like "Mike, Amy, and Kyle bought a box full of granola bars. The box has 20 granola bars. If each person wants an equal amount, how much will each person get?" I will have students try to devise a plan and approach the problem on their own, we will share out strategies, and we will, as a class, discuss and record the multiple strategies that we found to solve one problem. Students may approach the problem by sharing whole granola bars fairly and then dividing the remaining granola bars, or by dividing each granola bar into thirds and adding the pieces. As a group, we will discuss what the "whole" was in each person's answer and how their thought processes were similar and different. We will also discuss which would be a more reasonable solution in a real life situation. At the end of the activity, I will review how to divide equal pieces, ways to represent fractions using pictures and fraction bars, and how to represent a fraction greater than 1. Addendum 4 provides sample student work and prompts for discussion.

Activity 3

Students will learn a new strategy to compare and contrast fractions in this lesson. They will use a number line. I will model this strategy by using a piece of yarn to surround a drawn pizza (circle). I will then "cut" the pizza into 4 equal pieces, marking directly on the yarn. When I unfold the yarn, students will see the marks on it where the labels for the number line will be- ¼, 2/4, ¾ and 4/4 (1 whole). We will follow the same procedure for thirds, sixths, and eighths. This allows them to see the relationship of fractions to their place on a number line. It is to be noted that the number lines should always be the same size. After labeling number lines, students will complete greater than and less than questions. As a challenge, we will work with fractions greater than 1 and their placement. Students will attempt labeling fractions greater than 1 on a number line as well.

Activity 4

By the last activity, students should be able to compare the size of fractions. In activity 4, I want students to begin by reviewing previous fractions that we've worked with. We will begin to compare the size of those fractions using different approaches like fraction bars, pictures, and number lines. I will then pose a real world problem for them to solve like "Pro Painters used ¼ of a can of paint on Thursday and Real Great Painters used ¾ of a can of paint that same day. Who used more?" We will continue after attacking this problem with similar problems that have various size wholes. Addendum 5 shows possible problems that can be used to compare the sizes of fractions.

Extension Activities

For students showing proficiency through classroom observation, participation, and completed work, I will provide extension activities during our extra math period (30 minutes a day). Ideas include adding and subtracting fractions with like denominators, or finding fractions on a clock

Adding and subtracting on a number line address the 4th grade CCSS Math.Content.4.NF.B.3d which requires students to add and subtract fractions using visual representations and equations.[11] Finding fractions on a clock, by dividing a clock into fourths, allows students to use fractions to identify time terminology such as quarter past, half past, and quarter til.

Conclusion

This unit was designed to be taught in unison with my current curriculum, not to replace the existing resources. These activities and questions allow for more higher-order thinking about fractions and more time for student practice. At the conclusion of this unit, students will have the tools necessary to show proficiency on all 3rd grade math standards.

Bibliography

1. Empson, Susan B., and Linda Levi. *Extending children's mathematics: fractions and decimals*. Portsmouth, NH: Heinemann, 2011. I read this book to gain an understanding of how students learn fractions and the pacing at which they should acquire the concepts. The book also addressed common misconceptions and difficulties with fractions and how to address them.
2. Cai, J. (2008). Intended, implemented/planned, and achieved mathematical curricula in China and the United States: Some highlights from a decade of research. In Z. Usiskin & E. Willmore (Ed.), Mathematics Curriculum in Pacific Rim Countries – China, Japan, Korea, and Singapore. Charlotte, NC: Information Age Publishing. This research was provided in a class at the University of Delaware. The web provided was used as a guiding tool when discussing how students think about problem solving and all the factors that are involved with solving a mathematics problem.
3. "Common Core State Standards Initiative." Common Core State Standards Initiative. http://www.corestandards.org (accessed January 19, 2014). This website is a vital resource in my day-to-day lesson planning and the creation of this unit. The standards are easily accessible by grade level and strand.
4. Polya, George. *How to solve it*. 2nd ed. London: Penguin, 1990. This book was used to research problem solving in mathematics. The steps to successfully examining a math problem, as well as strategies for dealing with the reluctant learner were discussed.
5. "SMART Exchange." SMART Exchange. http://exchange.smarttech.com/search.html?q="5th%20grade%20math" (accessed January 18, 2014). This website is a search tool for lessons created for use on a Smartboard. The lessons are searchable by subject and grade level. There are many lessons on fractions for grades 3-8.
6. "Welcome to Discovery Education | Digital textbooks and standards-aligned educational resources." Welcome to Discovery Education | Digital textbooks and standards-aligned educational resources. http://www.discoveryeducation.com (accessed January 18, 2014). This website is only accessible by holding an account. However, the site has a vast collection of educational media on many topics. This site has a host of fraction videos, how-to lessons, and games.

Suggested Student Resources

Computer Access
Vocabulary List
Yarn/String
Permanent Marker
Construction Paper
Pencils
Scissors

Appendix A

Common Core State Standards

CCSS.Math.Content.3.NF.A.1: Understand the fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts. Understand the fraction *a/b* as the quantity formed by a parts of size 1/*b*.

CCSS.Math.Content.3.NF.A.2: Understand a fraction as a number on the number line, represent fractions on a number line diagram.

CCSS.Math.Content.3.NF.A.2a: Represent a fraction 1/*b* on a number line diagram by defininining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and the endpoint of the part based at 0 locates the number 1/*b* on the number line.

CCSS.Math.Content.3.NF.A.2b: Represent a fraction *a/b* on a number line diagram by marking off lengths 1/*b* from 0. Recognize that the resulting interval has size *a/b* and that its endpoint locates the number *a/b* on the number line.

CCSS.Math.Content.3.NF.A.3.: Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

CCSS.Math.Content.3.NF.A.3a: Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.

CCSS.Math.Content.3.NF.A.3b: Recognize and generate simple equivalent fractions, e.g., ½ = 2/4, 4/6 = 2/3. Explain why fractions are equivalent, e.g., by using a visual fraction model.

CCSS.Math.Content.3.NF.A.3c: Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.

CCSS.Math.Content.3.NF.A.3d: Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when two fractions refer to the same whole. Record the results of the comparisons with the symbols >, <. or = and justify the conclusions, e.g., by using a visual fraction model.

Mathematical Practice Standards

CCSS.Math.Practice.MP1: Make sense of problems and perservere in solving them.

CCSS.Math.Practice. MP2: Reason abstractly and quantitatively.

CCSS.Math.Practice.MP3: Construct viable arguments and critique the reasoning of others.

CCSS.Math.Practice.MP4: Model with Mathematics

CCSS.Math.Practice.MP5: Use appropriate tools strategically.

CCSS.Math.Practice.MP6: Attend to precision.

CCSS.Math.Practice.MP7: Look for and make use of a structure.

CCSS.Math.Practice.MP8: Look for and express regularity in repeated reasoning.

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[1] "Common Core State Standards," Common Core State Standards Initiative, accessed January 18, 2014, www.corestandards.org
[2] "Common Core State Standards"
[3] "Common Core State Standards"
[4] Jinfa Cai, Intended, implemented/planned, and achieved mathematical curricula in China and the United States: Some highlights from a decade of research. In Z. Usiskin & E. Willmore (Ed.), Mathematics Curriculum in Pacific Rim Countries – China, Japan, Korea, and Singapore. Charlotte, NC: Information Age Publishing.
[5] "Common Core State Standards"
[6] Susan B. Empson and Linda Levi, *Extending Children's Mathematics: fractions and decimals.*Portsmouth: Heinemann, 2011.
[7] George Polya, *How To Solve It.* London: Penguin, 1990.

Addendum 1

Name: _____ Date: _____

Fractions Pre-Test

1. Which is bigger 2/5 or 4/5? How do you know?

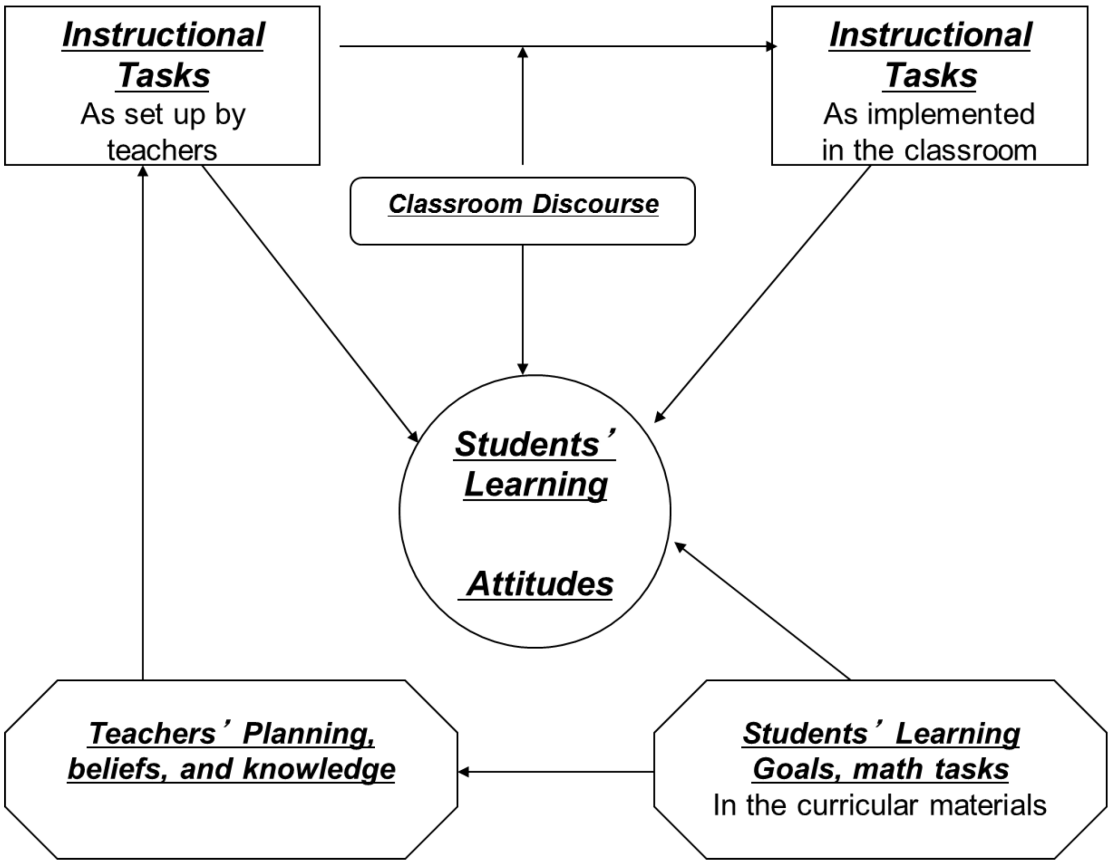
2. Using a number line label the fractions 2/4 and 4/8.
Which is larger? Justify your answer using evidence from the number line.

3. Nikki walked ¾ of a mile to school and ¾ of a mile back home. Roger walked ½ of a mile to school, ¼ of a mile to the store, and ¼ of a mile back home. Nikki exclaimed, "I am so tired. I walked so far today." "No way," said Roger,"I walked further than you." Who walked further? How can you show their total distance?

4. Ken ate ½ of a cookie, and Beth ate ½ of a cookie. Ken ate more than Beth. How is this possible?

5. 2/3 is a bigger fraction than ½. Keeping that in mind, how could the following be true? Jada used 2/3 of a bottle of nail polish to paint her nails black for Halloween. Her friend Gracie used ½ of a can of paint to paint the outside of her cardboard haunted house for a scary decoration. Why is ½ of the paint more than 2/3 of the nail polish?

Addendum 2



Addendum 3

Suggested Vocabulary List

Fraction

Numerator

Denominator

Half/ Halves

Thirds

Fourths

Equal Parts

Fair Shares

Number Line

Greater Than

Less Than

Equal/ Equivalent

Part

Whole

Addendum 4

When dividing a share of 18 objects, students may find 1/3 using some of the solutions below.

a. Students divide each picture into thirds. They then label the thirds A,B,C (or using letters/numbers they choose). After dividing all of the pictures into thirds, they then count the remaining pieces. As an example, finding 1/3 of 18, they would add 1/3 + 1/3 + 1/3… 18 times. This would leave them with the answer of 6 whole objects.

b. Students may draw all 18 objects. Rather than dividing the objects into thirds, they may circle groups of 3 and color in just 1 object (representing 1/3). The would follow this pattern for all 18 objects. In the end, 6 objects would be colored in.

c. Students may recognize that 18/3= 6.

Addendum 5

Third graders should be able to compare the size of fractions using drawings, a number line, and fraction bars. Once compared, they should be able to order fractions, and describe them using terms such as greater than, less than, and equal to/equivalent.

Sample problems include:

Comparing who ate more food using fractions such as ¾ of the pizza vs. ½ of a pizza or ½ of a candy bar compared to 1/6 of a candy bar

Would you rather questions using food (i.e. 2/3 or 6/8 of a pizza)

Questions challenging students to identify fractions larger than a benchmark fraction or equivalent to a benchmark fraction (i.e. Can you name 3 fractions bigger than 2/3? Can you name 2 fractions less than ½?)

Curriculum Unit Title	Fractions- Beyond Equal Parts	**Author**	Beth Vanaman

KEY LEARNING, ENDURING UNDERSTANDING, ETC.

Students can divide a whole into equal parts and write, represent, and compare fractions.

ESSENTIAL QUESTION(S) for the UNIT

How can I divide a whole into equal parts?
What models can be used to represent fractions?
How can you compare the size of fractions?

CONCEPT A	**CONCEPT B**	**CONCEPT C**
Equal Parts	Modeling Fractions	Comparing Fractions
ESSENTIAL QUESTIONS A	**ESSENTIAL QUESTIONS B**	**ESSENTIAL QUESTIONS C**
How can you divide an object into equal pieces?	How can a number line be used to show the size of fractions less than and greater than 1? What other representations can be used to compare the size of fractions?	How can you determine the larger of two fractions? What fractions are equivalent?
VOCABULARY A	**VOCABULARY A**	**VOCABULARY A**
Numerator, Denominator, Equal, Whole, Error Analysis Halves, Thirds, Fourths, Sixths, Eighths, etc.	Number Line, Fraction bars	Equivalent, compare, greater than, less than

ADDITIONAL INFORMATION/MATERIAL/TEXT/FILM/RESOURCES

Smart Board, Fraction Strips, Number Lines are recommended resources.

A pretest and sample student work is attached to the unit.

Websites such as www.corestandards.org and www.discoveryeducation.com are helpful for planning and implementing the lessons.