Appreciation for Fractals

Alicia Oleksy

Introduction

Conrad Schools of Science (CSS) is a magnet school geared towards science and mathematics serving students in grades 6-12. We are considered as an urban school on the outer edge of the city of Wilmington. My current teaching position is 8th grade Honors Geometry. These students are high achievers in the classroom. They are also exceptionally motivated to apply their math ability beyond the textbook. We are on block scheduling, so I see my student each day for 85 minutes for the entire year. This helps to accelerate the curriculum as well as enrich their experience of math outside of the regular course work. Prior to this year, my students had pre-algebra in sixth grade, Algebra 1 in 7th and then Geometry as they progressed through the honors program at the middle school level. Though the course of Geometry follows the traditional track of study as used in the high school, my students have the opportunity to be more hands on in this class much like they have been throughout the middle school program. Fractals seem to be quite appropriate for the students in this geometry class.

Rationale

What is a fractal? I am not quite sure that I fully understand the concept as of yet although I am constantly learning as I am researching this concept myself. Our curriculum at the current time briefly presents the ideas of fractals to students using a single worksheet and nothing else. The text does not allow for any further exploration on the topic and surely does not tap into how rich a topic this actually is. While initially researching the topic, it became clear that fractals deserve much more than a single worksheet to explore it. Also, the real world applications for fractals were even more evident. This proposed unit will allow the students to go beyond the text and have an appreciation of what a fractal really is as well as further my knowledge of the subject.

Students are always more engaged with a particular topic when they can manipulate things and use a hands-on approach. As students' progress from elementary into the middle school curriculum, it seems that this opportunity becomes less and less common. Certainly as they arrive to high school classes, this approach to learning is sometimes non existent. I see the topic of fractals as a means for students to go back to some of those hands-on experiences they have had in the younger grades leading up to the traditional Geometry class they are currently in. I also believe that this topic will help me reach some of my artistic types of students in that the work will revolve around either viewing drawings and depictions of fractals to having them create fractals.

The word "fractal" was coined in 1975 by the Polish/French/ American mathematician, Benoit Mandelbrot (b. 1924) to describe shapes which are detailed at all scales, suggesting fragmented, broken and discontinuous.[1] Fractals have actually been around for quite some time prior to the term fractal being used in 1975. Works by Georg Cantor in 1870 and Vaclav Sierpinski in 1916 were some of the earlier depictions of fractals. The age of computers was seen as a real jolt to the topic of fractals as works by Benoit Mandelbrot and Gaston Julia displayed the complexity of their designs. The early works of fractals and even the more modern versions are a good starting point for the students to step into the world of fractals.

By definition, a fractal is any geometric shape that is recursively constructed and has infinite complexity.[2] When magnified a fractal has a self-similar nature; it resembles the entire object. In mathematics, a fractal is a geometric object that has self similarity at any level. There is lots of detail if you look closely at a fractal, repeating a process over and over again can create fractals. Fractals can be found in real world, such as coastlines, clouds and mountains.

A fractal is a never-ending pattern. A fractal is a geometric shape that repeats indefinitely inside itself. The shape usually gets smaller. They deal with the concept of infinity. Georg Cantor made one of the first fractals in 1870. It was called Cantor's dust (Fig -1.1) Later, there was the Sierpiniski triangle, made by Vaclav Sierpinski in 1916. (Fig-1.2) Also, there is the von Koch curve, which Helge von Koch made in about 1900. (Fig-1.3) But the word fractal was not used until 1975, by Benoit Mandelbrot. He used the word for the different shapes that repeated themselves with an object. The word fractal comes from the Latin word for broken. Mandelbrot did not start his work till long after the first "fractaller" did. There have been many people through time to make fractals.[3]

Until computers came along, fractals were drawn by hand and not very interesting. People who were interested in different patterns in the nature made them. One natural fractal is the coastline fractal. Many people noticed this and just studied different coastline. The idea behind it is that if you measure a coastline in miles and draw a picture, it will be jagged and not have many details. If you measure it again with an inch ruler, it will have more details. You will see that each inlet has even smaller inlets. Also, many people have just used a straight line and manipulated it. One of those examples is the dragon. (Fig 2.1) The dragon is just a straight line that is twisted at 90-degree angles usually. The fractal gets smaller and smaller the more turns you add. Finally, it gets too small to see normally. It is a fractal because you can keep bending it for eternity. Also, there is the binary tree, which splits into 2 different branches forever (Fig-2.2). It is an early fractal, but is still important. These fractals usually seem boring compared to the computer fractals.

When computers came around, the fractals became more interesting. There are two very important fractals that are linked with computers. They are the Mandelbrot set (Fig-3.1) made by Benoit Mandelbrot, and the Julia set (Fig-3.2) made by Gaston Julia. The Julia set contains many fractals that are all equally famous, whereas the Mandelbrot set contains one fractal, which is more famous than other fractals including the Julia set. The only way to make these fractals is to use a computer to make them. The computer has really helped fractals.

There are many natural fractals. In fact that is what made many people start studying fractals. Some natural fractals are trees (Fig-4.1). Broccoli (Fig-4.2), ferns (Fig-4.3), coastlines (Fig-4.4), and other natural things. The broccoli has clusters that keep getting smaller, and the trees have braches that go into other branches. Also, the fern leaves repeat forever. The coastline, which I explained earlier, has smaller inlets. These are perfect fractals, but they are important. Arteries are also fractals. They branch off of each other and become small. These could someday be extremely important.

Fractals have many uses. Students need to use your imagination. One use of fractals is background for science fiction movies and commercials. Also, they can be used for a background on a desktop. Fractals are an efficient way of transporting material through a large area. This is why they are used in the human body. Fractals are used in fractal antenna. These antennas use a fractal patterns so that the minimum amount of metal can be put into a small area. Also fractals have to do with restricted population growth of successive generations of insects. Fractals are used to make music such as white sound. They are also used in architecture to make buildings better. They are ultimately used to depict different situations. Fractals have many uses and people are still discovering them today.

Fractals deal with the idea of infinity. This is because a fractal will repeat in itself forever. Also, the equations for some fractals have to do with tangents, sine, cosine, and logarithms. You need to know about trigonometry. There are also exponents and PI involved. Fractals have limits, many variables, and many points of data. Fractals are the same as they get smaller which is called self-similarity. This means that they are the same shape, or similar to itself, but not the same size, without math we could not have the cool computer fractals.

Fractals deal with measurements, spatial sense, algebra, and geometry. For example, students can calculate the area and perimeters in the Sierpiniski triangles. This unit on Fractals was written for a Middle School Geometry Class in addition to any regular curriculum. I plan to start the unit off by allowing the students to view many different examples of fractals. Students will come to recognize fractals in the real world, geometry, and algebra. The students can then begin to explore the Sierpinski's triangle which is a simple fractal created by repeatedly removing smaller triangles from the original shape. I would like them to color the triangle by how they see the pattern. This will allow me to see what types of patterns the students are viewing and also what types of abilities by viewing their patterns that they see from the patterns in the Sierpinski's Triangle. It starts with the basics of what a fractal is and lets students explore different fractal patterns. It will show the students how fractals are a geometric shape that repeats indefinitely inside itself. The initial introduction will look back at works from those mentioned previously and the significance of their works. As students are introduced to different simple fractals they begin to recognize the pattern and try to continue to expand it. They will explore similarity of the changing shape and perimeter in some cases. They will try to appreciate both the simplicity and complexity of the pattern leading them towards creating tables and equations represented by the fractals.

Fractals are very interesting after my research, so now let's begin to study a few of them I choose to look at a bit closer.

Objectives

The unit topic of fractals is sometimes thought of as an extension or an enrichment type of activity. As I mentioned previously, our current textbook offers a page exploration of the topic. It is brief, so it does not allow students to explore the topic and appreciate the topic. Currently, we are in the process this year of implementing the Common Core Curriculum into our classrooms. In looking at the Common Core Standards for Geometry, it is noted that the topic of fractals is not even mentioned in any of the guides concerning Common Core State Standards in Mathematics (CCSSM). This got me to think if the topic was even relevant to my curriculum. Upon further researching of the topic, I noticed that even though the word fractal is not even mentioned in the Common Core, there are many different activities that use fractals as a means of reaching other specific standards. Fractals become the vessel for leading us into a wide variety of standards. As students work with fractals, students will be involved in various exercises of congruence and similarity.

According to the Common Core for High School Geometry, the study of fractals will focus on the domain of Congruence. Specifically, students will be addressing the Common Core standard G-CO where students will experiment with transformations in the plane. This standard allows for students to work with different transformations that are present in the study of fractals. Students will work directly with rotation, reflection, and translations in the process of drawing fractals. Another domain that seems to be in line with that of fractals is in the area of similarity (G-SRT). Most notably students will understand similarity in terms of similarity transformations.

Prior to the unit, students should be familiar with geometric ideas and their relationship outside of the classroom as they present themselves in art, architecture, and in the real world. Students' works with flips, turns, and rotations in previous years will be built upon in this unit as they expand their understanding of those terms. Students should also have familiarity with rotational symmetry and line symmetry as described in the NCTM standards for middle school students. These fundamental skills are necessary for students to develop a better understanding of fractals and their connections to the world. Most students will have seen these terms and topics upon entering the middle school curriculum. I may be able to pre-assess these ideas via a warm up activity to see if it will be necessary to review briefly with the students. This will also be a good way to bridge the gap from previous understandings to the current topic of study.

Thinking and Reasoning Mathematically Through Solving Challenging Problems

During the seminar, it became quite clear that the problem solving process described below (Figure 1) provides people the means to make sense of challenging and everyday situations.

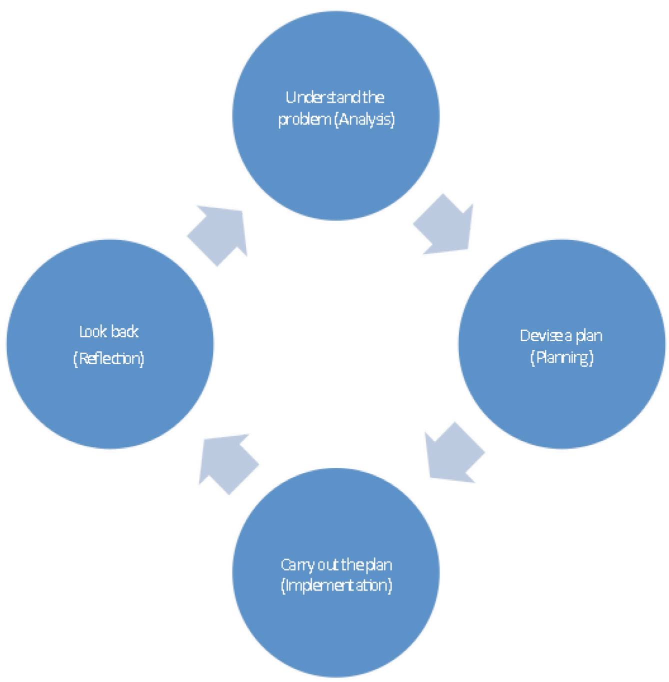

Figure 1

This process is an essential piece to the Common Core's eight Standards for Mathematical Practice. Pracitce number one states that students will be able to make sense of problems and persevere in solving them. The problem solving process as described in our discussion about Ploya duirng our seminar is in fact the very set of skills that are descibed in this mathematical practice. Understanding the problem involves many different things to consider. Students must be able to not only interpret the question being asked, but also consider the best method or strategey to inititiate the problem solving process.[4] As students carry out the plan, they must constantly monitor if their approach is leading to the correct solution and be able to make essential changes as they progress through the process. The reflection piece to me is actually something that is occuring throughout the process rather than at the end. As the Common Core describes, students should constantly be asking themselves, “Does this make sense?” This monitoring appraoch allows students to be flexible in their reasoning as they go through the problem solving process.

In the unit that I am presenting, the problem solving approach as it is intended in the Mathematical Practice, will be developed and practiced with my students. Fractal is a new and unfamiliar topic for the students so they must be able to make sense of what fractals are. The initial pursuit of this answer lies with the students ability to not feel overwhelmed with the topic, but to see it in terms that they will be able to understand. My first set of activites will make the entry into the topic less challenging to students and allow them to further develop their understanding of the topic.

Strategies

I plan to begin the unit with a basic study of fractals from the mathematical sense. Students will be presented with many visuals of fractals throughout history and in the real world to familiarize the students with what a fractal is as well as to introduce them to some of the pioneers of the topic. Upon inspection of these works, it is hoped that students will then be able to relate to the basic ideas of fractals.

Students will also be exposed to many opportunities for hands-on activities. One such activity will start with using cauliflower and the sense of self similarity. Another activity students could do is use hexagonal graph paper to create a Koch Curve fractal to help recognize and describe the symmetric characteristics of designs. By doing individual and group activities, students will be able to see how fractals are related to real world situations.

Upon being presented with the background knowledge of fractals and hands on explorations, students will then take on the task of creating their own fractals. Having the students create their own designs is a way for them to showcase their ability to apply what they have learned from the unit.

Classroom Activities

Lesson 1: Introduction to Fractals

Students will be asked the question of whether they know or have seen a fractal before. It will be presented to students the way that scientists use fractals in real life:

- earth scientists can predict the size, location, and timing of natural hazard such as forecast hurricanes, floods, earthquakes, volcanic eruptions, wildfires, and landslides using fractals
- doctors can use fractals to study how bacteria grow
- an economist may use fractals in the study of the behavior of the stock market

Students will be given a chance to do a hands-on introduction to fractals involving a piece of cauliflower. Groups of students will be given one cauliflower. They will be asked to note general characteristics of the whole cauliflower. Next, students will be asked to take note of one of the larger branches of the cauliflower and to notice how that branch looks similar to the whole cauliflower. Afterwards, students will be instructed to look at another small cluster in that branch. Again, students will be asked to take note of the how this smaller piece looks similar to the entire cauliflower. Students will be asked to look at another smaller piece and reflect back on its similarity to the whole. This process of self similarity is a great spot for students to look at other fractal situations in nature.

The following list of websites can be provided to students to self examine visual representations of fractals in the real world.

http://webecoist.momtastic.com/2008/09/07/17-amazing-examples-of-fractals-in-nature/

http://www.miqel.com/fractals_math_patterns/visual-math-natural-fractals.html

http://www.youtube.com/watch?v=QFE2n-OQSSk

Upon researching on their own, students will be asked to reflect back on what they viewed. Students will be asked to select a few representations and tell how they saw the process of self similarity being developed in each.

Lesson 2: The Sierpinksi Triangle

The Sierpinksi Triangle activity helps students see the idea of how fractals are built on the premise of repetition.

Each student will make their own fractal triangle by making smaller and smaller triangles. Students will be given the fractal triangle template.

Markers and crayons should be made available to the students. Students should connect the midpoints on the template and make a new downward facing triangle. Once they have made the triangle, they will be instructed to color in the downward facing triangle only. This will leave three upward facing triangles which are similar to the original, but half the width. They will then place midpoints on the three upward facing triangles. These midpoints are now to be connected making three smaller downward facing triangles. Students will now color in these three triangles. They will repeat this process for at least 3 iterations.[5]

When they are finished, students can cut their triangles out and join them with others in the class to form a larger version of the same shape. Triangles of 9, 27, or 81 work best.[6]

As a follow up to this activity, students will work on the Fractal Triangles student exercise in their groups.[7]

Lesson 3: Jurassic Park Fractal and Koch Curve

Fractals aren't actually on the part of the Common Core Curriculum for Geometry – but they do offer quite a good opportunity to look at different things such as limits, infinite sequences, complex numbers, and the relationship between math and art. One activity we are going to explore is the Koch snowflake. It will be a great hands one activity for the students to begin with an equilateral triangle and then replace the middle third of every line segment with a pair of line segments that form an equilateral "bump". In this students will do 2 different activities. One using the triangle and the other referring to the book or movie Jurassic Park.

Jurassic Park Fractal

Bring in a copy of Jurassic Park.

Have you ever read *Jurassic Park*? Most people have seen the movie where scientific technology was able to create an island full of living dinosaurs but did you read the book? If you have, then you may have noticed some interesting drawings on the chapter heading pages. They're labeled *First Iteration, Second Iteration, etc.*, and they get more and more complex with each iteration. This is in fact a fractal in its earliest stages that continues to grow as the book goes on.[8]

To construct this fractal, students are going to do a paper folding activity using a strip of construction paper 1- inch by 11- inches.

Students can follow along with the following "Dragon Curve" video clip stopping the video as they do an iteration at a time as they do the folding. The clip helps students make the connection with the book's chapter headings and the folding activity into the fractal.[9]

http://www.youtube.com/watch?v=wCyC-K_PnRY

Take the strip in both hands, fold the paper over end-to-end, right hand onto left, and crease. Now fold it again in the same way, right onto left, and crease, and again, right onto left and crease, and again right onto left and crease. You have folded it four times in all.

Before you open the paper out, can you imagine what it will look like unfolded? Let's unfold and see. Now you may be saying, this is just a mess of folded paper. How can this be a fractal? Well, we have to put some order to it. The video clip can be viewed from this point for students to start to see the pattern that is showing up.

Koch Snowflake Activity

In this activity, students will not only construct the Koch snowflake, but also examine how area and perimeter change throughout the progression.

Students will construct an equilateral triangle with side lengths of 9 inches in the center of their paper. Make sure that students have rulers and protractors available so that these constraints are met. Once this triangle is constructed, students should make observations about the triangle's area and perimeter and record this information onto the Observations page. After their measurements have been recorded, students will then divide each side of the triangle into 3 equal segments and draw 3 more equilateral triangles on the exterior of the original triangle using the middle segments as the bases of the new triangles as in the figure below (Figure 2).[10]

Figure 2

Students should shade in the new triangles the same color so that they may see the iteration (Figure 3) that has just taken place. Students will then compute the area and perimeter of these new triangles and add them to the Observation page. This process is continued and observations made as newer triangles are added to the original figure.

Figure 3

Observations WS

Name:________________________

	Triangles Added	Total Triangles	Sides Added	Total Sides	Perimeter Added	Total Perimeter	Area Added	Total Area
Original		1		3		27		35.07
1								
2								
3								
4								
5								

Discussion Questions

Students will use the information collected in the table to help them answer the discussion questions.

1. What would the area and perimeter be if another iteration was completed?

2. From this pattern, let's see if we can come up with a formula for finding the area and perimeter for any number of iterations.

3. What do the angles of the snowflake have in common?

4. What can we say about the perimeter?
5. What can we say about the area?

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Appendix A

CCSS G-CO Congruence: Experiment with transformation in the plane

CCSS G-CO Congruence: Understanding congruence in terms of rigid motions

CCSS G-SRT Similarity right triangles and trigonometry: understand similarity in terms of similar transformations

In this unit students will experiment with transformations in the plane. This standard allows the students to experiment with different transformations that are present with the study of fractals. Students will work directly with rotation, reflection, and translations in the process of drawing fractals.

Appendix B

The fractals from the report.

Fig 1.1

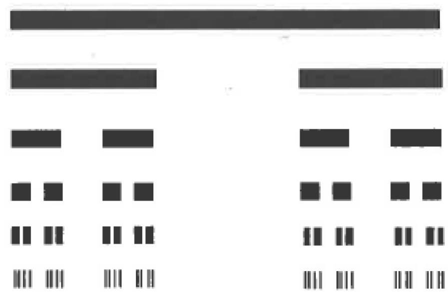


Fig 1.2

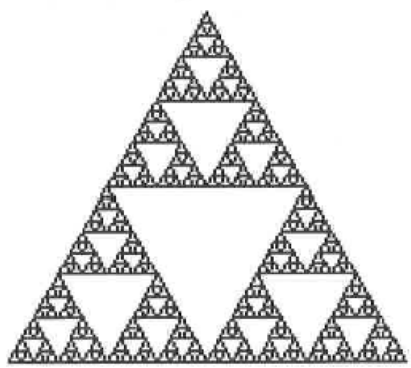


Fig 1.3

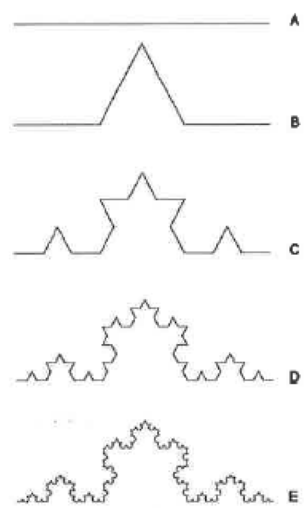


Fig 2.1

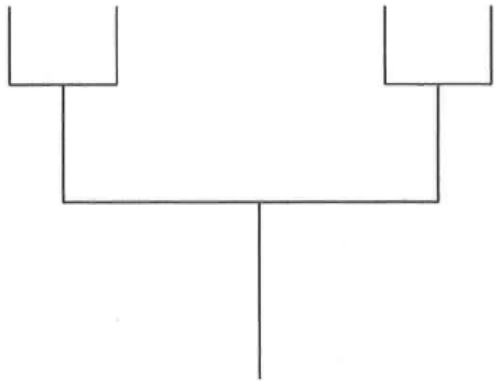


Fig 2.2

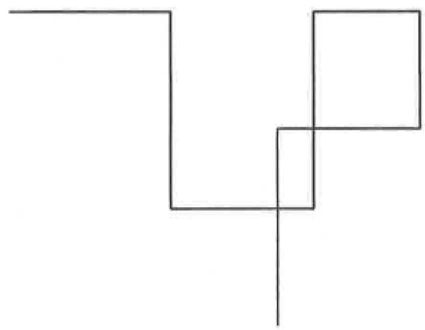


Fig 3.1

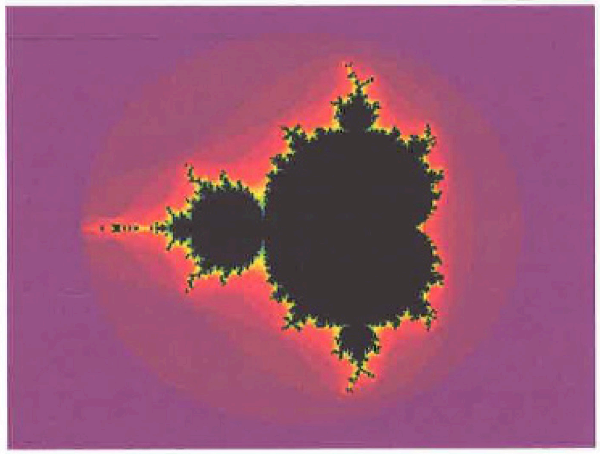


Fig 3.2

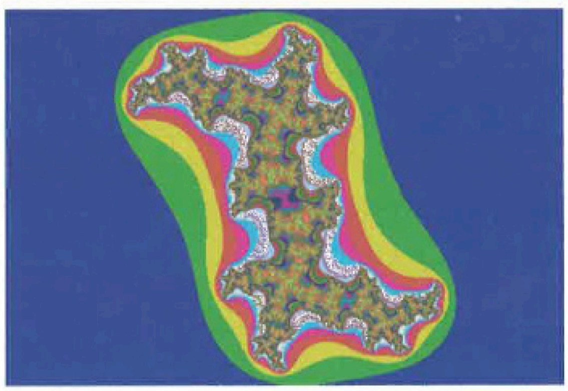
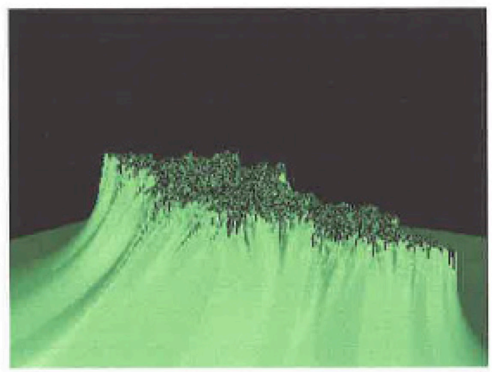


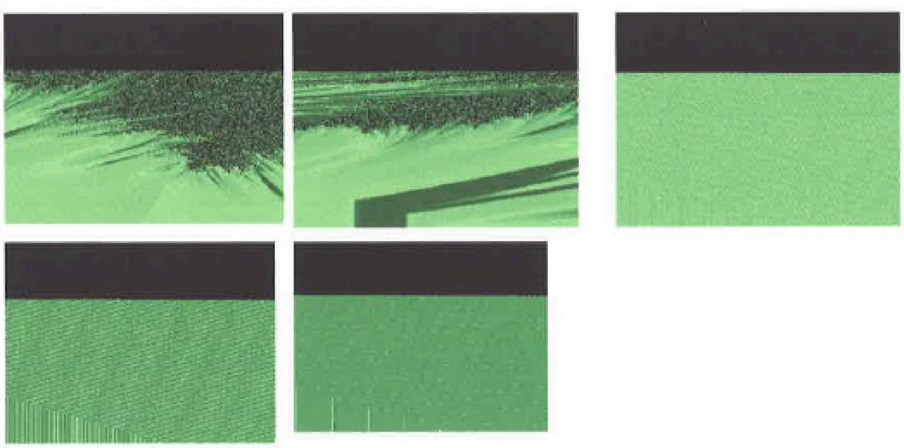
Fig 4.1

Fig 4.2

Fig 4.3

Fig 4.4





Fractal Triangle Template

Instructions:

The three dots on the sides of the triangles are called the "midpoints" of the sides, and they are half way from one corner to the other.

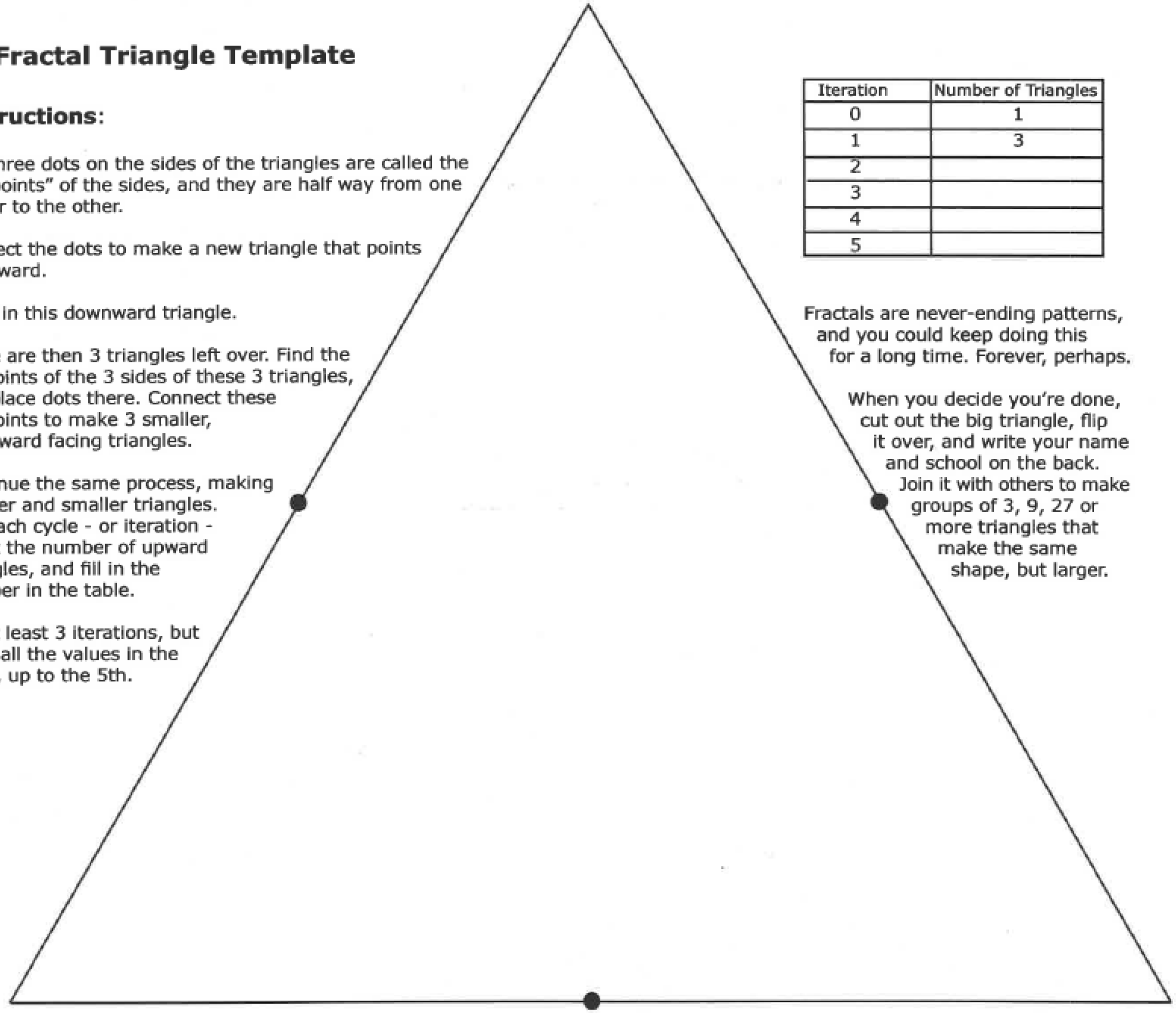
Connect the dots to make a new triangle that points downward.

Color in this downward triangle.

There are then 3 triangles left over. Find the midpoints of the 3 sides of these 3 triangles, and place dots there. Connect these midpoints to make 3 smaller, downward facing triangles.

Continue the same process, making smaller and smaller triangles. For each cycle - or iteration - count the number of upward triangles, and fill in the number in the table.

Do at least 3 iterations, but fill in all the values in the table, up to the 5th.



Iteration	Number of Triangles
0	1
1	3
2	
3	
4	
5	

Fractals are never-ending patterns, and you could keep doing this for a long time. Forever, perhaps.

When you decide you're done, cut out the big triangle, flip it over, and write your name and school on the back. Join it with others to make groups of 3, 9, 27 or more triangles that make the same shape, but larger.

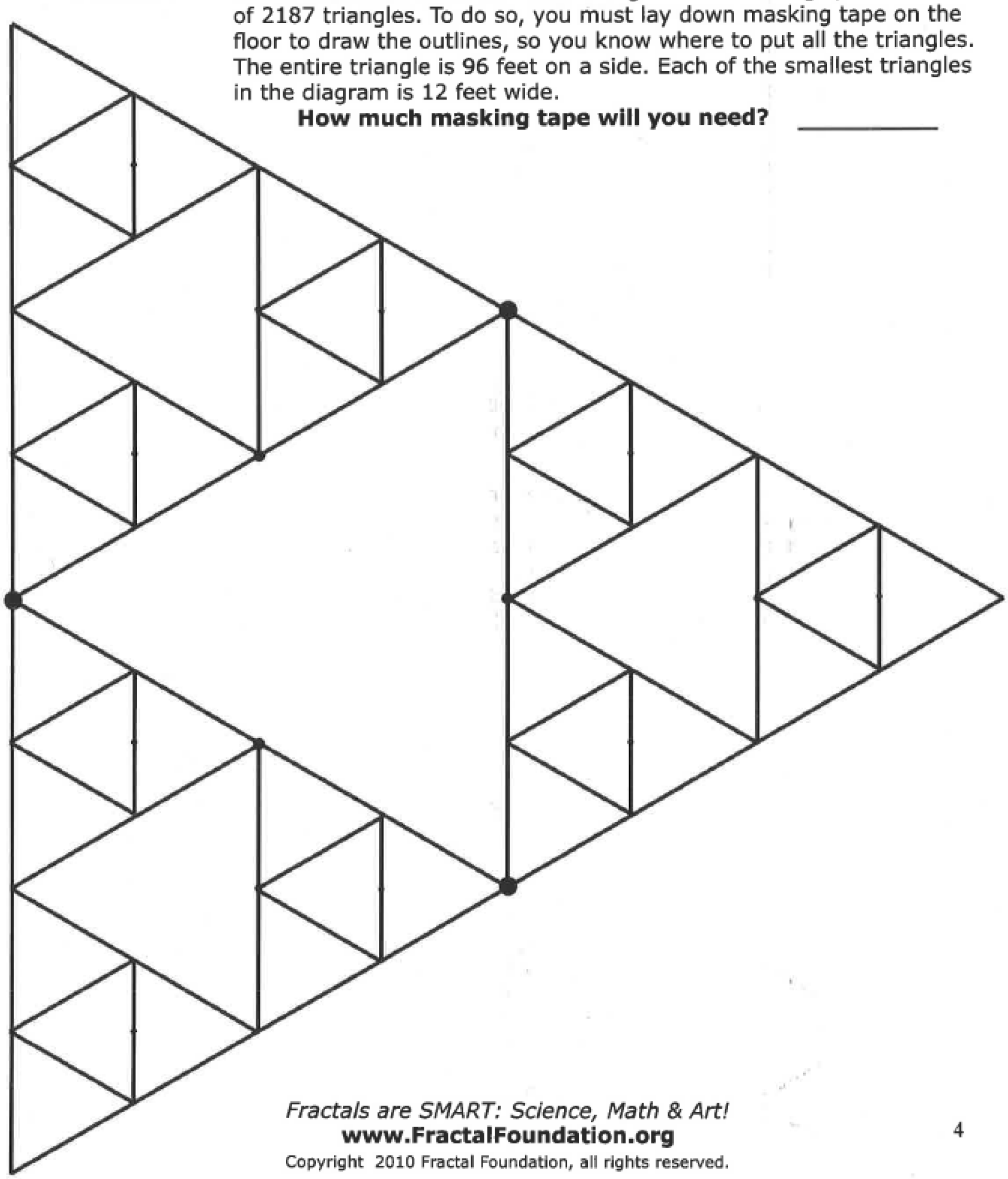
Fractals are SMART: Science, Math & Art!
www.FractalFoundation.org

Fractal Triangles

Student Exercise

Problem: You are about to build the world's largest fractal triangle, made out of 2187 triangles. To do so, you must lay down masking tape on the floor to draw the outlines, so you know where to put all the triangles. The entire triangle is 96 feet on a side. Each of the smallest triangles in the diagram is 12 feet wide.

How much masking tape will you need? __________



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KEY LEARNING, ENDURING UNDERSTANDING, ETC.

Self-similarity, patterns, congruence, transformations in the plane, symmetry

ESSENTIAL QUESTION(S) for the UNIT

How can we utilize the properties of fractals to discover their occurrence in real world situations?

CONCEPT A	CONCEPT B	CONCEPT C
Fractal	Repeating patterns	Area and perimeter
ESSENTIAL QUESTIONS A	**ESSENTIAL QUESTIONS B**	**ESSENTIAL QUESTIONS C**
What are the characteristics of fractals? How do smaller pieces of a fractal relate to the whole?	How are fractals built on the premise of repetition?	How are area and perimeter affected by repeated iterations?
VOCABULARY A	**VOCABULARY B**	**VOCABULARY C**
Fractal Self-Similarity Patterns	repetition Midpoint Iterations	Infinite sequences equilateral Area triangle perimeter

ADDITIONAL INFORMATION/MATERIAL/TEXT/FILM/RESOURCES

http://webecoist.momtastic.com/2008/09/07/17-amazing-examples-of-fractals-in-nature/

http://www.miqel.com/fractals_math_patterns/visual-math-natural-fractals.html

http://www.youtube.com/watch?v=QFE2n-OQSSk