Creating, Analyzing, and Interpreting Line Graphs

Beth Vanaman

Introduction and Rationale

Delaware is at a crossroads in education. During the current 2012-2013 school year, I am teaching to the Delaware Prioritized Standards. At the end of the year my students will be tested on these standards, and I will be held accountable for their test scores. However, by 2014, it is an expectation that all schools in Delaware will be teaching in accordance with Common Core State Standards (CCSS). With less than two years before transition to full implementation and national assessments, I am still learning the new standards myself, how they fit in with what I am currently teaching, and what I will need to supplement current lessons so that they are aligned with CCSS.

When sitting through a recent in-service, which reviewed our current practices and where the district falls short in alignment, I realized that many lessons and activities had been highlighted as covering both the CCSS and the Standards for Math Practice "Modeling with Mathematics." Any lesson using Unifix Cubes, tens frames, hundreds charts, array cards, etc. got the label. I did not, however, feel that it was just that simple. When the Delaware Teacher Institute offered a course on the topic, I jumped at the opportunity to learn about this expectation in-depth. For "Modeling with Mathematics" to be both a CCSS standard and Math Practice standard, I felt that understanding it was crucial to both planning and implementing lessons to address the standard. Over the course of the Fall 2012 seminar, I learned that modeling is not simply using manipulatives or using drawings to represent equations, but it is a complex cycle that takes the learner through observations and experimentation to develop the mathematics to explain experiments and phenomena.

In this unit, students will learn the foundations of Mathematical Modeling and begin to examine the Cycle of Inquiry that drives so many math lessons. By

> "Using the model method, teachers are able to present to students problems that demonstrate how concrete problem situations and accompanying algebraic ideas can emerge from real-world contexts without the use of formal algebraic notation. This approach helps promote several important aspects of algebraic thinking, such as the organization of commonalities among problem situations, the viewing of how change in one relationship affects change in another, and the overall ability to make important mathematical generalizations."[1]

My students will gather data, and use it to create line graphs. They will analyze and interpret their graphs and given graphs. Rather than writing three statements about the graph and stopping, which is all the current curriculum calls for, students will also study what happens when scales vary, axes are switched, and other factors are introduced. Video clips will supplement the lesson on scales, as well as a graph on racecar track and greyhound race times. A final project will offer students a variety of modalities from which they can display their understanding of the unit.

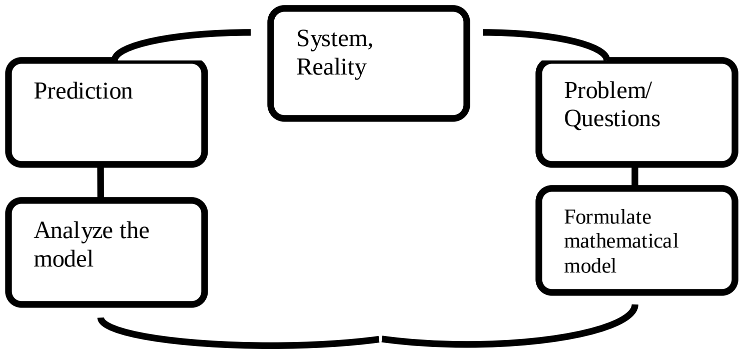
Demographics

Wilmington Manor Elementary School, of the Colonial School District, has approximately 475 students in grades K-5. Most of the students have little to no exposure to the world beyond their own neighborhood and a small stretch of highway known as Route 13 (fast food joints, schools, and drug stores). In this small school, there is a high population of Low Socio-Economic Status (LES) students and English Language Learners (ELL). 68.4% of students in the building receive free and reduced lunch. With so many students below poverty level and so many parents working multiple jobs, my students often report that they have no help afterschool with homework or assistance with reading. They are also unable to attend afterschool events or curriculum nights which offer guidance with math strategies, reading techniques, and so forth. 37.1% of students at Wilmington Manor Elementary School are classified as ELL. Many students are their family's only English speaker. Many parents cannot read my instructions and notes in English, and they are unable to help their students with their English work.

With Delaware Standards converting to CCSS, relying on a textbook and supplemental workbook pages will not suffice. Some textbooks have simply added 2-3 additional pages per unit to cover the state to national standards gap. For example, the TERC Investigative Series has provided teachers with book inserts (2-3 pages per unit) with supplemental lessons and student workbook pages. With limited resources, other means will have to be used.

Modeling with Mathematics

Modeling with Mathematics follows a cycle that is never-ending. If students are able to understand this cycle, students will be able to fluently adjust to a variety of math problems and situations. Projects and lessons created to introduce and sample this cycle at the elementary level will allow for exposure to critical skills as students mature mathematically. While there are many cycles, the one below is the one which was utilized in our seminar. It is the cycle that I refer to throughout my unit. This cycle can include an additional step, "Simplification," following "Formulate Mathematical Model"



The Mathematical Modeling Cycle

The first step of the model is System, Reality. At this starting point, students are able to observe the whole system. They may participate in an experiment, observe an experiment, or witness a natural occurrence. In the second step, students can begin to generate questions about what they witnessed, what they want to know/understand about the data, and then develop a problem which could be solved using mathematics. The third step, titled Formulate Mathematical Model, is one typically presented to students in math textbooks or by the teacher. However, by incorporating this step into a cycle, students are able to develop their own way of modeling, rather than having a specific way given to them.

> "During mathematics instruction, teachers usually give children well-defined problems to solve, problems whose solutions involve discovering a single well-defined answer… Finding their own problems to solve is important for children."[2]

Students create graphs, line plots, tables, and other ways to represent data. When Analyzing the Model, students make observations about the data representations, question whether results are duplicable, and discuss why the data is a best fit. By carefully following these steps, working and revising their model and mathematics, and creating a rich discussion forum in the classroom, students get to the last and most important step, Prediction. Models should be refined and the cycle repeated until the data and mathematical model allow for predictive power. When your model allows for prediction without having to repeat the experiment again and again, when your model lets you do something (predict) that you otherwise couldn't do, that is when your model has predictive value.

After completing the cycle, once or multiple times, the learner should be left with a deep understanding of the problem and mathematics used to explain it.

> "Mathematical reasoning develops in classrooms where students are encouraged to put forth their own ideas for examination. Teachers and students should be open to questions, reactions, and elaborations from others in the classroom. Students need to explain and justify their thinking and learn how to detect fallacies and critique others' thinking."[3]

Mathematical Modeling in the Elementary School

Mathematical Modeling looks different across grades. While students in elementary schools may not be ready to write algorithms, differentiate functions, or to devise a line of best fit using a graphing calculator, younger students are able to navigate the modeling cycle with guidance from a teacher. In Delaware, kindergarteners are introduced to comparing and contrasting. This skill continues on in first grade where students are expected to represent and interpret data. CC.1.MD.4 states

> "Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than another."[4]

Already, students are modeling mathematically and beginning to analyze a model; albeit basic, in the very early grades. Much like the leap between the kindergarten and first grade expectations, students' mathematical goals develop quickly throughout grades two thru five. By fifth grade, students will be asked to create and analyze line plots to solve real world problems i.e. *Given the measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*[5]

In this unit, students will be introduced to the mathematical modeling cycle by collecting and recording data about students' speeds while walking, creating line graphs, comparing line graphs, and analyzing related graphs.

Objectives

- Students will be able to gather data using a table.
- Students will be able to create a line plot with a title, labeled axes, and correctly placed data points.
- Students will be able to analyze both created and given line plots.
- Students will be able to predict further points on a line plot using points already plotted.
- Students will be able to compare and contrast variables in graphs and how they influence the graph's interpretation.

Essential Questions

What do I notice about the data collected?
How can I best represent the data that's been collected?
How can the line graph be described- constant speed, increasing speed, decreasing speed, etc.? Are there any stopping points (plateaus)?
How does scale influence the interpretation of a graph?
Will measuring the speed of other laps (i.e. racecars, animals) change the outcome?
Do the representations created have predictive value?

Strategies

I will utilize a variety of modalities to actively engage students in the unit. Students will rely on their background knowledge of previous math lessons to create and read line graphs. In addition classroom discussion and participation, along with teacher instruction will aid student understanding. Appropriate graph paper and provided graphs, as well as video clips will finalize the various teaching modalities utilized in this unit. This unit was created to span at least a week of instruction.

Background Knowledge

Students will need to know how to create a line graph in order to complete this unit. They have already learned (in Second Grade) how to create a line graph with an x- and y-axes, evenly spaced scales (counting by ones or twos), and to plot data points. The same skill was reviewed in fall of their third grade year. Students can also write basic statements about graphs such as numerical information (i.e. 8 students voted for hamburgers as their favorite food or 35 people like green the best) and generalizations like most people, very few people, and so on. While writing simple statements is all that is required in our curriculum, the state testing programs asks much more difficult questions requiring students to match graphs to written scenarios and to interpret sections of a graph.

Teacher Instruction

As a teacher of this unit, it is my job to foster an environment where students feel comfortable to learn, make mistakes and revisions, and to engage in discussion. I will carefully explain directions and ensure that all students understand their roles, my expectations, and the desired outcome. My guiding questions will have students interpreting graphs, questioning theirs and others' interpretations of graphs, and forcing them to reflect upon their representations.

Graphing

Students will be provided with tables to gather data and graph paper to create line plots. It will be up to students, however, to label the axes, decide on an appropriate scale, and plot the data points. Students will also be given teacher-created graphs to analyze and interpret. They are provided in the investigations section of this unit.

Video Clips

A Cyberchase video titled "Raising the Bar" from PBS will supplement a lesson on how scale influences graph interpretation. The video can be found at http://www.pbs.org/parents/cyberchase/episodes/season-2/. The video compares two exterminators who graph the amount of bugs they have killed. One exterminator uses a scale 0-100. The other uses a scale of 0-10. An explanation as to why scale is important is taught to children in a understandable and entertaining way with the video. Later in the unit, students will see clips of racecars completing laps at Dover Downs to explain a related graph and of greyhounds racing for the same purpose. Those videos can be found at http://www.youtube.com/watch?v=vPizUOy7DiA (Dover Downs Video from 00:41- 2:41) and http://www.youtube.com/watch?v=jculKwbtjvE (Greyhound Video from 00:45-1:45).

Classroom Activities

Investigation One

Students will begin by walking four laps around the school track and recording their data on a table. They will record the time for each lap in seconds. At the end of each lap, I will provide the time for them to write. Each lap will begin at 0 seconds.

	Lap 1	Lap 2	Lap 3	Lap 4
Time				

Upon coming inside, I will ask students to complete a T-Chart about their data.

"I Think, I Wonder" Chart

What I Notice	What I Wonder

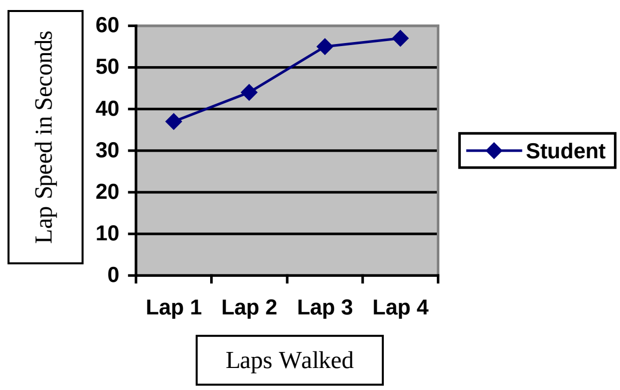
Using the t-chart, students will share out their thoughts and questions about the data collected. Below is a sample "I Notice, I Wonder" chart.

What I Notice	What I Wonder
I am faster than my friend. My first lap was my fastest.	Why are some people so much faster? I wonder if I I can go faster next time.

I will question students as to how they can then best represent the data so that they can analyze and interpret it further. I will ultimately lead them to a line plot; however, other options can be explored and eliminated if they are not working.

Students will be supplied with graph paper to create a line plot which represents their data. Axes should be labeled distance (x-axis) and time in seconds (y-axis). Students can discuss and work with those around them to decide what increments they need to use for time (increments of seconds will be needed). Once the graph is set-up, students will be expected to correctly plot their data points for the laps walked. Below is a sample graph with the lap times of 37, 44, 55, and 56 seconds.

Sample Student Graph



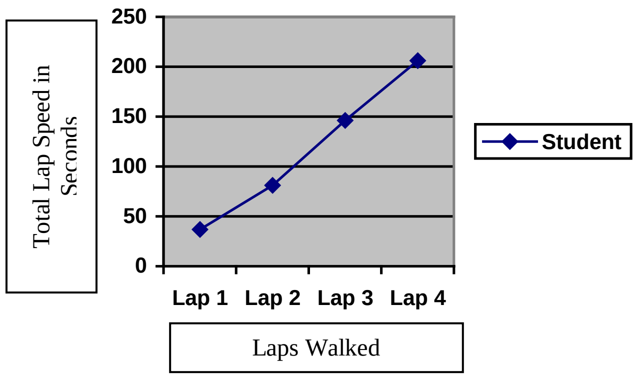
I expect that students will be able to correctly set-up the graph and accurately map the points. They should be able to recognize what scale to use so that all of their information fits on the graph.

Next, I will ask students to summarize their graph by describing the line. Is your line showing a constant speed (straight line)? Is your line increasing or decreasing? Students will also be asked if there is any way to predict the time of a fifth lap. They will be given time to discuss how to best predict. Students will be able to extend their graphs to show a hypothetical fifth lap. In a whole-class discussion, we will ponder why some people's data showed them slowing down (Were they tired?) or speeding up (Were you rushing to finish?).

Investigation Two

Students will use their tables to determine their total walk time. They will then be asked to create a line plot which shows the time that they finished each lap, as opposed to their previously created line plot where each lap began at 0 seconds. Once created, students will be asked if their new graph shows any new information. They will be asked why this new graph might be beneficial. What does it show? Compare and contrast questions will accompany this activity. Below is a sample graph. The times were plotted by adding together the lap times on the previous graph.

Sample Student Graph 2

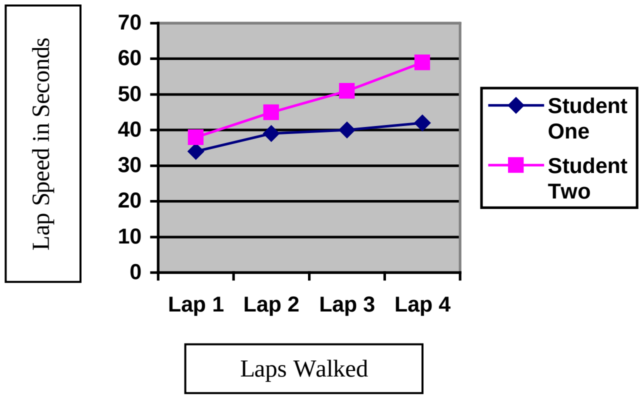


Again, I expect students to be able to correctly set-up a graph including scale and to accurately plot points. Students should be able to describe the direction of the line and answer whether it is similar or different to the previous graph, formulate reasons why the graphs are similar, and describe what new information comes from the graph (i.e. I can see how long the laps took me).

Investigation Three

Students will be presented with a graph comparing two students. One line will be that of a student who completed the laps quickly. The other line will be that of a student who completed the laps much more slowly. (A trial run of this lesson had students finishing in a span of 2 minutes). Below is the sample graph. Student One (the quick student) completed the laps in 34, 39, 40, and 42 seconds. Student Two (the slower student) completed the laps in 38, 45, 51, and 59 seconds.

Student One v. Student Two

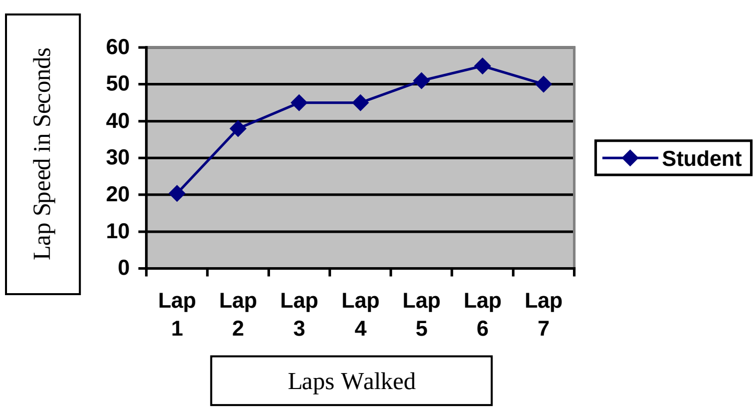


Students will examine the graph and complete the "I Notice/I Wonder" t-chart again. They will be expected to compare and contrast the two lines, explain the differences, and predict the difference between each graph's fifth lap. Upon completing those steps, students will be posed with the question: "Will Student B ever pass Student A?" Students will complete their short answers in sentence form using specific data from the graphs as evidence to support their argument.

Investigation Four

Students will be presented with a teacher-created graph that shows increasing and decreasing lines; as well as a plateau.

Hypothetical Graph 1



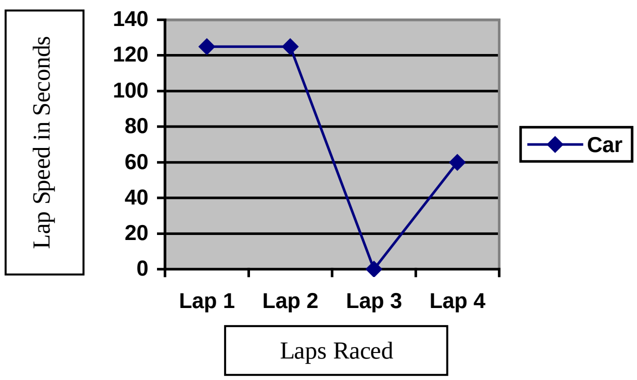
I will ask students to generate possible explanations for the plateau. Does the plateau represent a stop or slow down? How do you know? Students will then write a short paragraph to describe the graph with times listed to support their explanations.

Terms such as increasing, decreasing, constant speed, plateau, and stop will be expected in their detailed piece.

Investigation Five

Students will view a You Tube clip of a few laps of a Nascar Race at Dover Downs. The clip can be found at http://www.youtube.com/watch?v=vPizUOy7DiA. The section to be shown is 00:41- 2:41. In the background there are several of the racecars going in for a pit stop as well as a few race cars involved in a collision. Following the video clip, students will be presented with a teacher-created graph that shows the speed of the race car during each lap.

Nascar Graph

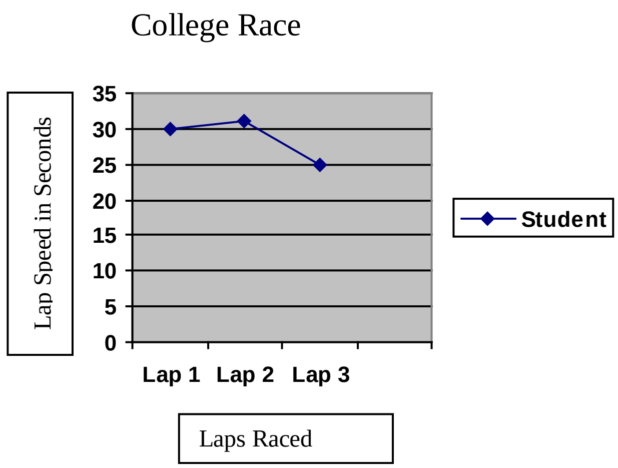


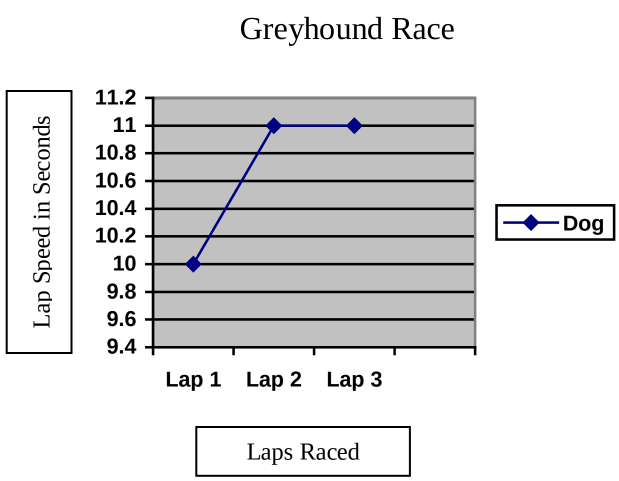
Students will be given lined paper and time to use evidence from the video to create a written explanation of the graph. They will be expected to explain why the sudden decrease in speed occurred. They will be required to use numerical data. I will listen to their discussions with peers while examining the graphs, and I will read their explanations to gauge student understanding that the graph shows the very fast racecars, the stop when the collision occurs, the slower speed when the car is leaving the race track.

As an extension activity, students will be asked to use graph paper and create a graph that shows lap speed on the y-axis (as opposed to car speed) and laps on the x-axis. They will then write an explanation as to why the graphs look different.

Investigation Six

Students will view a You Tube clip of a few laps (including the end) of a collegiate track race and a greyhound race. Students will match the correct graph with the correct clip. The collegiate race can be shown in its entirety at http://www.youtube.com/watch?v=xPtRI44kmSQ. The greyhound race can be found at http://www.youtube.com/watch?v=jculKwbtjvE. After the video, students will be asked to formulate reasons as to why the human racers sped up at the end (adrenaline rush to the finish) and why the greyhounds did not (faster than humans but no late burst of speed). We will generate a class list of possible explanations for the differences in the graphs. Students will ultimately need to think of the difference in behavior between humans and animals. Do the animals realize that they need to increase speed towards the end to cross the finish line first?





Investigation Seven

Students will be asked to create a graph and write an explanation for the graph as a culminating project for the unit. In the final project, students must collect data on a topic of their choice, create a line graph representing their data, and explain their data to others in a visual presentation or oral report. They will be scored on valid data collection, creating an accurate line graph, correctly plotting points on a line graph, and carefully explaining their data.

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Teacher Research Resources

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Schiro, Michael. *Integrating Children's Literature and Mathematics in the*

Classroom: Children as Meaning Makers, Problem Solvers, and Literary Critics. New York: Teacher's College Press, 1997.

Resources Used in Unit

"Dover Monster Mile NASCAR 2009." [2009]. Video clip. Accessed January 6, 2013. YouTube. *www.YouTube.com*, http://www.youtube.com/watch?v=vPizUOy7DiA.

"Raising the Bar." [2003]. Video clip. Accessed January 6, 2013. PBS. *www.pbs.org*, http://www.pbs.org/parents/cyberchase/episodes/season-2/.

"Wimbledon Greyhound Race 2009." [2009]. Video clip. Accessed January 6, 2013. YouTube. *www.YouTube.com*, http://www.youtube.com/watch?v=jculKwbtjvE.

Appendix A

Delaware Prioritized Standards

Standard 4: Quantitative Reasoning: Students will develop Quantitative Reasoning and an understanding of Data Analysis and Probability by solving problems in which there is a need to collect, appropriately represent, and interpret data; to make inferences or predictions and to present convincing arguments; and to model mathematical situations to determine the probability.

Collect: Collect categorical and numerical data to answer a question posed by a teacher or student.

Represent: Demonstrate a variety of informal and conventional techniques for representing and organizing categorical and numerical data.

Analyze: See and describe data as a whole describing the shape of distributions; reason about how individual pieces of data relate to the whole.

Probability: Describe the likelihood of an event based on experimental observations using simple randomizing devices (e.g. spinners, number cubes) and ideas such as certain, impossible, and equally likely.

Common Core State Standards

Mathematical Practice Standard 4, Modeling with Mathematics: Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision late. They are able to identify important quantities in a practical situation and map their relationships using tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Represent and interpret data:

3. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two- step “how many more” and “how many less” problems using information presented in scaled bar graphs. **CC.3.MD.3**

Curriculum Unit Title: Creating, Analyzing, and Interpreting Line Graphs

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KEY LEARNING, ENDURING UNDERSTANDING, ETC.

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Students will be able to analyze both created and given line plots.
Students will be able to predict further points on a line plot using points already plotted.
Students will be able to compare and contrast variables in graphs and how they influence the graph's interpretation.

ESSENTIAL QUESTION(S) for the UNIT

How can line graphs be used to represent data and provide information?
Do line graphs have predictive value for data not yet shown?

CONCEPT A	CONCEPT B	CONCEPT C
Creating Line Graphs	Analyzing and Interpreting Line Graphs	Predictive Value
ESSENTIAL QUESTIONS A	**ESSENTIAL QUESTIONS B**	**ESSENTIAL QUESTIONS C**
How can I best represent the data that's been collected? What scale should I use so that my graph portrays the accurate information that I want the reader to understand?	How can the line graph be described? How does scale influence the interpretation of a graph?	Will measuring the speed of different things (humans, racecars, dogs, etc.) change the outcome? Do the representations created have predictive value?
VOCABULARY A	**VOCABULARY A**	**VOCABULARY A**
Line graph Intervals Scale X- axis and Y-axis	Constant Increase and Decrease Plateau	

ADDITIONAL INFORMATION/MATERIAL/TEXT/FILM/RESOURCES

The unit requires access to PBS Cyberchase videos and www.YouTube.com.

[1] Greenes and Rubenstein, *Using a Model Approach to Enhance Algebraic Thinking in the Elementary School Mathematics Classroom* (Reston, VA: NCTM, 2008), 196.
[2]Brown and Walter, *The art of problem posing* (Hillside, NJ: Erlbaum, 1983), 34.
[3]Cuevas and Yeates, *Navigating through Algebra in Grades 3-5* (Reston, VA: NCTM, 2001), 68.
[4] National Governors Association Center for Best Practices, Council of Chief State School Officers, *Common Core State Standards* (Washington D.C.: National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010).
[5] National Governors Association Centers for Best Practices, Council of Chief State School Officers, *Common Core State Standards.*