Rolling Your Own

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Introduction

My unit will be on the topic of probability. Since I am only in my third year of teaching, there were so many things that I could focus in on to improve my teaching. However, I kept coming back to the same topic, probability and statistics. This seminar taught me how to take students through the modeling process, with the ultimate goal of making predictions about the world around them.

What better way to teach students modeling than through the study of probability and statistics? Unfortunately, in the past 2 years I had fallen into a rut of teaching my students to be casino junkies. They are able to calculate the odds of winning a game with number cubes, spinners, and cards, but are unable to relate those skills to the outside world. I did not want to continue this trend.

In my first year of teaching, a parent once brought up the subject of how I teach probability. He told me that he worked for a university and dealt with probability on a daily basis. He was concerned that the probability I was teaching was much different than the probability he used on a daily basis. This got me thinking, am I really preparing my students for probability in the real world? My unit attempts to remedy this by having students use the probability that they have learned to make real life conclusions. Common Core Standards require that I teach students how to calculate compound probability, sampling, experimental probability, and simulations. In my unit, the students will apply these skills to what they have learned in science about genetics to make a prediction about their school and town. It is my hopes that they will take these isolated skills that they have learned in the classroom to model the world around them.

Demographics

I teach Mathematics to 7^{th} graders at Louis L. Redding Middle School. We currently have 743 students at Redding Middle School, however, that population seems to grow with each passing month. Of those 743 students, 237 of them are in 7^{th} grade. Currently, there are 3 math teachers, as well as 2 special education teachers and one paraprofessional involved in the math education of these 7^{th} grade students. I am responsible for the math education of 125 of these students.

I teach a diverse population. Redding Middle School's student population consists of the following proportions. According to the Delaware Department of Education[1], 33.9% of Redding Middle School's students are African American, 3.4% are Asian, 7.3% are Hispanic, 53.4% are White, 1.9% are Multi-Racial. The students are also varied in terms of socio-economic status and ability. Students from Redding Middle School come from both high income and low income areas. 28.9% of the students are classified as low income. Many of the students receive free or reduced lunch as well as breakfast in the morning. In terms of ability, 17.6% of the students in my classroom are classified as Special Education and 24% of the students in my classroom are classified as Advanced.

With varying levels of abilities and backgrounds, it can be hard to accommodate all of the students. The students come from many different backgrounds and therefore come with varying degrees of background knowledge. This unit is based primarily upon information learned in class, as well as information about their school and town. Since many students are familiar with their school and town, they should be able to make the necessary connections.

Background Information

Compound Probability

Compound probability is probability caused by two or more events. For each event in the probability scenario, there are possibilities or outcomes. When calculating compound probability, you do not look at these outcomes separately, rather in combination. For example, if I roll a number cube and toss a coin, I have a set of outcomes for each event. On the number cube, I can roll a 1, 2, 3, 4, 5, or 6. On the coin, I can land on heads or tails. I can use a variety of strategies to figure out all of the possible combinations. See figure 1 for one possible strategy.

Figure 1

	1	2	3	4	5	6
Heads	{1, heads}	{2, heads}	{3, heads}	{4, heads}	{5, heads}	{6, heads}
Tails	{1, tails}	{2, tails}	{3, tails}	{4, tails}	{5, tails}	{6, tails}

When calculating probability, the formula below is used:

$$\text{Probability (event)} = \frac{\text{Number of Favorable Outcomes}}{\text{Number of Total Outcomes}}$$

Let's say I was looking at the probability of rolling a combination with a 4 in it. Figure 1 shows that there are 12 total outcomes, and that only 2 of them include a 4. Therefore, the probability would be $\frac{2}{12} = \frac{1}{6}$

Experimental Probability

We know that the world is not perfect. Therefore, our most educated predictions will not always come true. In probability, we can theoretically calculate the chance of something happening. For example, if we toss a coin, we can predict that the coin will land on heads 50% of the time and tails 50% of the time. However, we know that experimentally, this may not be true. In experimental probability, we base our conclusions or predictions on data. Experimental Probability is calculated using the formula below:

$$\text{Probability (event)} = \frac{\text{Number of Favorable Outcomes}}{\text{Number of Trials}}$$

If we toss a coin 20 times, our denominator would be 20. If we were interested in the experimental probability of the coin landing on tails, we would say landing on tails is a favorable outcome. Let's say of those 20 trials, our coin landed on tails 6 times. We would say the experimental probability of landing on tails is $\frac{6}{20} = \frac{3}{10}$. This is different from what we initially predicted to happen. However, the more times you do an experiment, the more data you have, the more accurate your results will be. If we conducted the experiment 500 times, we would expect our theoretical probability to be much closer to 50%.

Long Run Relative Frequency

The Law of Large Numbers says that the more times you do an experiment, the closer it will mirror your prediction. In my class, we do two experiments that relate to this. In the first, we look to see what happens to the experimental probability as you conduct an experiment more times. In the first experiment, students are split into groups. Each group is a assigned one specific experiment. One group is instructed to spin a spinner a total of 250 times and record the amount of times they land on a specific color. Another group is instructed to toss a coin a total of 250 times and record the number of times the coin lands on tails. The last group is instructed to toss a number cube a total of 250 times and record the number of times the number cube lands on a 3. Each group is conducting 250 trials and is each required to stop every 25 trials to record their results. To be more specific, each group will conduct their experiment 25 times, record their results, and calculate the experimental probability using 25 as their number of trials. They will repeat the experiment another 25 times, record their results, and calculate the experimental probability using 50 as their number of trials. They will repeat this process until they have reached 250 trials. Then, each group makes a line graph of the data from their experiment. Then the students do a gallery walk where they view each other's data and draw conclusions from the patterns they see.

The second experiment lends itself to the idea of random sampling. The students are given the distribution of the population of Delaware. They are told that 20% of Delawareans live in rural areas, while 80% live in an urban setting. They are given a bag with 10 pieces of paper. 8 slips have the word "urban" written on them, and 2 have the word "rural" written on them. The students are instructed to randomly choose 20 slips of paper, replacing after each pick, and record their results.[2] Then we put all of our data together as a class.

In both experiments, the students begin with a relatively small number of trials. Therefore, each year we tend to see a lot of variation in their results in the beginning. However, we see the trend change after awhile. In the experimental line graph that the students create, we see that the experimental probability tends to level out around the theoretical probability as the students approach 250 trials. In the sampling experiment, we wind up having approximately 300 total trials. We find that at this amount, our data almost always becomes very close to a distribution of 80% urban and 20% rural. See figure 2 and 3 for more information.

Figure 2[3]:

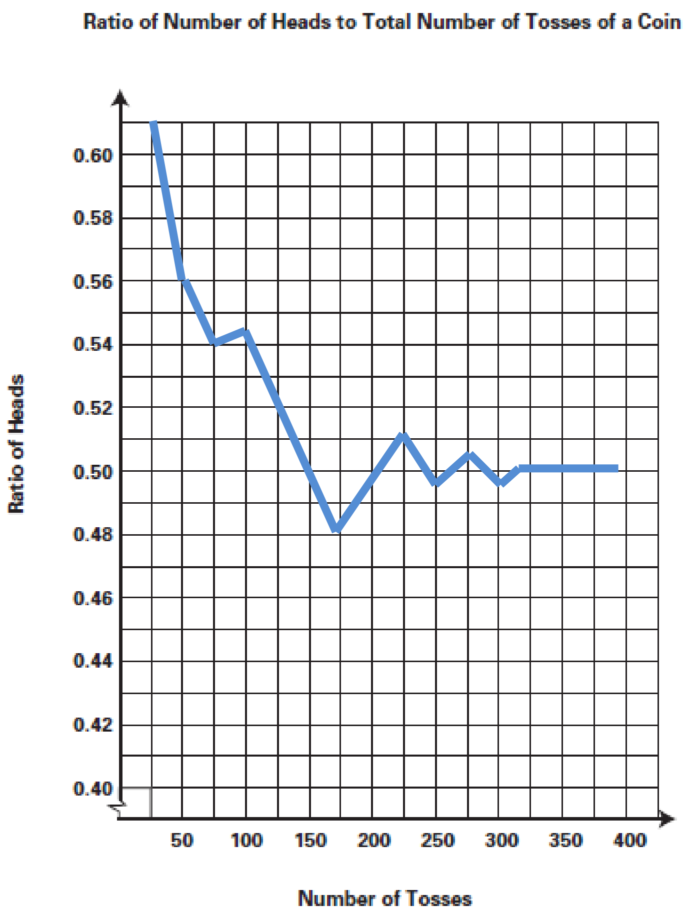


Figure 3:

Rural (20%)	Urban (80%)	Total: (100%)
9	11	20
5	15	20
4	16	20
8	12	20
6	14	20
3	17	20
0	20	20
5	15	20
4	16	20
1	19	20
3	17	20
4	16	20
2	18	20
2	18	20
56/280 = 20%	224/280= 80%	280=100%

What this means is that if there are only a few trials in a probability experiment, there may be some variance. The experimental probability may be a little higher or lower than what you expect. However, as we conduct the experiment more times and obtain more trial information, the probability will begin to converge to the expected probability. This is why is will be imperative to have enough trials in the Monte Carlo simulation(see Monte Carlo Simulation section for more information).

Sampling

In statistics, you study a population. A population is any group that you are interested in collecting data about, such as the United States. When you have a large population, it is sometimes hard to collect data from every individual. Therefore, scientists and statisticians choose a smaller group from the population to look at more in depth. This smaller group is called a sample.

You need to be very careful when choosing this sample, or else your conclusions can be rendered invalid. The first thing to take into consideration is, “Is my sample representative of the population?” In order for your sample to be representative, it needs to have a similar make-up. All subgroups of your population should be represented proportionally. Your sample should essentially be a smaller mirror image of your population.

The most valid form of sampling is random sampling. This means that a sampling method is used where each member of the population has the same chance of being selected. Random sampling helps to prevent bias in your sampling, where one group would have a higher chance of being selected. This creates prejudice in your sample and invalidates your results.[4]

Genetics

In reproduction, parents pass on traits to their offspring. Traits decide many things about our lives. Traits are what decides what color eyes we will have, the color of our hair, whether we have a widows peak, and much more. We receive all of our traits from our parents. Traits give us the original instructions for our appearance, behavior, and our medical predispositions. I say “original,” because there is always an ongoing tension between nature (the traits that were passed along to you) and nurture (the environment you grow up in). For example, genetics may give you a predisposition to being tall. However, without proper nutrition, your body will be unable to grow to its full potential. In this unit, we will focus on physical traits that are not affected by nurture.

These traits are transferred and become part of our genetic makeup. Each parent passes along an allele.[5] Therefore, our genes consist of a combination of 2 alleles for each trait. These alleles are like instructions for our cells. There are 2 types of alleles, dominant and recessive. Dominant alleles will always “take over” if you will. If a dominant allele is present, that characteristic will be represented in the offspring. A recessive allele will be represented in the offspring only if both parents provide a recessive allele.

Since each person has 2 alleles for each trait, each parent will have 2 alleles for each trait. They will only pass along one of these alleles to their offspring. Let’s say the parent has both a recessive and a dominant allele. They can pass along either the dominant or recessive allele. It is hard to determine which allele was passed along unless both parents pass along a recessive allele.

For example, tongue rolling is a characteristic that is determined by alleles that are passed down from your parents. In this case, a dominant allele determines the ability to roll the tongue and a recessive allele determines that you are not able to roll your tongue. If the child receives 2 recessive alleles, they will not be able to roll their tongue. If they receive either one or both dominant alleles, they will be able to roll their tongue.[6] Since there are three allele combinations that allow the offspring to roll their tongue, and only one that does not, it is hard to determine the allele combination. It is possible, however, for a geneticist to map your entire genome. Through this process, you will be able to know the exact make-up of your genes. However, the cost of this process is approximately $30,000.[7] As I am fairly sure that my principal will not approve a $3,750,000 request, we will have to settle for our best educated guess.

Monte Carlo Simulation

The Monte Carlo Simulation coined its name from the Monte Carlo casinos. Since this simulation has many connections to random probability situations such as a roulette wheel,[8] it was named after a place where you would most commonly find these types of devices. A Monte Carlo Simulation is a simulation that is formulated on the premise that you cannot formally calculate the probability of something occurring without a simulation. Usually, this is due to an excess of variables. When you have too many variables, it can be hard to combine them into one mathematical equation. Therefore, a simulation is carried out that takes all of these variables into consideration. Monte Carlo Simulations are very useful in the business industry, first being used in the airline industry.[9] The simulation took into account passenger numbers, average customer spend, fuel price inflation, and other costs. For each variable, they took into account the likelihood of each scenario occurring. Then they set up a probability model and conducted thousands of experiments. This allowed them to see the full range of possibilities as well as the likelihood of each occurring. They could then use this information to make predictions and adjust their business plans.

My unit requires you to calculate the theoretical distribution of students that are able to roll their tongue. Each student has a different genetic make up, and therefore will have different probabilities of inheriting the tongue rolling trait. Due to the amount of different probabilities, we cannot successfully calculate one single probability that someone in the class would have the trait. Therefore, each student is going to create a probability model that represents their unique genetic inheritance. They will each have one specific number assigned to them based on their probability (i.e.- .25 for a probability of ¼) In each trial, the students will conduct their unique experiment using their spinner and unique probability. We will conduct many trials. After, we will be able to see the range of possible outcomes as well as the most probable theoretical distribution.

Pedagogy

In order to teach this unit, a few things will need to be taken into consideration. The first is that students need essential background in order to complete this unit. Before completing the unit, make sure that students have a good understanding of theoretical probability, experimental probability, the Law of Large Numbers, alleles and genetics, and sampling. All of these topics are part of the 7th grade Common Core State Standards.

The second thing that should be taken into consideration is the backgrounds of your students. This unit requires students to gather information from their biological parents and grandparents. This activity may be sensitive for students that are adopted, in foster homes, or who are being raised by one parent. Make sure that you know the backgrounds of your students before choosing to teach this unit.

Content Information

Content Objectives

- Students will make predictions based on genetics and probability
- Students will use sampling to make predictions about a population

Content Standards

Common Core 7th Grade Statistics & Probability:
Use random sampling to draw inferences about a population.

1. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.
2. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words

from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.

Investigate chance processes and develop, use, and evaluate probability models.

5. Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

6. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.

7. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.8. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

Delaware Content Standards 7th Grade Reproduction, Heredity, and Development

K. Mendelian genetics can be used to predict genotypes and phenotypes of offspring resulting from sexual reproduction.

Common Core State Standards Mathematical Practices

4. Model with Mathematics

Modeling Process

In this seminar, we have spent many weeks studying the modeling process. According to the Common Core Standards Document, modeling helps us understand the world around us.[10] Modeling allows us to organize mathematical phenomena into equations, tables, and graphs with the end goal of making predictions. In this section of the document, I plan to take you through the many stages of the modeling process and explain how I will take my students through each step of the process during my unit.

Types of Models

In my unit, we will be using both Descriptive and Analytic Models. We will begin with a more Analytic approach. This means that we will begin on the basis of the proven laws of genetics. We will use these laws to make a simulation and gather data about our class. This will lead us to a Descriptive Modeling approach, where we will derive our model based on our class data.

Model

System, Reality

In this part of the modeling process, students' curiosity is invoked. I will display a statistic from the famous geneticist Alfred Sturtevant. The statistic is, "70% of people from European ancestry are able to roll up the lateral edges of the tongue, while the remaining 30% were unable to do so." [11]

Problem/Questions

This is an essential part of the modeling process that determines where the modeling process will go. The aim of every model is to answer some sort of question or solve a problem. This is where the class will determine which question they would like to investigate. To do this, students will be asked to generate a list of questions that come to their mind when they read the quote from Alfred Sturtevant. We will derive a class list together. Students will then be asked to determine which of their questions could be investigated inside the classroom. It is my hope that they will become curious about their town.

Formulate Mathematical Model

In this part of the process, students will model the mathematical information. To begin the process, students will be asked to predict the chances that they would be born with the ability to roll their tongue or not. Since it can be difficult to predict your allele combination unless you have the recessive trait, students will make a pedigree tree. They will question their parents and grandparents (if that information is available) and map the ability to roll the tongue. This will allow them to see the allele combination that they most likely have.

From there, students will create a spinner that represents the probability that they would be born with or without the ability to roll their tongue. We will then go to the computer lab where the students will create their individual spinner using Illuminations' technology.[12] In each trial, each student will spin their spinner once and record their result. We will gather a class set of data and find the theoretical distribution of our class.

The students will then create a bar graph displaying this information. On the same graph, they will show the actual distribution of the class. This will allow them to see the difference between the theoretical and experimental distribution of the class. They will be asked to describe the possible reasons for the variance in the data.

Prediction

In this part of the modeling process, the students will use their model to make predictions. The students will first be asked to make a prediction about the school based on their model. The neat part about this part of the prediction process is that we can actually test our prediction by collecting data from the school. Next, students will make a prediction about the town. The students will need to use what they have learned about the sampling process to decide whether our conclusions will be accurate. The students will begin by dissecting the town into different subgroups. We will then make a comparison between the subgroups that make up the town to the subgroups that make up our class. If there are many similarities, we will be able to conclude that our class is representative of the town.

An extension of the activity would be to explore whether the theoretical distribution of tongue rolling is specific to location. If possible, the teacher can get in contact with a class from another part of the world. We can use their data to go through the modeling process and explore whether their distribution varies from ours. Students can then pull in what they have learned in Social Studies to explore possible sources of difference.

Classroom Activities

Classroom Activity 1

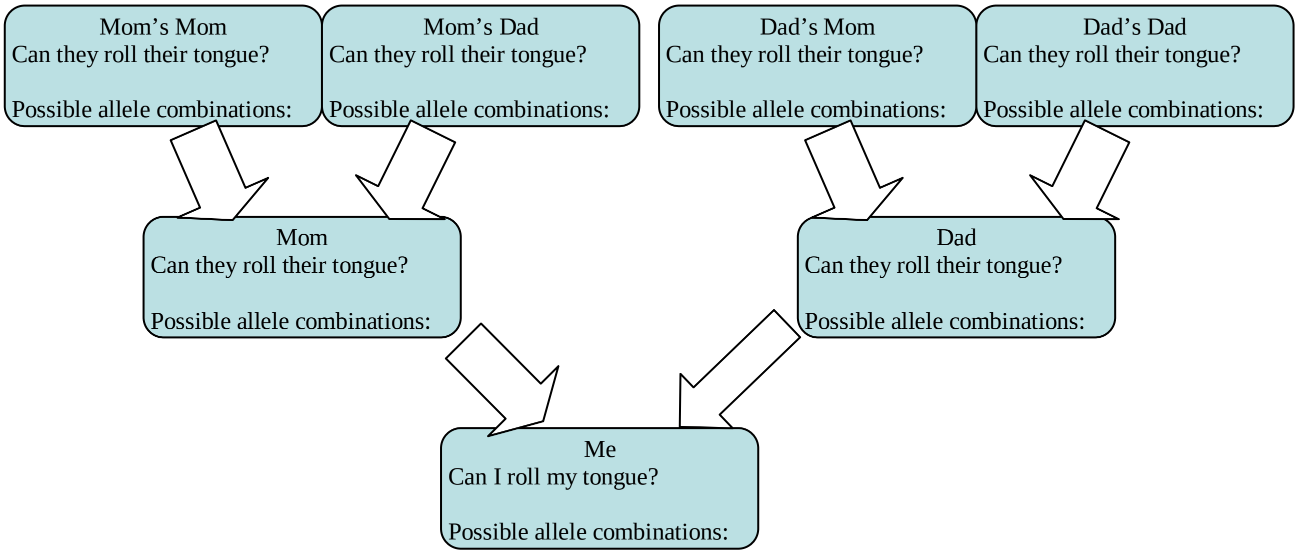
Alfred Sturtevant once said, “70% of people from European ancestry are able to roll up the lateral edges of the tongue, while the remaining 30% were unable to do so.”

What do you notice about this statement? What do you wonder?

Brainstorm a list of questions that come to mind when you read this quote.

Classroom Activity 2

Directions: Complete the following chart to the best of your ability. Then answer the questions below using the chart.



a) What are the possible allele combinations for my Mom:

b) What are the possible allele combinations for my Dad:

c) Circle the most likely allele combination for your mom. Explain why you chose this combination.

____

____

d) Circle the most likely allele combination for your dad. Explain why you chose this combination.

____

____

e) Based on what you believe to be the most likely allele combinations for your parents, use a strategy that we have learned in class to show all of possible allele combinations that could have been passed along to you.

f) What was the probability that you would be born with the ability to roll your tongue?

g) What was the probability that you would be born without the ability to roll your tongue?

h) Draw a spinner that would represent the probability that you would versus would not be able to roll your tongue.

1) Go to http://illuminations.nctm.org/ActivityDetail.aspx?ID=79

2) Create a spinner that represents the probability that you would be born with the ability to roll your tongue.

3) When your teacher gives you the signal, spin your spinner. Then record your results in your table. You will write a yes or a no in the appropriate column. Your teacher will fill in the class table at the end of the experiment.

Trial	Student 1	Student 2	Student 3	Student 4
1				
2				
3				
4				
	Total Yes:	Total Yes:	Total Yes:	Total Yes:

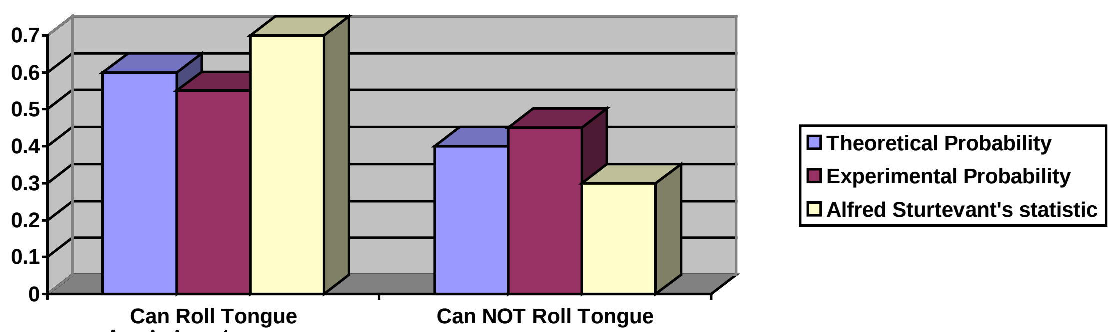
4) Now that you have the class data, calculate the probability that someone in the class would be born with the ability to roll their tongue.

Theoretical Probability:

5) Calculate the experimental probability that someone in the class has the ability to roll their tongue.

Experimental Probability:

6) Make a bar graph comparing the theoretical and experimental probability that someone in the class has the ability to roll their tongue. Also graph Alfred Sturtevant's statistic.



Classroom Activity 4

1) Examine our class data. What was similar about the sets of data? What was different? Explain any discrepancy between the three sets of data. Why do you think the data sets were the same or different?

2) In this experiment, we looked at only our class. Could we apply what we have learned from our sample (us) to make a prediction about Redding Middle School or the town of Middletown? Could we make a prediction about the amount of students in Redding Middle School or Middletown that have the ability to roll their tongue? Why or why not?

3) Make a prediction. How many students in Redding Middle School do you think have the ability to roll their tongue?

4) Make a prediction. How many students in Middletown do you think have the ability to roll their tongue?

5) Would we be able to test our prediction? Why or why not?

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Curriculum Unit Title: Rolling Your Own

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KEY LEARNING, ENDURING UNDERSTANDING, ETC.

Probability and genetics can be used to make predictions about the world around us.

ESSENTIAL QUESTION(S) for the UNIT

How can probability and genetics be used to make predictions about the world around us?

CONCEPT A	CONCEPT B	CONCEPT C
Traits	Probability	Making predictions
ESSENTIAL QUESTIONS A	**ESSENTIAL QUESTIONS B**	**ESSENTIAL QUESTIONS C**
How can you predict the genetic make-up of offspring?	How can you use theoretical probability to make predictions about a given sample? What are the possible sources of difference between theoretical probability and experimental probability?	How can conclusions from a sample be used to make predictions about a population.
VOCABULARY A	**VOCABULARY A**	**VOCABULARY A**
Trait, Punnett Square, Allele	Theoretical Probability, Experimental Probability, Monte Carlo Simulation	Population, sample

ADDITIONAL INFORMATION/MATERIAL/TEXT/FILM/RESOURCES

[1] Delaware Dept Education Website
http://profiles.doe.k12.de.us/SchoolProfiles/School/Default.aspx?checkSchool=16&districtCode=29
[22]Math in Context books-Great Predictions
[3] http://mathincontext.eb.com/content/books/activities/data_prob_second_chance_student_activity_4.pdf
[4] "statistics." *Compton's by Britannica*. 01 Aug 2011: n.p. *SIRS Discoverer*. Web. 11 Nov 2012.
[5] http://learn.genetics.utah.edu/content/begin/traits/
[6] http://learn.genetics.utah.edu/content/begin/traits/
[7] http://singularityhub.com/2011/03/05/costs-of-dna-sequencing-falling-fast-look-at-these-graphs/
[8] http://www.fm-magazine.com/feature/depth/monte-carlo-simulation
[9] http://go.galegroup.com/ps/retrieve.do?sgHitCountType=None&sort=DA-SORT&inPS=true&prodId=ITOF&userGroupName=dove10524&tabID=T003&searchId=R3&resultListType=RESULT_LIST&contentSegment=&searchType=BasicSearchForm&currentPosition=7&contentSet=GALE|A282514310&&docId=GALE|A282514310&docType=GALE&role=
[10] http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf
[11] http://learn.genetics.utah.edu
[12] http://illuminations.nctm.org/ActivityDetail.aspx?ID=79