Modeling with Probability in First Grade

Nicole DiRocco

Introduction

When I first began reading and learning about modeling with mathematics I thought that it only applied to middle and high school math. I was under the impression that all models were formulas and functions. I now understand that modeling is something that all students should use from grades K-12. In the article *Uncovering Variables in the Context of Modeling Activities* the authors describe modeling as "a process that is useful when working with real-world phenomena and when solving real-world problems." [1] A model can be as simple as a bar graph for elementary students, and as complex as a function created to model how and when a bubble will pop. Models can be empirical, models that are derived from data, or fundamental, models derived from physical law.

The first step in the modeling process is having a question or problem relating to the real world. This is perfect for first grade math, because first graders are very inquisitive. They seem to have a never ending amount of questions about their environment and the things in it. After coming up with a question that needs to be answered, you must then identify the variables and create a model that represents the relationship among the variables. Modeling is a skill that students will use throughout their education and it will be beneficial for my first graders to begin using models at such a young age. My students will work as a whole group, in small groups and in cooperative learning groups to investigate questions pertaining to the real world.

My unit focuses on sorting, graphing and probability. Sorting is a skill that first graders should have learned and mastered in kindergarten. Graphing and probability are first grade skills. The graphs used in this unit will be simple bar graphs. I am having my students use simple bar graphs because first grade students have a limited ability to make and read graphs beyond simple, bar graphs.

In this unit my students will be sorting objects that they have seen and eaten before. I chose to use M&Ms, Alpha-Bits, and Animal Crackers because I think it is important for my students to ask and answer questions about things often seen in their environment.

Throughout the unit, my students will sort and categorize various heterogeneous samples. My students will use animal crackers to sort and gather data about how many of each animal is in a box of animal crackers. They will use Alpha-Bits cereal to sort and gather data about how many of each letter are found in a box of Alpha-Bits cereal, and M&Ms to sort and gather data about how many of each color are found in a bag of M&Ms. They will then graph their data and be able to analyze it by asking and answering questions about probability. My students will expand their thinking and further describe the data by creating various number sentences based on the data.

Demographics

Red Clay Consolidated School Districted is located in New Castle County, Delaware. Red Clay consists of 28 schools in both urban and suburban settings. Red Clay has over 16,000 students. Of those students, 27% are African American, 4% are Asian, 20% are Hispanic, and 49% are White. Red Clay provides almost 15% of students with Special Education, and 10% of students with English Language support. In addition, 41% of students come from low income families.

Highlands Elementary is a small, urban school in the City of Wilmington. We service on average about 320 students in grades K-5. Minority populations make up 86% of our student body, and 81% of our students come from families with a low socio-economic status. I am a first grade teacher with a class size that varies between 18-24 students.

The Modeling Cycle

When modeling with mathematics students should be progressing through The Mathematical Modeling Cycle. The modeling cycle described in the modeling seminar consists of five steps. Each step in the cycle can and should be repeated several times. The five steps are: the system or reality, the question or problem, formulating the model, analyzing the model, and making predictions. Although the steps will most likely be used in this order, they do not necessarily have to be. The modeling cycle is outlined in Figure 1.

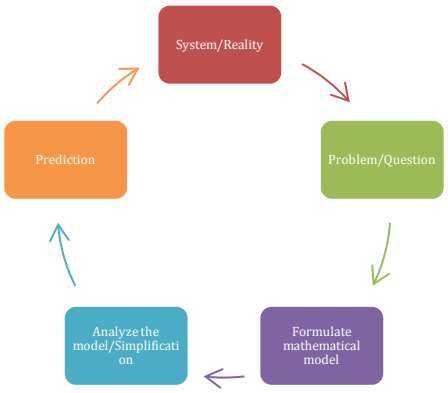


Figure 1

As mentioned above, the first step in the modeling cycle is the system or reality. This is the actual experiment or activity that is taking place. In my unit, the systems or reality are the food objects.

The second step in the modeling cycle is to determine a question to answer or a problem to solve. This is perfect for first grade math, because first graders are very inquisitive. They seem to have a never ending amount of questions about their environment and the things in it. Before I begin each lesson I will show the class the box or bag of whatever it is that we will be working with that day. I will ask the students a series of questions about how we can sort the objects and what they want to find out by sorting and counting the objects. I will guide my students into the idea of sorting samples according to color, alphabet or animals and trying to find out answers to questions such as "Which category will have the most?" That will be the problem or question that we will eventually create a model to answer.

The next step in the modeling cycle is to formulate a mathematical model. In my unit the model will be the graphs that my students create. In each lesson my students will complete data tables and create graphs that reflect their individual samples and we will also create a class data table and graph in each lesson. The class graph will reflect a collection of all the data each student or pair collected. We will combine the data they found for each sample and compile it into a class graph. The graphs will model probability and be used to make predictions about each object.

After the model is created, it is time to analyze the model and use the model to make predictions. My students will be analyzing the class graph that we create using the data they find from the samples.

To further analyze the model and incorporate more first grade math skills into the unit I will have my students use the models to create number sentences. In order to keep the

number sentences simple enough for first graders, the students will use their individual graphs for this step, not the class graph.

The Problem

The first step in each lesson will be for my students to determine what it is they want to find out about the M&Ms, animal crackers or cereal. I will show them the box and ask them to come up with some questions they have about the object. According to Joshua Paul Abrams, "The process of modeling is best learned when students pose and explore their own questions about their world".[2] I assume that at this point in the unit, students will ask static questions. Static questions ask about the steady state of the system. Dynamic questions ask about the dynamic aspect of the system, when things are moving. I will record their questions on the SMART board and then discuss which question we should investigate. I expect they will ask questions such as the following:

- How do they make M&Ms?
- How many animal crackers are in a box?
- Are there dogs in the box of animal crackers?
- What is your favorite color of M&M?
- How many lions are in the box of animal crackers?
- Can we eat the cereal?

After I have recorded all student questions, I will guide them towards the following quantitative questions:

- Which color M&M appears the most in a bag of M&Ms?
- Which color appears the least in the bag of M&Ms?
- Which animal appears the most in the box of animal crackers?
- Which animal appears the least in the box of animal crackers?
- Which letter appears the most in a box of Alpha-Bits Cereal?
- Which letter appears the least in a box of Alpha-Bits Cereal?

I will then ask them to think about ways that we can find answers to these questions. First graders will most likely say that we should dump out the whole box and count them. I will then introduce the idea of taking samples and explain that it will be easier if we all count some and then add our data together. I will explain that samples are helpful because they can accurately reflect the larger amount but that samples are easier to work with. I will also ask them to make predictions about which color, animal, or letter they think will appear most, and which color, animal or letter they think will appear the least. In the article *Using Probability Experiments to Foster Discourse,* Edwards and Hensien (2000) explain that students should make predictions before completing a probability experiment because, "When students examine their hunches, the mathematical concepts embedded in the experiments will emerge naturally."[3]

The Systems

Once my students have determined the questions they want to answer it is time to sort and count the samples. I will provide a sorting mat for each student pair to make sure they are keeping the pieces together. The sorting mat can be as simple as a piece of paper that the students are required to keep their pieces on, or a teacher created paper with separate sections to help the students sort the objects according to the different categories. I will ask students to determine what they think would be the best way to count their samples, by 1's, 2's, 5's or 10's. After they have sorted and counted their samples I will ask them to count their total number. I will encourage them count the totals for the cereal and M&Ms by ten since the totals will be a larger number.

M&Ms

I will use a 19.2 oz bag of M&Ms and give each student pair a ¼ cup sample. This works out to about 11 samples with around 60 M&Ms in each. This is perfect for my class because I can make 11 student pairs and each pair will get an equal sample. Also, first graders are expected to be able to count up to 120 so 60 is a perfect amount to use at the beginning, middle or end of the year. There are 6 colors of M&Ms: blue, orange, yellow, green, brown and red. When I did the experiment, I found that orange was the most frequent color, with 158 M&Ms total, and yellow was the least frequent with 67 M&Ms total.

After completing the experiment myself, I thought it was important to find out the actual percentage of each color in each bag of M&Ms. I was able to contact Lynn, a representative from Mars Chocolate. She was able to answer my questions about M&Ms. I found out that Mars Chocolate tries their best to get as close to a specific ratio of each color as they can. The aimed percentage of each color in a bag of M&Ms is as follows: 24% blue, 20% orange, 16% green, 14% yellow, 13% red, 13% brown.

Animal Crackers

There are various kinds of animal crackers that you could use for this unit. I am going to use a 13 oz box of My Essential Shortbread Zoo Cookies. I will use one-cup samples of animal crackers which come out to be about 23 crackers per sample, not including the broken crackers. I will ask the students to take the broken crackers and set them aside as they will not be counted in the sample. Each box contains about four, 1 cup samples. Therefore, I will use three boxes to make 11, 1 cup samples. There are 13 different types of animals in a box of Zoo Cookies: apes, sheep, elephants, camels, rhinos, hippos, seals, lions, tigers, bears, donkeys, kangaroos, and giraffes. When I did the experiment hippos were the most frequently found animal with a total of 13 in the box. I was unable to get the information regarding the actual ratio of each type of animal in a box of cookies.

Because it can be difficult to determine which animal is which, I will create a slide on the SMART board that will label each animal and show what that animal looks like in cracker form.

Alpha-Bits Cereal

Alpha-Bits Cereal is the most difficult of the three items to sort. The pieces are tiny and the letters can be hard to determine. However, I am confident that first grade students will be able to work with their samples.

I will use an 11.5 oz box of Post Alpha-Bits Cereal and give each student pair a 1/3 cup sample. The pieces of cereal are rather small so in a 1/3 cup sample they will have to count and sort about 97 pieces, not counting the broken pieces that the students will remove before sorting. Just like the animal crackers, I will have the students set aside the broken pieces. Because the samples are only 1/3 cup, the entire box will not be counted and sorted. I have to keep the samples small in order for my first graders to be able to work with them.

I found that not all letters are made in Alpha-Bits Cereal. The only letters I found were the following: A, B, D, E, F, H, I, J, K, L, O, P, Q, R, S, T, X, Y, and Z. This means the following letters were not found in the box: C, G, M, N, U, V, and W. I found that the letter A appeared the most with 163 in the 11 samples.

After completing the experiment I was able to contact Mike, a representative from Post Cereal. Mike informed me that in fact, all 26 letters are made in Alpha-Bits Cereal. Because the letters are created through a shaper, they have no control over what letters make it into each box. It is completely random.

The Model

After the students have sorted and counted their samples the students will begin to create their models. The models used in this unit will be descriptive or empirical models, models that are derived from data. For each lesson, the students will first be required to record their data onto a data table. Then, we will discuss how and why it will be easier to look at the data if they create a graph. The students will then transfer their data onto a graph. I will create a data table and a blank bar graph for each lesson that the students will have to complete. After they have completed their data tables and graphs the students will then analyze their individual models in various ways.

After they have worked with their sample data, we will begin creating a class graph. Since creating an accurate bar graph that goes up to 158 will be hard for first graders to accomplish, I will simplify this process. Each pair will receive strips of paper with blank squares. Each strip will represent a category, so for the M&Ms they will receive six

strips, one for each color. The strips of paper will have blank squares. The students will color the squares according to their data. So if they had 19 blue M&Ms in their sample they will color 19 squares and then cut the remaining squares off. Then they will take turns going to the board and taping the strips of paper onto our class graph. Each strip will connect to the last strip, creating a bar graph.

Analyzing the Data

My students will first work with their individual sample data, analyzing it and using it to create different number sentences. I will ask them to look at their data and determine which type they had the most of, which type they had the least of, and if they had the same amount of any type. They will also be asked to write number sentences based on their data. For example, add your blue M&Ms and your yellow M&Ms together and find out how many you have altogether, or take your total number of animal crackers and take away your giraffes and find out how many you have left. After they have analyzed their individual data they will create the class graph and then analyze the class data.

Making Predictions

Models are only beneficial if they have predictive power. A good, functional model must be useful in making predictions about the system. After we have analyzed the data, we will use the model to make predictions. My students will use the data to make the following predictions:

- If I stick my hand in the bag or box and pull out an item which type will it most likely be?
- If I stick my hand in the bag or box and pull out an item which type will it least likely be?
- How many bags/boxes would I need to get x amount of a certain type?

Lesson Outline and Activities

In each lesson, students will completely progress through the modeling cycle. The goal of each lesson is for students to use the food objects to create models and make predictions about probability. The lessons are outlined in Figure 2.

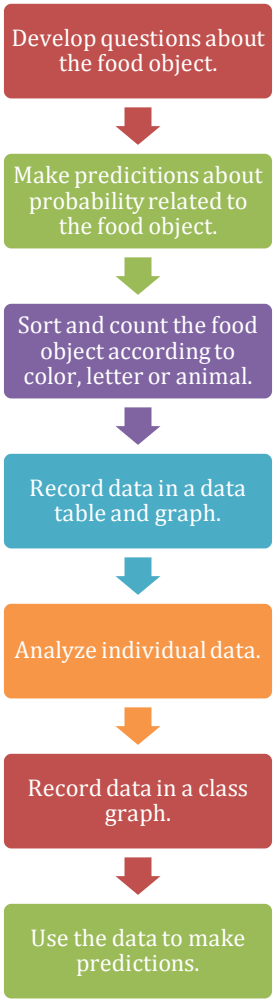


Figure 2

Conclusion

In my unit, each step in the modeling cycle will be used once in each lesson. The modeling cycle serves as a guide to my entire unit. I think it is essential to both understand and use the modeling cycle anytime modeling is used in the classroom.

Appendix A- Implementing Common Core State Standards

My unit is going to be based on the skills first graders are expected to learn according to the Common Core State Standards.

Common Core Practice Standard 4 Model With Mathematics- Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life,

society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the contexts of the situation and reflect on whether the results make sense, possibly improving the model if it has not served purpose.

CC.1.MD.4 Organize, represent and interpret data with up to three categories: ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.

The above standard will be addressed quite frequently throughout this unit. My students will collect samples and organize it in a way that will make it easy to count and interpret. Students will use data tables and graphs to organize their data and then they will carefully analyze the data by asking and answering questions about each variable and comparing the results.

CC.1.NBT.1 Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.

An important part of this unit will be for students to be able to accurately count samples. Students will be expected to count samples up to 120 using different strategies such as counting on, counting by 1's, 2's, 5's and 10's. After they have counted their samples, they will have to record their data using data tables and graphs.

CC.1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings

In order to further analyze the data, students will create number sentences to represent the different variables. They will also use number sentences to compare the results.

Appendix B- Student Worksheets

M&M Data Table

Color	**Amount**
Red	
Yellow	
Orange	
Blue	
Purple	
Green	

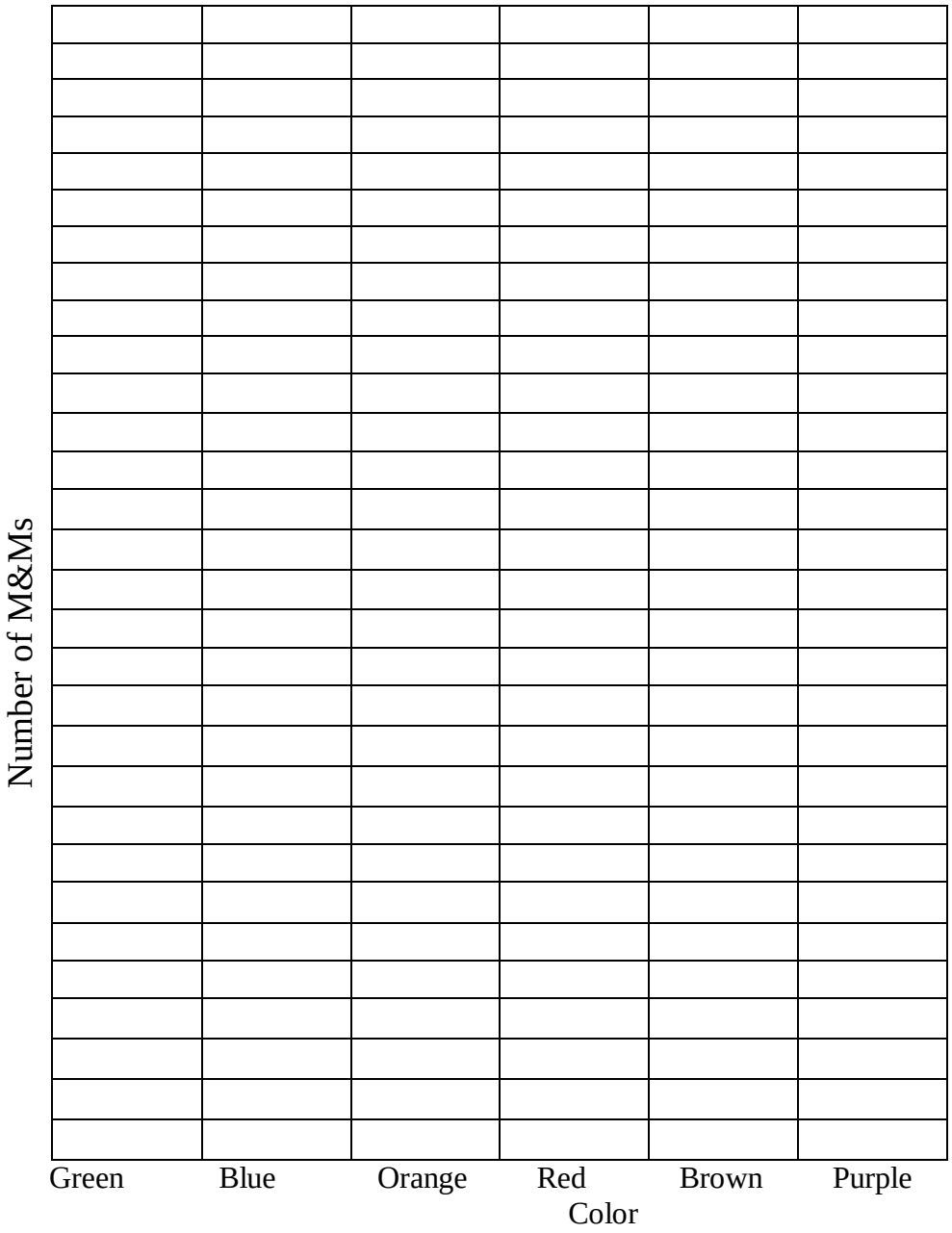
Alpha-Bits Cereal Data Table

Letter	Amount
A	
B	
C	
D	
E	
F	
G	
H	
I	
J	
K	
L	
M	
N	
O	
P	
Q	
R	
S	
T	
U	
V	
W	
X	
Y	
Z	

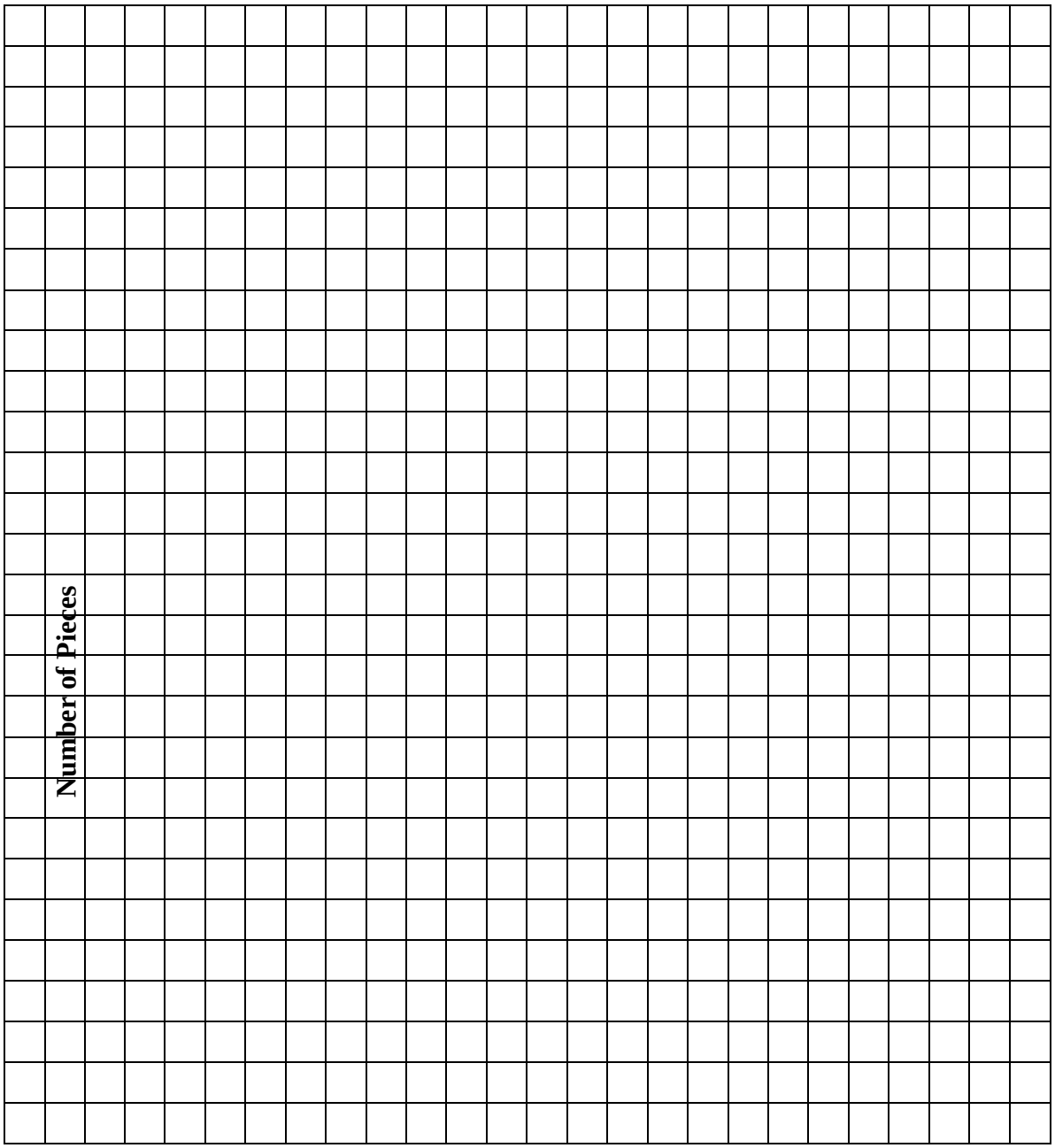
Animal Crackers Data Table

Animal	**Amount**
Apes	
Sheep	
Elephants	
Camels	
Rhinos	
Hippos	
Seals	
Lions	
Tigers	
Bears	
Donkeys	
Kangaroos	
Giraffes	

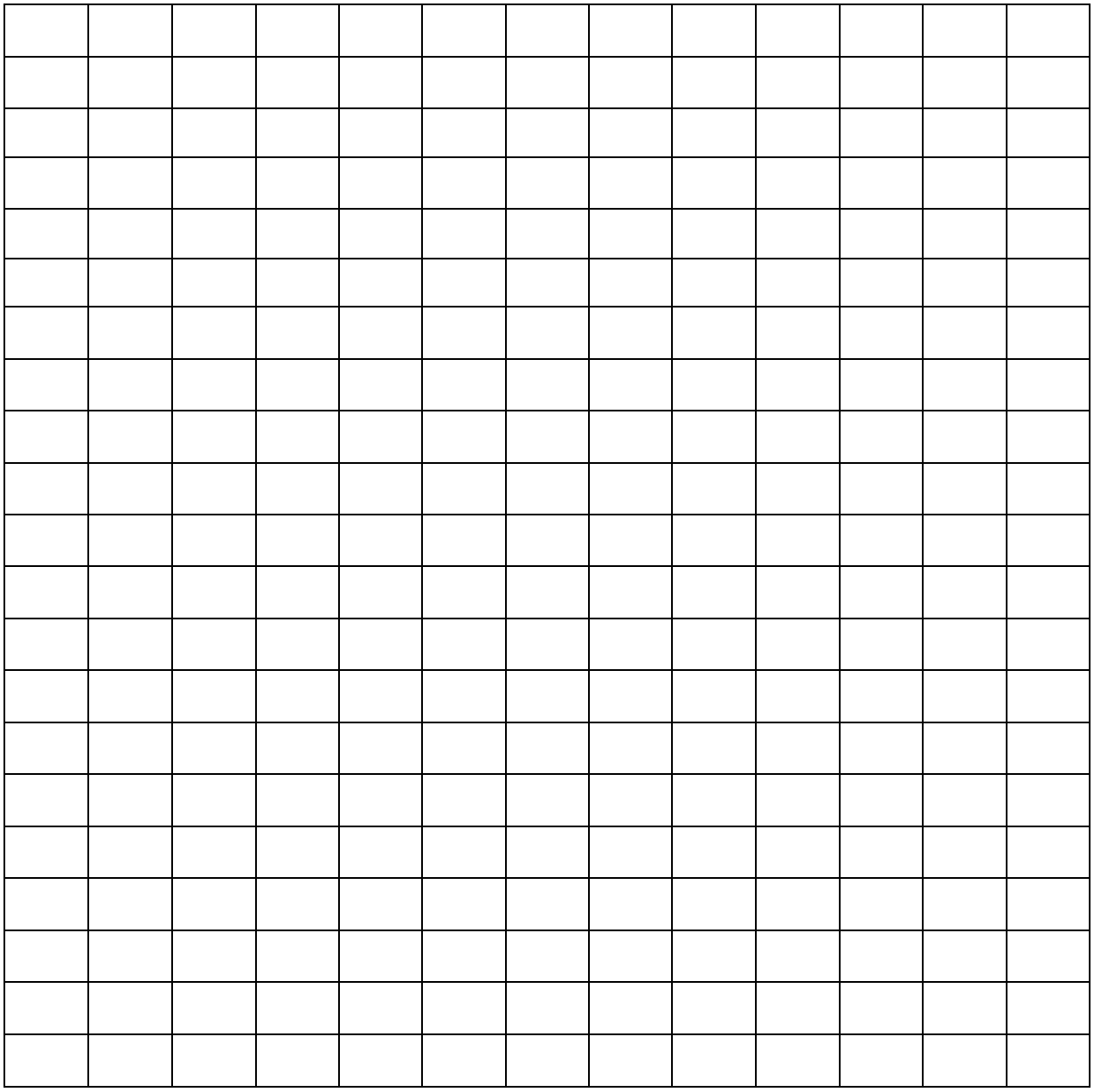
M&M Graph



Alpha-Bits Cereal Graph



Animal Cracker Graph



Ape Sheep Elephant Camel Rhino Hippo Seal Lion Tiger Bear Donkey Kangaroo Giraffe

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Curriculum Unit Title	Modeling with Probability in First Grade	Author	Nicole DiRocco

KEY LEARNING, ENDURING UNDERSTANDING, ETC.

Measuring and modeling probability can allow us to make predictions about probability.

ESSENTIAL QUESTION(S) for the UNIT

How do we measure probability? How do we model probability? How can we represent and analyze data? How can we use data to make predictions?

CONCEPT A	CONCEPT B	CONCEPT C
Modeling	Measuring Probability	Data Collection and Recording
ESSENTIAL QUESTIONS A	**ESSENTIAL QUESTIONS B**	**ESSENTIAL QUESTIONS C**
How do we model probability?	How do we measure probability?	How can we represent and analyze data? How can we use data to make predictions?
VOCABULARY A	**VOCABULARY A**	**VOCABULARY A**
Bar graph	Probability Ratio	Data table

ADDITIONAL INFORMATION/MATERIAL/TEXT/FILM/RESOURCES

Materials: M&Ms, Animal Crackers, Alpha-Bits Cereal, teacher created worksheets

[1] A. Susan Gay and Alyson R. Jones, "Uncovering Variables in the Context of Modeling Activities," Algebra and Algebraic Thinking in School Mathematics 15: 211

[2] Joshua Paul Abrams, "Teaching Mathematical Modeling and the Skills of Representation," The Roles of Representation in School Mathematics 21: 270

[3] Thomas G. Edwards and Sarah M. Hensien, "Using Probability Experiments to Foster Discourse," Teaching Children Mathematics 6.8: 524, accessed December 3, 2012, http://go.galegroup.com/ps/retrieve.do?sgHitCountType=None&sort...