Building a Solid Foundation in Geometry

Gretchen Wolfe

Introduction

Geometry is everywhere in our environment. My first grade students enter the classroom each year with the remarkable ability to identify two-dimensional shapes all around the classroom, playground, and surrounding community. These two-dimensional shapes have become an integral part of how they describe what they see and interact with in their environment. They often describe things using phrases such as: it looks like a square, or round like a circle. When the children enter first grade they have had many experiences making patterns using attribute blocks, constructing pictures with attribute blocks, and building with blocks.

As strong as the children's enthusiasm is for using and recognizing shapes, when entering first grade misconceptions often become apparent when solids are introduced in the mathematics classroom. Once the children are introduced to three-dimensional shapes they sometimes have difficulty understanding the difference between plane figures and solids. Often the children confuse solids with plane figures and will call a ball a circle or a cube a square. Each year this becomes a slight stumbling block on the road to understanding geometry for my students. I have often wrestled with creating different ways to approach introducing solids that would help move the children beyond their awareness of plane figures into a better understanding of solids. When I saw that the Delaware Teachers Institute was offering a seminar in geometry I was thrilled. Finally I would have the opportunity to talk to teachers from other grades and have the guidance of a college professor to help figure out how to hurdle the stumbling block!

The goal of this unit is to help build a foundation for understanding solids in geometry and prevent possible misconceptions in identifying and defining solids. The children for whom this unit has been constructed are my 23 first-grade students in my self-contained classroom at Henry M. Brader Elementary School in the Christina School District. Christina School District is located in New Castle County, Delaware and serves both the urban and suburban populations of Wilmington and the surrounding areas. There are approximately 17,000 pre-kindergarten through high school students in Christina's thirty-one schools. Brader is one of twenty-two elementary schools and serves its surrounding community in the city of Newark. Brader has approximately 600 kindergarten through fifth-grade students. Forty-two percent of Brader's student population qualifies for free or

reduced lunch. There are four first grade classrooms at Brader Elementary, I am part of a team of five talented first-grade teachers.

Overview

Henry M. Brader Elementary School in the Christina School District of Delaware has been traditionally a well performing elementary school. Every year since Delaware has been ranking each public school's performance Brader has earned a superior school status for its students' overall testing performance. However, the mathematics scores at Brader have frequently fallen below the school's performance scores in reading and science. Mathematics is also an area where little professional development time has been devoted in order to improve performance. With the implementation of Response to Intervention requirements in reading more time has been taken from mathematics instruction and intervention and dedicated to reading instruction. Brader Elementary has used the Trailblazers mathematics program published by Kendall Hunt, for ten years. During that time the students' performance levels in state testing situations have reached a plateau and remained relatively stagnant in the last four years. To revamp the mathematics program our district and school have adopted the Math Connects program published by Macmillan McGraw-Hill, to serve as the core mathematics curriculum for the approaching school year.

When teaching geometry to first grade students, using Trailblazers as our core mathematics program I began the year with a unit titled Exploring Shapes. The unit consisted of 5 lessons taught over seven days where children would identify, describe, and compare two-dimensional shapes. Then the children would create one shape using other shapes and fill in an outline of a figure with shapes. Throughout the year daily practice problems would arise occasionally to allow the children to practice analyzing and manipulating shapes. Toward the end of the year a unit titled Exploring 3-D Shapes, consisting of four lessons taught over 5 days, provided experiences for the children to explore the attributes of, compare, and classify three-dimensional shapes. Although the program provided opportunities for the children to work with solids during the 5 days teaching the unit, there was little opportunity for reviewing three-dimensional shapes as the unit was taught near the end of the year. In my initial preview of the Math Connects program, I found that it includes one unit with nine lessons taught over 9 days focusing on both two and three-dimensional shapes. In the Math Connects unit children will identify, describe, and compare two and three-dimensional shapes. Therefore less time will be allocated to the teaching of geometry in our new mathematics core curriculum.

I began to wonder how I could teach more geometry in less time. How can concepts be introduced in a powerful manner that will allow children a deep and lasting understanding of the geometric concepts of solids? Through my participation in the Delaware Teachers Institute "Reasoning and Sense Making through Geometry" seminar,

I was able to develop my understanding of geometry, and began to form a better understanding of how very young children develop their understandings.

As part of our geometry seminar, to build our background knowledge, we read parts of Euclid's Elements. Euclid wrote 13 books of geometry over 2,000 years ago that include geometric definitions and propositions. Together we worked through some of his propositions. We discussed our classroom experiences and the knowledge and misconceptions our students bring with them as they move through each grade level. I searched the literature to find information about children's earliest stages for understanding geometric concepts. I looked to the National Council of Teachers of Mathematics (NCTM) for some insight as well as other mathematical journals and books. I have synthesized this information in order to create a unit of instruction for my first grade students to build an understanding of solids.

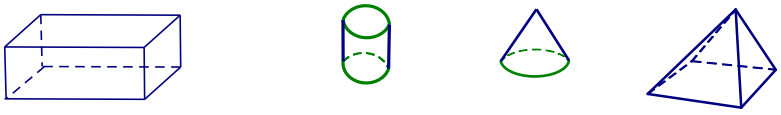
What is a Solid?

To understand a solid we must first define a two-dimensional or plane figure. Euclid defines a surface as: that which has length and breadth only, and a plane surface as: a surface which lies evenly with the straight lines on itself.
[1] Two-dimensional figures have length and width and are shapes that can be drawn on a plane. These figures include rectangles, triangles, and circles. Attribute blocks that are used to introduce young children to shapes, are technically not plane figures because they have thickness. Perhaps this leads to one of the misconceptions that children have about plane figures and solids being interchangeable.

Examples of two-dimensional shapes: rectangle, triangle, hexagon, trapezoid

With three-dimensional or solid figures we add the dimension of height. There are two types of solids: polyhedral and non-polyhedral. Polyhedral solids have flat faces, non-polyhedral shapes have one or more curved faces. Polyhedral solids include prisms and pyramids. Non-polyhedral shapes include cones, spheres, and cylinders.

Examples of three-dimensional shapes: prism, cylinder, cone, pyramid



Children's Early Geometric Thinking

To best understand how to introduce geometric concepts, I needed to identify the understandings that the children bring with them to first grade. I also needed to know their developmental readiness for these concepts. NCTM identified the foundational components of geometric thinking as: shape, location, transformations, and visualization.[2]

Shape

Young children come to school with concepts of shape based on the shape's general appearance.[3] Building upon this understanding, recognizing and naming the attributes of various shapes and building a mathematical vocabulary should be the main focus of instructional activities. In our seminar discussions we talked about activities that young children should engage in during their early school years in order for the children to be able to define a shape in a way that their definitions are not limited and causing misconceptions in later years. Attribute blocks show one type of triangle, equilateral, when presented with a scalene triangle will a child recognize it as a triangle having had experiences with only equilateral triangles? This is where defining shape becomes essential to developing geometric concepts of shape. The shapes I intend to have my first grade students define are: circle, square, triangle, rectangle, sphere, cone, pyramid, prism, and cylinder. A young child's definition of a triangle should include it having three straight sides, connected, with three points or corners.[4] In seminar we determined that opportunities to play with shapes, build, sort, solve puzzles, fold, cut, and manipulate shapes were meaningful experiences for our young geometers to analyze the characteristics and properties of shapes.

Location

For young children the concepts of location, distance, direction, and position are developed in relation to other objects. First children are introduced to location words (e.g., in front of, behind, next to). Then distance and direction words are introduced (e.g., two steps to the left of). Once position or orientation is explored children begin mapping and diagramming to document spatial relationships.[5] These are the necessary foundations for the concepts of coordinate geometry that will be taught later in middle and high school.

Transformations

Translations, reflections, and rotations are introduced to young children as slides, flips, and turns. Translations, or slides, are moving an object across a plane without changing its shape or size, revolving, or turning it over. Reflections, or flips, are flipping an object

over a line where it faces the opposite direction and becomes a mirror image of itself. Rotations, or turns, are turning around a fixed point. Children bring many experiences with translations to first grade through game boards and puzzle play. Puzzle pieces have to be turned and flipped to fit into place; pieces are moved around game boards through slides and turns. Experiences with transformations in the early grades lead children to understanding that transformations will change the location of an object but not its shape.

Visualization

Creating mental images of objects, how the objects might appear from different perspectives, and being able to visualize a three-dimensional object from a two-dimensional picture are necessary foundational experiences for young geometers. In the upper grades the children will be using spatial visualization to calculate the volume or area of shapes. Although visualization is more abstract and young children are more concrete, experiences with creating shapes from tangrams, reading and drawing maps, and physically manipulating objects will provide opportunities for visualization to be developed.

I continued my search for information about how to approach teaching young developing geometers. Upon searching the literature using the keywords geometry and first grade, I found several articles describing different approaches to developing spatial reasoning. One article described two studies where geometry was taught to primary children using a storytelling context and puzzle problems.[6] The children participated in a reading of a book about shapes and used shapes to fill in a puzzle outline. Another article described an approach to building spatial skills and reasoning by having the children build and describe polyhedrons.[7] One article described using a social studies context to analyze and recreate the patterns in cultural garments.[8] The children analyzed kilts, kenta cloth, and kimonos to identify shapes and patterns then used shapes to create the patterns. Unit blocks were used with first grade students in another approach where the students were presented with challenges and problems to solve using unit blocks to build shapes.[9] (Andrews 1999) All of the approaches described in these articles have the children working with manipulatives and analyzing figures and shapes. Most of the approaches included children decomposing shapes or building with shapes. The literature suggests that providing experiences building is a powerful way to approach the foundational learning in geometry.

Objectives

In the Christina School District we are currently using our state standards but are also in the process of phasing in the Common Core standards. Therefore I used the following

state standards, enduring understandings, essential questions, Common Core standards, and NCTM standards to guide the creation of the unit:

Delaware State Standards, Enduring Understandings, and Essential Questions

Standard 3 (K–4) – Geometric Reasoning: Students will develop Geometric Reasoning and an understanding of Geometry and Measurement by solving problems in which there is a need to recognize, construct, transform, analyze properties of, and discover relationships among geometric figures; and to measure to a required degree of accuracy by selecting appropriate tools and units.

Enduring Understandings: Two- and three-dimensional objects can be described, classified, and analyzed by their attributes. An object in a plane or in space can be oriented in an infinite number of ways while maintaining its size or shape. An object's location on a plane or in space can be described quantitatively.

Essential Questions: Why do we compare, contrast, and classify objects? How do decomposing and recomposing shapes help us build our understanding of mathematics? How can transformations be described mathematically?

Common Core Standards

Reason with shapes and their attributes.

- 1.G.1. Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.
- 1.G.2. Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.
- 1.G.3. Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

The NCTM Standards outline that first grade students should analyze characteristics and properties of two- and three-dimensional shapes, specify and describe spatial relationships, recognize transformations, and record and communicate their mathematical thinking.

In developing this unit I designed experiences where the children will, at the information level 1 of Bloom's taxonomy: define, label, match, select and organize shapes by their attributes. Then at the conceptual understanding level 2 the children will compare, illustrate, show, explain, classify, and select shapes to plan a design to build objects. Next at the application level 3 the children will apply, build, and construct three-dimensional objects. Finally, at the working expertise level 4 the children will analyze their created objects.

Teaching Strategies

The teaching approaches that are imbedded in this unit include:

Cooperative learning: in which the children work collaboratively, in pairs or groups to complete tasks and activities. Working collaboratively allows children to work as a team and assume different roles while learning.

Comparing/Contrasting: children compare and contrast objects using a graphic organizer to record their observations.

Indirect instruction: in which students are presented with a question to answer or problem to solve. The children observe, investigate, and draw conclusions while the teacher supports their investigation.

Learning Centers: children work collaboratively to complete activities or play games that reinforce content previously taught.

Classroom Activities

To introduce shapes to the children begin with having the children participate in activities in which they will recognize, name, compare and sort different two-dimensional figures. Then introduce three-dimensional figures having the children name, compare, sort, and build with three-dimensional figures.

Investigation Activity 1
Objective: The students will identify, describe, and sort two-dimensional shapes by their attributes.
Materials: Sets of attribute blocks , Attributes of Two-Dimensional Shapes table in appendix A, markers
Procedure: First introduce the attributes' terms using one of the shapes. Describe the vertices as the corners and the straight edges as the sides. Gather students into groups of four or five. Distribute a set of attribute blocks to each group of students. Introduce each

shape to the students. As you name the shape have the students work together to identify the number of each face and the number of vertices. Record these in the Attributes of Two-Dimensional Shapes table. Have the students sort the shapes by their attributes.

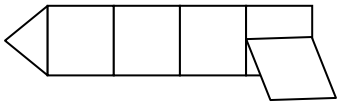
Investigation Activity 2, Introducing 2-D Shapes
Objective: The students will describe, compare, and sort two-dimensional shapes.
Materials: Attribute blocks, Frayer Model in appendix C
Procedure: Gather students into collaborative pairs. Distribute a set of attribute blocks to each pair. Have each student select a block from the set. Choose one student to begin in each pair by naming the shape he or she selected and telling one thing that is the same about the selected shape and his or her partner's selected shape. Then have the child tell one thing that is different. Have the students sort the shapes by their attributes.

Investigation Activity 3, Introducing 3-D Shapes
Objective: The students will identify, describe, and sort three-dimensional shapes by their attributes.
Materials: Sets of Geoblocks or other three-dimensional blocks, Attributes of Three-Dimensional Shapes table in appendix B, markers
Procedure: First introduce the attributes' terms using one of the shapes. Describe the face as the sides of each shape, the edges as where the sides meet, and the vertices as the corners. Gather students into groups of four or five. Distribute a set of Geoblocks to each group of students. Introduce each shape to the students. As you name the shape have the students work together to identify the shape of each face, the number of each kind of face, the number of vertices, and the number of edges. Record these in the Attributes of Three-Dimensional Shapes table. Have the students sort the shapes by their attributes.

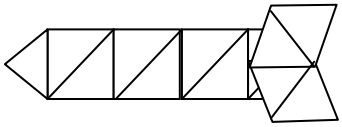
Learning Center Activities

Learning Centers allow the children to explore concepts and engage in hands-on activities in pairs or small groups. Children should rotate throughout all the learning centers during the course of the unit. Each learning center procedure should be modeled for the students before they engage in the center. While groups are engaged in the learning center the teacher may conduct small group instruction with students needing additional support with geometry concepts.

Create a Shape
Objective: The students will put together two-dimensional shapes to create a design.
Materials: Two-dimensional die-cut shapes, paper, glue
Procedure: Ask each student to create a different design such as a rocket, pinwheel, or a tree using the two-dimensional die-cut shapes. Each child creates a design and glues it to the paper.



Then the other students in the group should look at another child's design and try to recreate it using different shapes to make the same design. The rocket above could be recreated as the figure below:



Guess the Shape

Objective: The students will name, describe, and compare two-dimensional shapes.

Materials: Set of attribute blocks, shoe box

Procedure: One child reaches into the box and selects a shape without revealing the shape to the other children in the group. The child describes the shape using attribute words (edges, corner, faces), one attribute at a time. The other children in the group take turns guessing the shape after each attribute is revealed. When a child guesses the shape it becomes his or her turn to reach in the box and select a shape.

Match Me

Objective:

The students will identify, describe, and draw a 3-D shape by its attributes.

Materials:

A set of Geoblocks, brown bag, file folders, drawing paper, pencils

Procedure:

Put the set of Geoblocks into a brown bag. Have students sit side by side with a file folder standing between them to block the view of each other's papers. Have one student reach into the bag and choose one 3-D shape. He or she should describe the shape to the

other students by its attributes. The other students guess and draw the shape based on the description. Have the children take turns being the describer.

Culminating Activity

For the culminating activity students will use their knowledge of three dimensional shapes and their attributes to work in pairs to build a structure using three dimensional shapes they have created with straws and gumdrops. First the students will create a front view picture of a structure such as a castle or skyscraper using two-dimensional shape cut die-cuts. Then the children will create the three-dimensional shapes needed to create the structure using straws as the edges and gumdrops as the vertices. The children will build the structure and display it. The children will participate in a gallery walk of the structures and analyze each structure for its strength and stability.

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Endnotes

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[2] Findell, Carol R. *Navigating through geometry in prekindergarten-grade 2*. Reston, VA: National Council of Teachers of Mathematics, 2001. 2.

[3] Ibid.

[4] Hannibal, Mary Anne, "Young Children's Developing Understanding of Geometric Shapes," 356.

[5] Findell, Carol R. *Navigating through geometry in prekindergarten-grade 2*. 5.

[6] Beth Casey, Sumru Erkut, Ineke Ceder, Jessica Mercer Young. "Use of a storytelling context to improve girls' and boys' geometry skills in kindergarten." *Journal of Applied Developmental Psychology*, 2008: 29-48.

[6] Falkner, Rebecca C. Ambrose and Karen. "Developing Spatial Understanding through Building Polyhedrons." *Teaching Children Mathematics*, 2002: 442-447.

[6] White, Dorothy Y. "Kenta, Kilts, and Kimonos: Exploring Cultures and Mathematics through Fabrics." *Teaching Children Mathematics*, 2001: 354-359.

[6] Andrews, Angela Giglio. "Solving Geometric Problems by Using Unit Blocks." *Teaching Children Mathematics*, 1999: 318-323.

Appendix A

Shape	Number of Sides	Number of Vertices
Triangle		
Square		
Trapezoid		
Rectangle		
Hexagon		

Shape	Number of Sides	Number of Vertices
Triangle	3	3
Square	4	4
Trapezoid	4	4
Rectangle	4	4
Hexagon	6	6

Appendix B

Attributes of Three-Dimensional Shapes

3-D Shape	Number and Shape of Faces	Total Number of Faces	Number of Edges	Number of Vertices
Rectangular Prism				
Cube				
Pyramid				
Cylinder				
Cone				

Attributes of Three-Dimensional Shapes

3-D Shape	Number and Shape of Faces	Total Number of Faces	Number of Edges	Number of Vertices
Rectangular Prism	6 rectangles	6	12	8
Cube	6 squares	6	12	8
Pyramid	4 triangles 1 square	5	8	5
Cylinder	2 bases are circular	2 (curved side is not a face)	0 (faces do not meet)	0
Cone	1 circular base	1 (curved side is not a face)	0	1

Appendix C

Shape _____

Definition	**Characteristics**
Examples	**Non-Examples**

Appendix D

Implementing Standards

Delaware State Standards, Enduring Understandings, and Essential Questions

Standard 3 (K–4) – Geometric Reasoning: Students will develop Geometric Reasoning and an understanding of Geometry and Measurement by solving problems in which there is a need to recognize, construct, transform, analyze properties of, and discover relationships among geometric figures; and to measure to a required degree of accuracy by selecting appropriate tools and units.

Common Core Standards

Reason with shapes and their attributes.

- 1.G.1. Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.
- 1.G.2. Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.
- 1.G.3. Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves, fourths,* and *quarters,* and use the phrases *half of, fourth of,* and *quarter of*. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

NCTM Standards

Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships
Pre-K–2 Expectations: In pre-K through grade 2 all students should–

- recognize, name, build, draw, compare, and sort two- and three-dimensional shapes;
- describe attributes and parts of two- and three-dimensional shapes;
- investigate and predict the results of putting together and taking apart two- and three-dimensional shapes.

Specify locations and describe spatial relationships using coordinate geometry and other representational systems
Pre-K–2 Expectations: In pre-K through grade 2 all students should–

- describe, name, and interpret relative positions in space and apply ideas about relative position;

- describe, name, and interpret direction and distance in navigating space and apply ideas about direction and distance;
- find and name locations with simple relationships such as "near to" and in coordinate systems such as maps.

Apply transformations and use symmetry to analyze mathematical situations

Pre-K–2 Expectations: In pre-K through grade 2 all students should–

- recognize and apply slides, flips, and turns;
- recognize and create shapes that have symmetry.

Use visualization, spatial reasoning, and geometric modeling to solve problems

Pre-K–2 Expectations: In pre-K through grade 2 all students should–

- create mental images of geometric shapes using spatial memory and spatial visualization;
- recognize and represent shapes from different perspectives;
- relate ideas in geometry to ideas in number and measurement;
- recognize geometric shapes and structures in the environment and specify their location.

Process Standards

Reasoning and Proof

Instructional programs from prekindergarten through grade 12 should enable all students to—

- Recognize reasoning and proof as fundamental aspects of mathematics
- Make and investigate mathematical conjectures
- Develop and evaluate mathematical arguments and proofs
- Select and use various types of reasoning and methods of proof

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KEY LEARNING, ENDURING UNDERSTANDING, ETC.

Two- and three-dimensional objects can be described, classified, and analyzed by their attributes.
An object in a plane or in space can be oriented in an infinite number of ways while maintaining its size or shape.

ESSENTIAL QUESTION(S) for the UNIT

How do we describe, compare, and sort two-dimensional shapes?
How do decomposing and recomposing shapes help us build our understanding of mathematics?
How do we identify and describe three-dimensional shapes?

CONCEPT A	**CONCEPT B**	**CONCEPT C**
Characteristics and properties of two-dimensional shapes.	Characteristics and properties of three-dimensional shapes.	Visualize and describe characteristics and properties of three-dimensional shapes.
ESSENTIAL QUESTIONS A	**ESSENTIAL QUESTIONS B**	**ESSENTIAL QUESTIONS C**
How do we describe, compare, and sort two-dimensional shapes?	How do we identify and describe three-dimensional shapes?	How do decomposing and recomposing shapes help us build our understanding of mathematics?
VOCABULARY A	**VOCABULARY B**	**VOCABULARY C**
Triangle, square, hexagon, trapezoid, rectangle, face, vertices, edges	Cone, cylinder, prism, pyramid, cube, sides, vertices, edges, curved	Stability, strength, structure

ADDITIONAL INFORMATION/MATERIAL/TEXT/FILM/RESOURCES