?? Analytic Hierarchy Process (AHP)

Model under conditions of certainty where we know all of the info but we need to trade-off many factors.
We are comparing several alternatives on the basis of the same set of attributes.

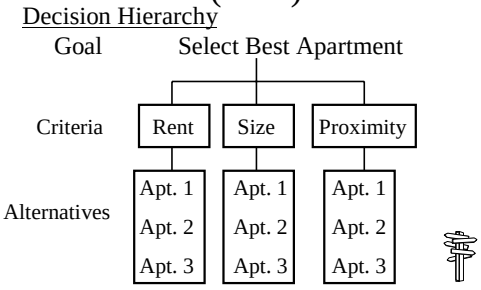
Attributes →
Alternatives ↓

?? DM with multiple alternatives and attributes

Apt	Rent	Size	Proximity	Safety	Reputation	Noise	#BRs
1	325	Lg.	20 mins.	Safe	Poor	Low	2
2	400	V.Lg.	30 mins.	V.Safe	Good	Avg	2
3	275	Lg.	35 mins.	Safe	Fair	Low	1
4	175	V. Sm	5 mins.	V.Safe	Good	High	St.
5	250	Avg	15 mins.	S. Risk	Exc.	High	1
6	275	Lg	15 mins.	V. Risk	Good	High	2
7	300	Avg	10 mins.	Safe	Fair	Avg	1
8	185	Sm	5 mins.	Safe	Poor	Avg	1
9	210	Avg	25 mins.	S.Risk	Fair	Low	1
10	200	Sm	20 mins.	V.Safe	Poor	Avg	St.

?? Analytic Hierarchy Process (AHP)

Decision Hierarchy



?? Analytic Hierarchy Process (AHP)

Major question is how to assign relative weights across alternatives, as well as for the attributes (assigning weights is the crux of AHP).

People have been found to be more consistent when they do pairwise comparisons than when they just try to assign relative weights.

?? Analytic Hierarchy Process (AHP)

Steps in AHP:

1. Make pairwise comparisons
2. Synthesize judgments
3. Check for consistency

?? Analytic Hierarchy Process (AHP)

Pairwise Comparison Scale for AHP Preferences

Verbal Judgement of Preferences	Numerical Rating
Extremely preferred	9
Very strongly to extremely	8
Very strongly preferred	7
Strongly to very strongly	6
Strongly preferred	5
Moderately to strongly	4
Moderately preferred	3
Equally to moderately	2
Equally preferred	1

Analytic Hierarchy Process

1. Make pairwise comparisons (for each attribute)

Rent

	Apt. 1	Apt. 2	Apt. 3
Apt. 1	1	4	1/3
Apt. 2	1/4	1	1/7
Apt. 3	3	7	1

Analytic Hierarchy Process (AHP)

Size

	Apt. 1	Apt. 2	Apt. 3
Apt. 1	1	1/6	1
Apt. 2	6	1	6
Apt. 3	1	1/6	1

Analytic Hierarchy Process (AHP)

Proximity

	Apt. 1	Apt. 2	Apt. 3
Apt. 1	1	5	8
Apt. 2	1/5	1	3
Apt. 3	1/8	1/3	1

?? Analytic Hierarchy Process (AHP)

1. Make pairwise comparisons (for the criteria)

	Criterion		
	Rent	Size	Proximity
Rent	1	5	6
Size	1/5	1	1/3
Proximity	1/6	3	1

?? Analytic Hierarchy Process (AHP)

Steps in AHP:

1. Make pairwise comparisons
2. Synthesize judgments
3. Check for consistency

?? Analytic Hierarchy Process

Synthesis

Step 1: Sum values in each column of pairwise comparison matrix
Step 2: Divide each element by its column total (gives *normalized pairwise comparison matrix*)
Step 3: Compute average of elements in each row (gives estimate of *relative priorities* of elements being compared)

?? Analytic Hierarchy Process (AHP)

Rent

	Apt. 1	Apt. 2	Apt. 3
Apt. 1	1	4	1/3
Apt. 2	1/4	1	1/7
Apt. 3	3	7	1
	17/4	12	31/21

Sum values in each column of PC matrix
Divide each element by column total

?? Analytic Hierarchy Process (AHP)

Relative priorities

	Apt. 1	Apt. 2	Apt. 3	Average
Apt. 1	.235	.334	.226	.266
Apt. 2	.059	.083	.097	.080
Apt. 3	.706	.583	.677	.654
				1.00

Normalized pairwise comparison matrix

?? Analytic Hierarchy Process (AHP)

Relative priorities for Rent

Apt. 1	.266
Apt. 2	.080
Apt. 3	.654

What do these mean?

?? Analytic Hierarchy Process (AHP)

Relative priorities for Rent

Apt. 1	.266
Apt. 2	.080
Apt. 3	.654

These mean that with respect to Rent, Apt. 3 is preferred first (65%), then Apt. 1 (26%), then Apt. 2 (8%)

?? Analytic Hierarchy Process (AHP)

Steps in AHP:

1. Make pairwise comparisons
2. Synthesize judgments
3. Check for consistency

?? Analytic Hierarchy Process

3. Check for Consistency

Step 1: Multiply pairwise comparison matrix by relative priorities
Step 2: Divide weighted sum vector elements by associated priority value
Step 3: Compute average (denoted λ_{max}) of the values from Step 2.
Step 4: Compute consistency index (CI)
Step 5: Compute consistency ratio (CR)

?? Analytic Hierarchy Process (AHP)

Step 1: Multiply pairwise comparison matrix (for rent) by relative priorities

$$.266 \begin{pmatrix} 1 \\ 1/4 \\ 3 \end{pmatrix} + .08 \begin{pmatrix} 4 \\ 1 \\ 7 \end{pmatrix} + .654 \begin{pmatrix} 1/3 \\ 1/7 \\ 1 \end{pmatrix} =$$

?? Analytic Hierarchy Process

1. Make pairwise comparisons (for each attribute)

Rent

	Apt. 1	Apt. 2	Apt. 3
Apt. 1	1	4	1/3
Apt. 2	1/4	1	1/7
Apt. 3	3	7	1

?? Analytic Hierarchy Process (AHP)

Step 1: Multiply pairwise comparison matrix by relative priorities

$$\begin{pmatrix} .265 \\ .066 \\ .795 \end{pmatrix} + \begin{pmatrix} .32 \\ .08 \\ .56 \end{pmatrix} + \begin{pmatrix} .215 \\ .093 \\ .653 \end{pmatrix} = \begin{pmatrix} .80 \\ .239 \\ 2.008 \end{pmatrix}$$

Weighted Sum Vector

?? Analytic Hierarchy Process (AHP)

Step 2: Divide weighted sum vector elements by associated priority value

.80/.266 = 3.019

.239/.08 = 2.988

2.008/.654 = 3.075

?? Analytic Hierarchy Process (AHP)

Relative priorities for Rent

Apt. 1	.266
Apt. 2	.080
Apt. 3	.654

These mean that with respect to Rent, Apt. 3 is preferred first (65%), then Apt. 1 (26%), then Apt. 2 (8%)

?? Analytic Hierarchy Process (AHP)

Step 3: Compute average (denoted λ_{max}) of the values from Step 2.

λ_{max} = (3.019 + 2.988 + 3.075) / 3 = 3.027

?? Analytic Hierarchy Process (AHP)

Step 4: Compute consistency index (CI)
(where n = # items being compared)

CI = (λ_{max} - n) / (n-1) = (3.027 - 3) / 2

= .0135

This is called the *consistency index*

?? Analytic Hierarchy Process (AHP)

Step 5: Compute consistency ratio (CR)

CR = CI / RI

RI = random index (CI of randomly generated pairwise comparison matrix)

?? Analytic Hierarchy Process (AHP)

Value of RI is based on n

n	3	4	5	6	7	8
RI	0.58	0.90	1.12	1.24	1.32	1.41

?? Analytic Hierarchy Process (AHP)

Step 5: Compute consistency ratio (CR)

CR = CI / RI = .0135 / .58 = .023

If CR <= .10, then consistency is acceptable

?? Analytic Hierarchy Process (AHP)

Develop Overall Priority Ranking

To do this, we need to have the Relative Priorities for each of the attributes for each apartment

?? Analytic Hierarchy Process

Synthesis - repeat for SIZE

Step 1: Sum values in each column of pairwise comparison matrix
Step 2: Divide each element by its column total (gives *normalized pairwise comparison matrix*)
Step 3: Compute average of elements in each row (gives estimate of *relative priorities* of elements being compared)

?? Analytic Hierarchy Process (AHP)

Size

	Apt. 1	Apt. 2	Apt. 3
Apt. 1	1	1/6	1
Apt. 2	6	1	6
Apt. 3	1	1/6	1
	8	4/3	8

Sum values in each column of PC matrix
Divide each element by column total

?? Analytic Hierarchy Process (AHP)

Relative priorities

	Apt. 1	Apt. 2	Apt. 3	Average
Apt. 1	.125	.125	.125	.125
Apt. 2	.75	.75	.75	.75
Apt. 3	.125	.125	.125	.125
				1.00

Normalized pairwise comparison matrix

?? Analytic Hierarchy Process (AHP)

Relative priorities for Size

Apt. 1	.125
Apt. 2	.75
Apt. 3	.125

These mean that with respect to Size, Apt. 2 is preferred first (75%), then Apt. 1 (12.5%), and Apt. 3 (12.5%).

?? Analytic Hierarchy Process (AHP)

Relative priorities

	Size	Proximity	Criteria
Apt. 1	.125	.737	.702
Apt. 2	.75	.186	.101
Apt. 3	.125	.076	.197

?? Analytic Hierarchy Process (AHP)

Develop Overall Priority Ranking

Overall Apartment 1 priority =

.702(.265) + .101(.125) + .197(.737) = .344

Overall Apartment 2 priority =

.702(.08) + .101(.75) + .197(.186) = .169

Overall Apartment 3 priority =

.702(.653) + .101(.125) + .197(.076) = .486

Rent Size Proximity