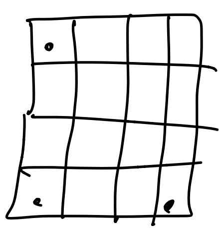
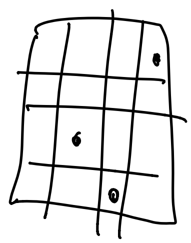
Entropy   Isolated System

$$S = k_B \ln W$$

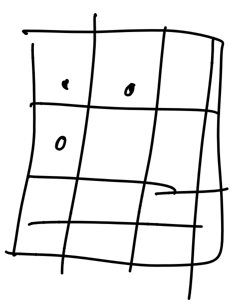
"ways"

1) a system can be found to exist
2) # "microstates" available to a system



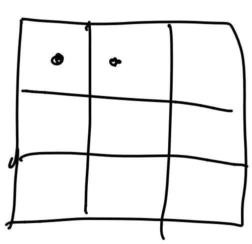


microstates
configurations



what is probability of this microstate?

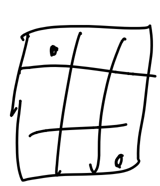
2 particles

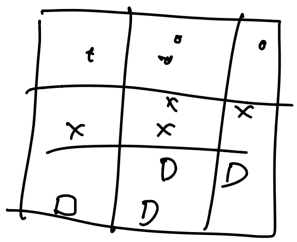


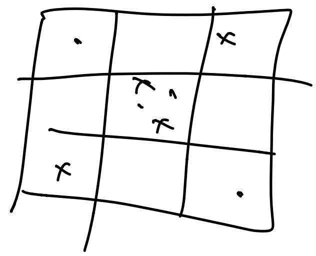
$N = 9$

$m = 2$

$$W = \frac{N!}{m!(N-m)!} = \frac{9!}{2!\,7!} = 36$$



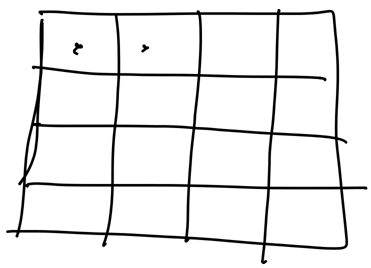




\# 32

$$P_{dead} = \frac{32}{36} \sim \text{close to } 1$$

$m = 2$



$N = 16$

$$W = \frac{16!}{2!\,14!} = 8 \times 15 = 120$$

$$P_{dead} = \frac{60}{120} \sim 0.5$$

$m = 10^{23}$

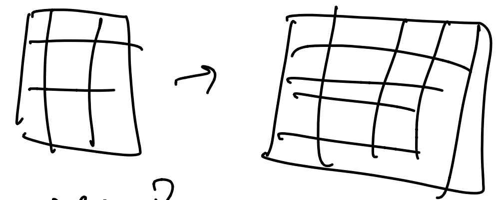
$N \gg\gg\gg m$

P_{dead} = infinitesimal

non-zero, but "small"

$10^{10^{10}}$

N changes



What about $\underline{\underline{m}}$ changes ?

Think about how W changes if m changes

$$W = \frac{N!}{m!\,(N-m)!}$$

$$m \rightarrow 2m$$

$$2m \rightarrow m$$

Entropy is Probability

Isolated System

→ W ways

Probability of a particular microstate

$$p_i = \frac{1}{W}$$

$$\ln p_i = \ln\left(\frac{1}{W}\right) = -\ln(W)$$

? Average of $\ln(p_i)$?

$$\langle \ln p \rangle = \boxed{\sum_{\substack{\text{states} \\ i=1}}^{W} p_i \ln(p_i)} = -\ln W$$

average

$$= -\sum_{i=1}^{W} \left(\frac{1}{W}\right) \ln W$$

$$= -W\left(\frac{1}{W}\right) \ln W$$

$$= -\ln W$$

$$\boxed{-k_B \sum_{i=1}^{W} p_i \ln(p_i) = k_B \ln W}$$

Entropy is related to a probability distribution