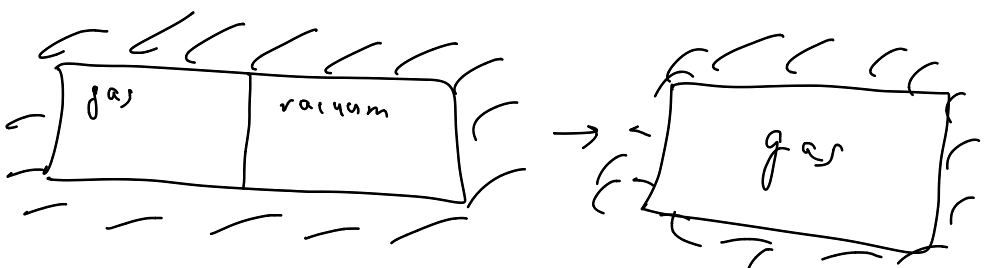
$$dS = \frac{dq_{rev}}{T} \; ; \quad \text{State function} : \int_1^2 dS = \Delta S = S_2 - S_1$$

S = entropy

prescription for calculating ΔS

∴ "make up" your own process as long as it's Reversible

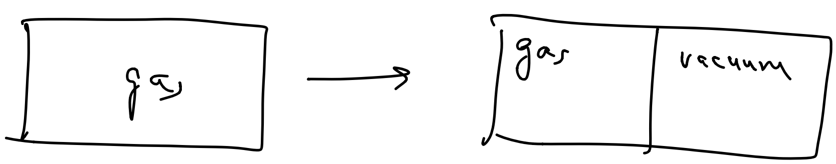
Free Expansion of I. G. (Isolated)



$dq = 0$

can't use $dS = \frac{dq_{rev}}{T}$

make up another path



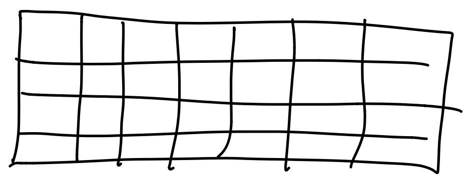
Isot : Reversible

$$\Delta S_{expansion} = -\Delta S_{compression} = nR \ln\left(\frac{V_{uncompressed}}{V_{compressed}}\right)$$

$\Delta S > 0$ IRREVERSIBLE

Spontaneous

$$\boxed{\Delta S = nR\ln 2} \quad \text{if } V_{uncmp} = 2V_{compressed}$$



N bins

m particles

Indistinguishable

how many unique ways

$$W_1 = \frac{N!}{m!\,(N-m)!}$$

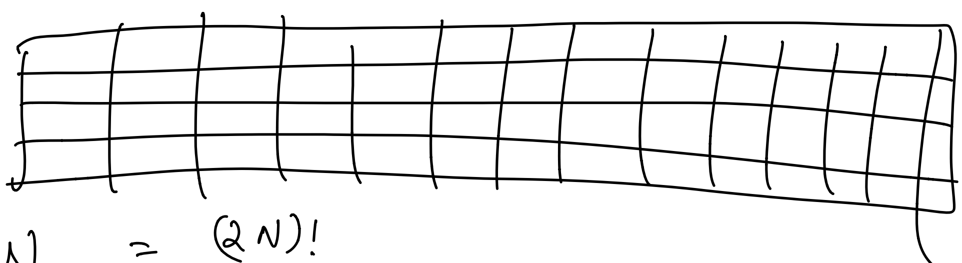
$5! = 5\times4\times3\times2\times1$

$N >>>> m$

Expansion

2N bins

m particles



$$W_2 = \frac{(2N)!}{m!\,(2N-m)!}$$

let $N, m \rightarrow \infty$ (become very large)

$\frac{m}{N}$ = constant = density

Consider $\ln W_1$ and $\ln W_2$

$$W_1 = \frac{N!}{m!(N-m)!}$$

$N, m \to$ large $\to \infty$

$N >>> m$

$$\ln W_1 = \ln(N!) - \ln(m!) - \ln((N-m)!)$$

$$\ln(N!) \cong N \ln N - N$$

$$\ln W_1 = N \ln N - N - [m \ln m - m] - [(N-m)\ln(N-m) - (N-m)]$$

$$= N \ln N \cancel{- N} - m \ln m \cancel{+ m} - (N-m)\ln(N-m) \cancel{+ N} \cancel{- m}$$

$$= \cancel{N \ln N} - m \ln m \cancel{- \underbrace{N \ln (N-m)}_{\approx N \ln N}} + m \ln (N-m)$$

$$\boxed{\ln W_1 = -m \ln(m) + m \ln(N-m)}$$

$$\ln W_2 = \ln \underbrace{\left[\frac{(2N)!}{m!(2N-m)!}\right]}_{W_2}$$

$$\ln W_2 = \ln((2N)!) - \ln(m!) - \underbrace{\ln((2N-m)!)}$$

$$= (2N)\ln(2N) \cancel{- (2N)} - [m \ln m \cancel{- m}]$$

$$- [(2N-m)\ln(2N-m) - (\cancel{2N} \cancel{- m})]$$

$$= (2N)\ln(2N) - m \ln m - (2N-m)\ln(2N-m)$$

$$= \cancel{\underbrace{(2N)\ln(2N)}} - m \ln m \cancel{- \underbrace{2N \ln(2N-m)}} + m \ln(2N-m)$$

$$\ln W_2 = -m \ln m + m \ln 2 + m \ln N \Leftarrow$$

$$\ln W_1 = -m \ln m + m \ln(N-m)$$

Now: $\ln W_2 - \ln W_1$

$$\ln W_2 - \ln W_1 = m \ln 2$$

$$pV = nRT$$

$m = nN_A$

$n = \frac{m}{N_A}$

$pV = m$

$$pV = nN_A RT$$

$$pV = \frac{m}{N_A} RT$$

$$pV = m\left(\frac{R}{N_A}\right) T$$

$$k_B \equiv \frac{R}{N_A}$$

Boltzmann Constant

$$\ln W_2 - \ln W_1 = m \ln 2$$
$$= nN_A \ln 2$$

$k_B = \frac{R}{N_A}$

$N_A = \frac{R}{k_B}$

$$\ln W_2 - \ln W_1 = n\frac{R}{k_B} \ln 2$$

$$k_B\left[\ln W_2 - \ln W_1\right] = \underline{nR \ln 2}$$

$$\Delta S = S_{final} - S_{initial} = k_B \ln W_2 - k_B \ln W_1$$

$$S_{initial} = k_B \ln W_1$$
$$S_{final} = k_B \ln W_2$$

$$\boxed{S = k_B \ln W}$$!!!!!

Ω

$$S = k_B \ln \Omega$$