nag_ref_vec_multi_normal (g05eac)

1. Purpose

nag_ref_vec_multi_normal (g05eac) sets up a reference vector for a multivariate Normal distribution with mean vector a and variance-covariance matrix C, so that **nag_ref_vec_multi_normal (g05eac)** may be used to generate pseudo-random vectors.

2. Specification

```
#include <nag.h>
#include <nagg05.h>

void nag_ref_vec_multi_normal(double a[], Integer n, double c[],
            Integer tdc, double eps, double **r, NagError *fail)
```

3. Description

When the variance-covariance matrix is non-singular (i.e., strictly positive-definite), the distribution has probability density function

$$f(x) = \sqrt{\frac{|C^{-1}|}{(2\pi)^n}} \exp\{-(x-a)^T C^{-1}(x-a)\}$$

where n is the number of dimensions, C is the variance-covariance matrix, a is the vector of means and x is the vector of positions.

Variance-covariance matrices are symmetric and positive semi-definite. Given such a matrix C, there exists a lower triangular matrix L such that $LL^T = C$. L is not unique, if C is singular.

nag_ref_vec_multi_normal decomposes C to find such an L. It then stores n, a and L in the reference vector r for later use by nag_return_multi_normal (g05ezc). nag_return_multi_normal (g05ezc) generates a vector x of independent standard Normal pseudo-random numbers. It then returns the vector $a + Lx$, which has the required multivariate Normal distribution.

It should be noted that this routine will work with a singular variance-covariance matrix C, provided C is positive semi-definite, despite the fact that the above formula for the probability density function is not valid in that case. Wilkinson (1965) should be consulted if further information is required.

4. Parameters

a[n]

Input: the vector of means, a, of the distribution.

n

Input: the number of dimensions, n, of the distribution.
Constraint: **n** > 0.

c[n][tdc]

Input: the variance-covariance matrix of the distribution. Only the upper triangle need be set.

tdc

Input: the second dimension of the array **c** as declared in the function from which nag_ref_vec_multi_normal is called.
Constraint: **tdc** $\geq$ **n**.

eps

Input: the maximum error in any element of C, relative to the largest element of C.
Constraint: $0.0 \leq$ **eps** $\leq 0.1/$**n**.

r

Output: reference vector for which memory will be allocated internally. This reference vector will subsequently be used by nag_return_multi_normal (g05ezc). If no memory is allocated to **r** (e.g. when an input error is detected) then **r** will be NULL on return.

fail

The NAG error parameter, see the Essential Introduction to the NAG C Library.

5. Error Indications and Warnings

NE_INT_ARG_LT

On entry, **n** must not be less than 1: **n** = ⟨*value*⟩.

NE_2_INT_ARG_LT

On entry, **tdc** = ⟨*value*⟩ while **n** = ⟨*value*⟩. These parameters must satisfy **tdc** ≥ **n**.

NE_REAL_ARG_LT

On entry, **eps** must not be less than 0.0: **eps** = ⟨*value*⟩.

NE_2_REAL_ARG_GT

On entry, **eps** = ⟨*value*⟩ while 0.1/**n** = ⟨*value*⟩. These parameters must satisfy **eps** ≤ 0.1/**n**.

NE_ALLOC_FAIL

Memory allocation failed.

NE_NOT_POS_SEM_DEF

Matrix C is not positive semi-definite.

6. Further Comments

The time taken by the routine is of order n^3.

It is recommended that the diagonal elements of C should not differ too widely in order of magnitude. This may be achieved by scaling the variables if necessary. The actual matrix decomposed is $C + E = LL^T$, where E is a diagonal matrix with small positive diagonal elements. This ensures that, even when C is singular, or nearly singular, the Cholesky Factor L corresponds to a positive-definite variance-covariance matrix that agrees with C within a tolerance determined by **eps**.

6.1. Accuracy

The maximum absolute error in LL^T, and hence in the variance-covariance matrix of the resulting vectors, is less than $(n \times \max(\mathbf{eps}, \varepsilon) + (n+3)\varepsilon/2)$ times the maximum element of C, where ε is the ***machine precision***. Under normal circumstances, the above will be small compared to sampling error.

6.2. References

Knuth D E (1981) *The Art of Computer Programming (Vol 2)* (2nd Edn) Addison-Wesley.
Wilkinson J H (1965) *The Algebraic Eigenvalue Problem* Clarendon Press, Oxford.

7. See Also

nag_random_init_repeatable (g05cbc)
nag_random_init_nonrepeatable (g05ccc)
nag_random_normal (g05ddc)
nag_return_multi_normal (g05ezc)

8. Example

The example program prints five pseudo-random observations from a bivariate Normal distribution with means vector

$$\begin{bmatrix} 1.0 \\ 2.0 \end{bmatrix}$$

and variance-covariance matrix

$$\begin{bmatrix} 2.0 & 1.0 \\ 1.0 & 3.0 \end{bmatrix},$$

generated by nag_ref_vec_multi_normal and nag_return_multi_normal (g05ezc) after initialisation by nag_random_init_repeatable (g05cbc).

8.1. Program Text

```
/* nag_ref_vec_multi_normal(g05eac) Example Program
 *
 * Copyright 1991 Numerical Algorithms Group.
 *
 * Mark 2, 1991.
 *
 * Mark 3 revised, 1994.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nagg05.h>

#define N 2
#define TDC N

main()
{
  Integer i, j;
  double a[N], c[N][TDC], z[N];
  double *r = (double *)0;
  double eps = 0.01;

  Vprintf("g05eac Example Program Results\n");
  a[0] = 1.0;
  a[1] = 2.0;
  c[0][0] = 2.0;
  c[1][1] = 3.0;
  c[0][1] = 1.0;
  c[1][0] = 1.0;
  g05cbc((Integer)0);
  g05eac(a, (Integer)N, (double *)c, (Integer)TDC,
         eps, &r, NAGERR_DEFAULT);
  for (i=1; i<=5; i++)
    {
      g05ezc(z, r);
      for (j=0; j<2; j++)
        Vprintf("%10.4f",z[j]);
      Vprintf("\n");
    }
  NAG_FREE(r);
  exit(EXIT_SUCCESS);
}
```

8.2. Program Data

None.

8.3. Program Results

```
g05eac Example Program Results
    1.7697    4.4481
    3.2678    3.0583
    3.1769    2.3651
   -0.1055    1.8395
    1.2933   -0.1850
```