

nag_1d_spline_interpolant (e01bac)

1. Purpose

nag_1d_spline_interpolant (e01bac) determines a cubic spline interpolant to a given set of data.

2. Specification

```
#include <nag.h>
#include <nage01.h>

void nag_1d_spline_interpolant(Integer m, double x[], double y[],
                               Nag_Spline *spline, NagError *fail)
```

3. Description

This function determines a cubic spline $s(x)$, defined in the range $x_1 \leq x \leq x_m$, which interpolates (passes exactly through) the set of data points (x_i, y_i) , for $i = 1, 2, \dots, m$, where $m \geq 4$ and $x_1 < x_2 < \dots < x_m$. Unlike some other spline interpolation algorithms, derivative end conditions are not imposed. The spline interpolant chosen has $m - 4$ interior knots $\lambda_5, \lambda_6, \dots, \lambda_m$, which are set to the values of x_3, x_4, \dots, x_{m-2} respectively. This spline is represented in its B-spline form (see Cox (1975)):

$$s(x) = \sum_{i=1}^m c_i N_i(x)$$

where $N_i(x)$ denotes the normalised B-Spline of degree 3, defined upon the knots $\lambda_i, \lambda_{i+1}, \dots, \lambda_{i+4}$, and c_i denotes its coefficient, whose value is to be determined by the routine.

The use of B-splines requires eight additional knots $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_{m+1}, \lambda_{m+2}, \lambda_{m+3}$ and λ_{m+4} to be specified; the function sets the first four of these to x_1 and the last four to x_m .

The algorithm for determining the coefficients is as described in Cox (1975) except that *QR* factorization is used instead of *LU* decomposition. The implementation of the algorithm involves setting up appropriate information for the related function **nag_1d_spline_fit_knots** (e02bac) followed by a call of that function. (For further details of **nag_1d_spline_fit_knots** (e02bac), see the function document.)

Values of the spline interpolant, or of its derivatives or definite integral, can subsequently be computed as detailed in Section 6.

4. Parameters

m

Input: m , the number of data points.
Constraint: $m \geq 4$.

x[m]

Input: **x**[$i - 1$] must be set to x_i , the i th data value of the independent variable x , for $i = 1, 2, \dots, m$.
Constraint: **x**[i] < **x**[$i + 1$], for $i = 0, 1, \dots, m - 2$.

y[m]

Input: **y**[$i - 1$] must be set to y_i , the i th data value of the dependent variable y , for $i = 1, 2, \dots, m$.

spline

Output: Pointer to structure of type **Nag_Spline** with the following members:

n - Integer

Output: the size of the storage internally allocated to **spline.lamda** and **spline.c**. This is set to $m + 4$.

lamda - double *

Output: pointer to which storage of size **spline.n** is internally allocated.

spline.lamda[$i - 1$] contains the i th knot, for $i = 1, 2, \dots, m + 4$.

c - double *

Output: pointer to which storage of size **spline.n** - 4 is internally allocated. **spline.c**[$i - 1$] contains the coefficient c_i of the B-spline $N_i(x)$, for $i = 1, 2, \dots, m$.

fail

The NAG error parameter, see the Essential Introduction to the NAG C Library.

5. Error Indications and Warnings

NE_INT_ARG_LT

On entry, **m** must not be less than 4: **m** = $\langle \text{value} \rangle$.

NE_NOT_STRICTLY_INCREASING

The sequence **x** is not strictly increasing: **x**[$\langle \text{value} \rangle$] = $\langle \text{value} \rangle$, **x**[$\langle \text{value} \rangle$] = $\langle \text{value} \rangle$.

NE_ALLOC_FAIL

Memory allocation failed.

6. Further Comments

The time taken by the function is approximately proportional to m .

All the x_i are used as knot positions except x_2 and x_{m-1} . This choice of knots (see Cox (1977)) means that $s(x)$ is composed of $m - 3$ cubic arcs as follows. If $m = 4$, there is just a single arc space spanning the whole interval x_1 to x_4 . If $m \geq 5$, the first and last arcs span the intervals x_1 to x_3 and x_{m-2} to x_m respectively. Additionally if $m \geq 6$, the i th arc, for $i = 2, 3, \dots, m - 4$ spans the interval x_{i+1} to x_{i+2} .

After the call

```
e01bac(m, x, y, &spline, &fail)
```

the following operations may be carried out on the interpolant $s(x)$.

The value of $s(x)$ at $x = \mathbf{xval}$ can be provided in the variable **sval** by calling the function

```
e02bbc(xval, &sval, &spline, &fail)
```

The values of $s(x)$ and its first three derivatives at $x = \mathbf{xval}$ can be provided in the array **sdif** of dimension 4, by the call

```
e02bcc(derivs, xval, sdif, &spline, &fail)
```

Here **derivs** must specify whether the left- or right-hand value of the third derivative is required (see nag_1d_spline_deriv (e02bcc) for details).

The value of the integral of $s(x)$ over the range x_1 to x_m can be provided in the variable **sint** by

```
e02bdc(&spline, &sint, &fail)
```

6.1. Accuracy

The rounding errors incurred are such that the computed spline is an exact interpolant for a slightly perturbed set of ordinates $y_i + \delta y_i$. The ratio of the root-mean-square value of the δy_i to that of the y_i is no greater than a small multiple of the relative **machine precision**.

6.2. References

Cox M G (1975) An Algorithm for Spline Interpolation *J. Inst. Math. Appl.* **15** 95–108.

Cox M G (1977) A Survey of Numerical Methods for Data and Function Approximation *The State of the Art in Numerical Analysis* D A H Jacobs (ed) Academic Press, London pp 627–668.

7. See Also

nag_1d_spline_fit_knots (e02bac)

nag_1d_spline_evaluate (e02bbc)

nag_1d_spline_deriv (e02bcc)

nag_1d_spline_intg (e02bdc)

8. Example

The following example program sets up data from 7 values of the exponential function in the interval 0 to 1. `nag_1d_spline_interpolant` is then called to compute a spline interpolant to these data.

The spline is evaluated by `nag_1d_spline_evaluate` (`e02bbc`), at the data points and at points halfway between each adjacent pair of data points, and the spline values and the values of e^x are printed out.

8.1. Program Text

```
/* nag_1d_spline_interpolant(e01bac) Example Program
 *
 * Copyright 1991 Numerical Algorithms Group.
 *
 * Mark 2, 1991.
 */

#include <nag.h>
#include <stdio.h>
#include <math.h>
#include <nag_stdlib.h>
#include <nage01.h>
#include <nage02.h>

#define MMAX 7
main()
{
    static double x[MMAX] = {0.0, 0.2, 0.4, 0.6, 0.75, 0.9, 1.0};
    Integer i, j;
    double y[MMAX], fit, xarg;
    Nag_Spline spline;
    Integer m = MMAX;

    Vprintf("e01bac Example Program Results\n");
    for (i=0; i<m; ++i)
        y[i] = exp(x[i]);
    e01bac(m, x, y, &spline, NAGERR_DEFAULT);
    Vprintf("\nNumber of distinct knots = %ld\n", m-2);
    Vprintf("Distinct knots located at \n\n");
    for (j=3; j<m+1; j++)
        Vprintf("%8.4f%s", spline.lamda[j], (j-3)%5==4 || j==m ? "\n" : " ");
    Vprintf("\n\n      J      B-spline coeff c\n\n");
    for (j=0; j<m; ++j)
        Vprintf("      %ld %13.4f\n", j+1, spline.c[j]);
    Vprintf("\n      J      Abscissa      Ordinate      Spline\n\n");
    for (j=0; j<m; ++j)
    {
        e02bbc(x[j], &fit, &spline, NAGERR_DEFAULT);
        Vprintf("      %ld %13.4f      %13.4f      %13.4f\n", j+1, x[j], y[j], fit);
        if (j < m-1)
        {
            xarg = (x[j] + x[j+1]) * 0.5;
            e02bbc(xarg, &fit, &spline, NAGERR_DEFAULT);
            Vprintf("      %13.4f      %13.4f\n", xarg, fit);
        }
    }
    exit(EXIT_SUCCESS);
}
```

8.2. Program Data

None.

8.3. Program Results

e01bac Example Program Results

Number of distinct knots = 5

Distinct knots located at

0.0000 0.4000 0.6000 0.7500 1.0000

J	B-spline coeff c
1	1.0000
2	1.1336
3	1.3726
4	1.7827
5	2.1744
6	2.4918
7	2.7183

J	Abscissa	Ordinate	Spline
1	0.0000	1.0000	1.0000
	0.1000		1.1052
2	0.2000	1.2214	1.2214
	0.3000		1.3498
3	0.4000	1.4918	1.4918
	0.5000		1.6487
4	0.6000	1.8221	1.8221
	0.6750		1.9640
5	0.7500	2.1170	2.1170
	0.8250		2.2819
6	0.9000	2.4596	2.4596
	0.9500		2.5857
7	1.0000	2.7183	2.7183
