

nag_1d_quad_inf_wt_trig (d01asc)

1. Purpose

nag_1d_quad_inf_wt_trig (d01asc) calculates an approximation to the sine or the cosine transform of a function g over $[a, \infty)$:

$$I = \int_a^\infty g(x) \sin(\omega x) dx \quad \text{or} \quad I = \int_a^\infty g(x) \cos(\omega x) dx$$

(for a user-specified value of ω).

2. Specification

```
#include <nag.h>
#include <nagd01.h>

void nag_1d_quad_inf_wt_trig(double (*g)(double x), double a,
                             double omega, Nag_TrigTransform wt_func,
                             Integer maxintervals, Integer maxsubints_per_interval,
                             double epsabs, double *result, double *abserr,
                             Nag_QuadSubProgress *qpsub, NagError *fail)
```

3. Description

nag_1d_quad_inf_wt_trig is based upon the QUADPACK routine QAWFE (Piessens *et al* (1983)). It is an adaptive routine, designed to integrate a function of the form $g(x)w(x)$ over a semi-infinite interval, where $w(x)$ is either $\sin(\omega x)$ or $\cos(\omega x)$. Over successive intervals

$$C_k = [a + (k - 1) \times c, a + k \times c], \quad k = 1, 2, \dots, \mathbf{qpsub.intervals}$$

integration is performed by the same algorithm as is used by **nag_1d_quad_wt_trig (d01anc)**. The intervals C_k are of constant length

$$c = \{2[|\omega|] + 1\}\pi/|\omega|, \quad \omega \neq 0,$$

where $[|\omega|]$ represents the largest integer less than or equal to $|\omega|$. Since c equals an odd number of half periods, the integral contributions over succeeding intervals will alternate in sign when the function g is positive and monotonically decreasing over $[a, \infty)$. The algorithm, described by Piessens *et al* (1983), incorporates a global acceptance criterion (as defined by Malcolm and Simpson (1976)) together with the ϵ -algorithm (Wynn (1956)) to perform extrapolation. The local error estimation is described by Piessens *et al* (1983).

If $\omega = 0$ and **wt_func** = **Nag-Cosine**, the routine uses the same algorithm as **nag_1d_quad_inf (d01amc)** (with **epsrel** = 0.0).

In contrast to most other functions in Chapter d01, **nag_1d_quad_inf_wt_trig** works only with a user-specified absolute error tolerance (**epsabs**). Over the interval C_k it attempts to satisfy the absolute accuracy requirement

$$EPSA_k = U_k \times \mathbf{epsabs},$$

where $U_k = (1 - p)p^{k-1}$, for $k = 1, 2, \dots$ and $p = 0.9$.

However, when difficulties occur during the integration over the k th interval C_k such that the error flag **qpsub->interval_flag[k-1]** is non-zero, the accuracy requirement over subsequent intervals is relaxed. See Piessens *et al* (1983) for more details.

4. Parameters

g

The function **g**, supplied by the user, must return the value of the function g at a given point. The specification of **g** is:

```
double g(double x)
```

x

Input: the point at which the function g must be evaluated.

a

Input: the lower limit of integration, a .

omega

Input: the parameter ω in the weight function of the transform.

wt_func

Input: indicates which integral is to be computed:

if **wt_func** = **Nag_Cosine**, $w(x) = \cos(\omega x)$;

if **wt_func** = **Nag_Sine**, $w(x) = \sin(\omega x)$;

Constraint: **wt_func** = **Nag_Cosine** or **Nag_Sine**.

maxintervals

Input: an upper bound on the number of intervals C_k needed for the integration.

Suggested value: **maxintervals** = 50 is adequate for most problems.

Constraint: **maxintervals** ≥ 3 .

maxsubints_per_interval

Input: the upper bound on the number of sub-intervals into which any of the intervals, C_k , may be divided by the function.

Suggested value: a value in the range 200 to 500 is adequate for most problems.

Constraint: **maxsubints_per_interval** ≥ 1 .

epsabs

Input: the absolute accuracy required. If **epsabs** is negative, the absolute value is used. See Section 6.1.

result

Output: the approximation to the integral I .

abserr

Output: an estimate of the modulus of the absolute error, which should be an upper bound for $|I - \text{result}|$.

qpsub

Pointer to structure of type Nag_QuadSubProgress with the following members:

intervals – Integer

Output: the number of intervals C_k actually used for the integration.

fun_count – Integer

Output: the number of function evaluations performed by nag_1d_quad_inf_wt_trig.

subints_per_interval – Integer

Output: the maximum number of sub-intervals actually used for integrating over any of the intervals C_k .

interval_error – double *

Output: the error estimate corresponding to the integral contribution over the interval C_k , for $k = 1, 2, \dots, \text{qpsub.intervals}$.

interval_result – double *

Output: the corresponding integral contribution over the interval C_k for $k = 1, 2, \dots, \text{qpsub.intervals}$.

interval_flag – Integer *

Output: the error flag corresponding to **qpsub.interval_result**, for $k = 1, 2, \dots, \text{qpsub.intervals}$. See also Section 5.

fail

The NAG error parameter, see the Essential Introduction to the NAG C Library.

Users are recommended to declare and initialise **fail** and set **fail.print** = TRUE for this function.

5. Error Indications and Warnings**NE_INT_ARG_LT**

On entry, **maxsubints_per_interval** must not be less than 1: **maxsubints_per_interval** = $\langle value \rangle$.

On entry, **maxintervals** must not be less than 3: **maxintervals** = $\langle value \rangle$.

NE_BAD_PARAM

On entry, parameter **wt_func** had an illegal value.

NE_ALLOC_FAIL

Memory allocation failed.

NE_QUAD_MAX_SUBDIV

The maximum number of subdivisions has been reached: **maxsubints_per_interval** = $\langle value \rangle$.
The maximum number of subdivisions within an interval has been reached without the accuracy requirements being achieved. Look at the integrand in order to determine the integration difficulties. If the position of a local difficulty within the interval can be determined (e.g. a singularity of the integrand or its derivative, a peak, a discontinuity, etc.) you will probably gain from splitting up the interval at this point and calling **nag_1d_quad_inf_wt_trig** on the infinite subrange and an appropriate integrator on the finite subrange. Alternatively, consider relaxing the accuracy requirements specified by **epsabs** or increasing the value of **maxsubints_per_interval**.

NE_QUAD_ROUNDOff_ABS_TOL

Round-off error prevents the requested tolerance from being achieved: **epsabs** = $\langle value \rangle$.

The error may be underestimated. Consider relaxing the accuracy requirements specified by **epsabs**.

NE_QUAD_BAD_SUBDIV

Extremely bad integrand behaviour occurs around the sub-interval ($\langle value \rangle$, $\langle value \rangle$).

The same advice applies as in the case of **NE_QUAD_MAX_SUBDIV**.

NE_QUAD_ROUNDOff_EXTRAPL

Round-off error is detected during extrapolation.

The requested tolerance cannot be achieved, because the extrapolation does not increase the accuracy satisfactorily; the returned result is the best which can be obtained. The same advice applies as in the case of **NE_QUAD_MAX_SUBDIV**.

NE_QUAD_BAD_SUBDIV_INT

Bad integration behaviour has occurred within one or more intervals.

NE_QUAD_MAX_INT

Maximum number of intervals allowed has been achieved. Increase the value of **maxintervals**.

NE_QUAD_EXTRAPL_INT

The extrapolation table constructed for convergence acceleration of the series formed by the integral contribution over the integral does not converge.

In the cases where **fail.code** = **NE_QUAD_BAD_SUBDIV_INT**, **NE_QUAD_MAX_INT** or **NE_QUAD_EXTRAPL_INT**, additional information about the cause of the error can be obtained from the array **qpsub->interval_flag**, as follows:

qpsub.interval_flag[$k - 1$] = 1

The maximum number of subdivisions (= **maxsubints_per_interval**) has been achieved on the k th interval.

qpsub.interval_flag[$k - 1$] = 2

Occurrence of round-off error is detected and prevents the tolerance imposed on the k th interval from being achieved.

qpsub.interval_flag[$k - 1$] = 3

Extremely bad integrand behaviour occurs at some points of the k th interval.

qpsub.interval_flag[$k - 1$] = 4

The integration procedure over the k th interval does not converge (to within the required accuracy) due to round-off in the extrapolation procedure invoked on this interval. It is assumed that the result on this interval is the best which can be obtained.

qpsub.interval_flag[$k - 1$] = 5

The integral over the k th interval is probably divergent or slowly convergent. It must be noted that divergence can occur with any other value of **qpsub.interval_flag**[$k - 1$].

If users declare and initialise **fail** and set **fail.print** = **TRUE** as recommended then

NE-QUAD-NO-CONV

The integral is probably divergent or slowly convergent. Please note that divergence can occur with any error exit other than **NE-INT-ARG-LT**, **NE-BAD-PARAM** or **NE-ALLOC-FAIL**.

may be produced supplemented by messages indicating more precisely where problems were encountered by the function. However, if the default error handling, **NAGERR-DEFAULT**, is used then one of the following errors may occur. Please note the program will terminate when the first of such errors is detected.

NE-QUAD-MAX-SUBDIV-SPEC-INT

The maximum number of subdivisions has been reached,
maxsubints_per_interval = $\langle value \rangle$ on the $\langle value \rangle$ interval.
interval_flag [$\langle value \rangle$] = $\langle value \rangle$ over sub-interval ($\langle value \rangle$, $\langle value \rangle$).

NE-QUAD-ROUND-OFF-TOL-SPEC-INT

Round-off error prevents the requested tolerance from being achieved on the $\langle value \rangle$ interval.
interval_flag [$\langle value \rangle$] = $\langle value \rangle$ over sub-interval ($\langle value \rangle$, $\langle value \rangle$).

NE-QUAD-BAD-SPEC-INT

Bad integrand behaviour occurs at some points of the $\langle value \rangle$ interval.
interval_flag [$\langle value \rangle$] = $\langle value \rangle$ over sub-interval ($\langle value \rangle$, $\langle value \rangle$).

NE-QUAD-NO-CONV-SPEC-INT

The integral has failed to converge on the $\langle value \rangle$ interval.
interval_flag [$\langle value \rangle$] = $\langle value \rangle$ over sub-interval ($\langle value \rangle$, $\langle value \rangle$).

NE-QUAD-DIVERGENCE-SPEC-INT

The integral is probably divergent on the $\langle value \rangle$ interval.
interval_flag [$\langle value \rangle$] = $\langle value \rangle$ over sub-interval ($\langle value \rangle$, $\langle value \rangle$).

6. Further Comments

The time taken depends on the integrand and on the accuracy required.

6.1. Accuracy

The function cannot guarantee, but in practice usually achieves, the following accuracy:

$$|I - \text{result}| \leq |\text{epsabs}|.$$

where **epsabs** is the user-specified absolute error tolerance. Moreover it returns the quantity **abserr** which, in normal circumstances, satisfies

$$|I - \text{result}| \leq \text{abserr} \leq |\text{epsabs}|.$$

6.2. References

- Malcolm M A and Simpson R B (1976) Local Versus Global Strategies for Adaptive Quadrature *ACM Trans. Math. Softw.* **1** 129–146.
- Piessens R, De Doncker-Kapenga E, Überhuber C and Kahaner D (1983) *QUADPACK, A Subroutine Package for Automatic Integration* Springer-Verlag.
- Wynn P (1956) On a Device for Computing the $e_m(S_n)$ Transformation *Math. Tables Aids Comput.* **10** 91–96.

7. See Also

nag_1d_quad_inf (d01amc)
nag_1d_quad_wt_trig (d01anc)

8. Example

To compute

$$\int_0^\infty \frac{1}{\sqrt{x}} \cos(\pi x/2) dx.$$

8.1. Program Text

```
/* nag_1d_quad_inf_wt_trig(d01asc) Example Program
 *
 * Copyright 1991 Numerical Algorithms Group.
 *
 * Mark 2, 1991.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <math.h>
#include <nagd01.h>
#include <nagx01.h>

#ifdef NAG_PROTO
static double g(double x);
#else
static double g();
#endif

main()
{
    double a;
    double omega;
    double epsabs, abserr;
    Nag_TrigTransform wt_func;
    double result;
    Nag_QuadSubProgress qpsub;
    Integer maxintervals, maxsubint_per_int;
    static NagError fail;

    Vprintf("d01asc Example Program Results\n");
    epsabs = 0.001;
    a = 0.0;
    omega = X01AAC * 0.5;
    wt_func = Nag_Cosine;
    maxintervals = 50;
    maxsubint_per_int = 500;
    d01asc(g, a, omega, wt_func, maxintervals, maxsubint_per_int,
           epsabs, &result, &abserr, &qpsub, &fail);
    Vprintf("a      - lower limit of integration = %10.4f\n", a);
    Vprintf("b      - upper limit of integration = infinity\n");
    Vprintf("epsabs - absolute accuracy requested = %9.2e\n\n", epsabs);
}
```

```

if (fail.code != NE_NOERROR)
    Vprintf("%s\n", fail.message);
if (fail.code != NE_INT_ARG_LT)
{
    Vprintf("result - approximation to the integral = %9.5f\n", result);
    Vprintf("abserr - estimate of the absolute error = %9.2e\n", abserr);
    Vprintf("qpsub.fun_count - number of function evaluations = %4ld\n",
        qpsub.fun_count);
    Vprintf("qpsub.intervals - number of intervals used = %4ld\n",
        qpsub.intervals);
    Vprintf("qpsub.subints_per_interval - \n\
maximum number of subintervals used in any one interval = %4ld\n",
        qpsub.subints_per_interval);
    exit(EXIT_SUCCESS);
}
else
    exit(EXIT_FAILURE);
}

#ifdef NAG_PROTO
static double g(double x)
#else
    static double g(x)
    double x;
#endif
{
    return (x > 0.0) ? 1.0/sqrt(x) : 0.0;
}

```

8.2. Program Data

None.

8.3. Program Results

```

d01asc Example Program Results
a      - lower limit of integration =      0.0000
b      - upper limit of integration = infinity
epsabs - absolute accuracy requested =  1.00e-03

result - approximation to the integral =  1.00000
abserr - estimate of the absolute error =  5.92e-04
qpsub.fun_count - number of function evaluations =  380
qpsub.intervals - number of intervals used =      6
qpsub.subints_per_interval -
    maximum number of subintervals used in any one interval =      8

```
