nag_1d_quad_wt_trig (d01anc)

1. Purpose

nag_1d_quad_wt_trig (d01anc) calculates an approximation to the sine or the cosine transform of a function g over $[a, b]$:

$$I = \int_a^b g(x) \sin(\omega x)\, dx \quad \text{or} \quad I = \int_a^b g(x) \cos(\omega x)\, dx$$

(for a user-specified value of ω).

2. Specification

```
#include <nag.h>
#include <nagd01.h>

void nag_1d_quad_wt_trig(double (*g)(double x), double a, double b,
               double omega, Nag_TrigTransform wt_func, double epsabs,
               double epsrel, Integer max_num_subint, double *result,
               double *abserr, Nag_QuadProgress *qp, NagError *fail)
```

3. Description

nag_1d_quad_wt_trig is based upon the QUADPACK routine QFOUR (Piessens *et al* (1983)). It is an adaptive routine, designed to integrate a function of the form $g(x)w(x)$, where $w(x)$ is either $\sin(\omega x)$ or $\cos(\omega x)$. If a sub-interval has length

$$L = |b - a| 2^{-l}$$

then the integration over this sub-interval is performed by means of a modified Clenshaw-Curtis procedure (Piessens and Branders (1975)) if $L\omega > 4$ and $l \leq 20$. In this case a Chebyshev-series approximation of degree 24 is used to approximate $g(x)$, while an error estimate is computed from this approximation together with that obtained using Chebyshev-series of degree 12. If the above conditions do not hold then Gauss 7-point and Kronrod 15-point rules are used. The algorithm, described in Piessens *et al* (1983), incorporates a global acceptance criterion (as defined in Malcolm and Simpson (1976)) together with the ϵ-algorithm (Wynn (1956)) to perform extrapolation. The local error estimation is described in Piessens *et al* (1983).

4. Parameters

g

The function **g**, supplied by the user, must return the value of the function g at a given point. The specification of **g** is:

```
double g(double x)
```

x
Input: the point at which the function g must be evaluated.

a

Input: the lower limit of integration, a.

b

Input: the upper limit of integration, b. It is not necessary that $a < b$.

omega

Input: the parameter ω in the weight function of the transform.

wt_func

Input: indicates which integral is to be computed:
if **wt_func** = **Nag_Cosine**, $w(x) = \cos(\omega x)$;
if **wt_func** = **Nag_Sine**, $w(x) = \sin(\omega x)$;
Constraint: **wt_func** = **Nag_Cosine** or **Nag_Sine**.

epsabs
Input: the absolute accuracy required. If **epsabs** is negative, the absolute value is used. See Section 6.1.

epsrel
Input: the relative accuracy required. If **epsrel** is negative, the absolute value is used. See Section 6.1.

max_num_subint
Input: the upper bound on the number of sub-intervals into which the interval of integration may be divided by the function. The more difficult the integrand, the larger **max_num_subint** should be.
Suggested value: a value in the range 200 to 500 is adequate for most problems.
Constraint: **max_num_subint** ≥ 1.

result
Output: the approximation to the integral I.

abserr
Output: an estimate of the modulus of the absolute error, which should be an upper bound for $|I-$**result**$|$.

qp
Pointer to structure of type Nag_QuadProgress with the following members:

num_subint – Integer
Output: the actual number of sub-intervals used.

fun_count – Integer
Output: the number of function evaluations performed by nag_1d_quad_wt_trig.

sub_int_beg_pts – double *
sub_int_end_pts – double *
sub_int_result – double *
sub_int_error – double *
Output: these pointers are allocated memory internally with **max_num_subint** elements. If an error exit other than **NE_INT_ARG_LT**, **NE_BAD_PARAM** or **NE_ALLOC_FAIL**, occurs, these arrays will contain information which may be useful. For details, see Section 6. If nag_1d_quad_wt_trig is to be called repeatedly, then the user should free the storage allocated by these pointers before any subsequent call is made.

fail
The NAG error parameter, see the Essential Introduction to the NAG C Library.
Users are recommended to declare and initialize **fail** and set **fail.print** = TRUE for this function.

5. Error Indications and Warnings

NE_INT_ARG_LT
On entry, **max_num_subint** must not be less than 1: **max_num_subint** = ⟨*value*⟩.

NE_BAD_PARAM
On entry, parameter **wt_func** had an illegal value.

NE_ALLOC_FAIL
Memory allocation failed.

NE_QUAD_MAX_SUBDIV
The maximum number of subdivisions has been reached: **max_num_subint** = ⟨*value*⟩.
The maximum number of subdivisions has been reached without the accuracy requirements being achieved. Look at the integrand in order to determine the integration difficulties. If the position of a local difficulty within the interval can be determined (e.g. a singularity of the integrand or its derivative, a peak, a discontinuity, etc.) you will probably gain from splitting up the interval at this point and calling the integrator on the sub-intervals. If necessary, another integrator, which is designed for handling the type of difficulty involved, must be used. Alternatively, consider relaxing the accuracy requirements specified by **epsabs** and **epsrel**, or increasing the value of **max_num_subint**.

NE_QUAD_ROUNDOFF_TOL
Round-off error prevents the requested tolerance from being achieved: **epsabs** = ⟨*value*⟩, **epsrel** = ⟨*value*⟩.
The error may be underestimated. Consider relaxing the accuracy requirements specified by **epsabs** and **epsrel**.

NE_QUAD_BAD_SUBDIV
Extremely bad integrand behaviour occurs around the sub-interval (⟨*value*⟩, ⟨*value*⟩).
The same advice applies as in the case of **NE_QUAD_MAX_SUBDIV**.

NE_QUAD_ROUNDOFF_EXTRAPL
Round-off error is detected during extrapolation.
The requested tolerance cannot be achieved, because the extrapolation does not increase the accuracy satisfactorily; the returned result is the best that can be obtained. The same advice applies as in the case of **NE_QUAD_MAX_SUBDIV**.

NE_QUAD_NO_CONV
The integral is probably divergent or slowly convergent.
Please note that divergence can also occur with any error exit other than **NE_INT_ARG_LT**, **NE_BAD_PARAM** or **NE_ALLOC_FAIL**.

6. Further Comments

The time taken by the function depends on the integrand and the accuracy required.

If the function fails with an error exit other than **NE_INT_ARG_LT**, **NE_BAD_PARAM** or **NE_ALLOC_FAIL**, then the user may wish to examine the contents of the structure **qp**. These contain the end-points of the sub-intervals used by nag_1d_quad_wt_trig along with the integral contributions and error estimates over the sub-intervals.

Specifically, for $i = 1, 2, \ldots, n$, let r_i denote the approximation to the value of the integral over the sub-interval $[a_i, b_i]$ in the partition of $[a, b]$ and e_i be the corresponding absolute error estimate.

Then, $\int_{a_i}^{b_i} g(x)w(x)\,dx \simeq r_i$ and **result** $= \sum_{i=1}^{n} r_i$ unless nag_1d_quad_wt_trig terminates while testing for divergence of the integral (see Piessens *et al* (1983), Section 3.4.3). In this case, **result** (and **abserr**) are taken to be the values returned from the extrapolation process. The value of n is returned in **num_subint**, and the values a_i, b_i, r_i and e_i are stored in the structure **qp** as

$$
\begin{aligned}
a_i &= \textbf{sub\_int\_beg\_pts}[i-1],\\
b_i &= \textbf{sub\_int\_end\_pts}[i-1],\\
r_i &= \textbf{sub\_int\_result}[i-1] \quad \text{and}\\
e_i &= \textbf{sub\_int\_error}[i-1].
\end{aligned}
$$

6.1. Accuracy

The function cannot guarantee, but in practice usually achieves, the following accuracy:

$$|I - \textbf{result}| \leq tol$$

where

$$tol = \max\{|\textbf{epsabs}|, |\textbf{epsrel}| \times |I|\}$$

and **epsabs** and **epsrel** are user-specified absolute and relative error tolerances. Moreover it returns the quantity **abserr** which, in normal circumstances, satisfies

$$|I - \textbf{result}| \leq \textbf{abserr} \leq tol.$$

6.2. References

Malcolm M A and Simpson R B (1976) Local Versus Global Strategies for Adaptive Quadrature *ACM Trans. Math. Softw.* **1** 129–146.

Piessens R and Branders M (1975) Algorithm 002. Computation of Oscillating Integrals *J. Comput. Appl. Math.* **1** 153–164.

Piessens R, De Doncker-Kapenga E, Überhuber C and Kahaner D (1983) *QUADPACK, A Subroutine Package for Automatic Integration* Springer-Verlag.

Wynn P (1956) On a Device for Computing the $e_m(S_n)$ Transformation *Math. Tables Aids Comput.* **10** 91–96.

7. See Also

nag_1d_quad_gen (d01ajc)

8. Example

To compute

$$\int_0^1 \ln x \sin(10\pi x)\, dx.$$

8.1. Program Text

```
/* nag_1d_quad_wt_trig(d01anc) Example Program
 *
 * Copyright 1991 Numerical Algorithms Group.
 *
 * Mark 2, 1991.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <math.h>
#include <nagd01.h>
#include <nagx01.h>

#ifdef NAG_PROTO
static double g(double x);
#else
static double g();
#endif

main()
{

  double a, b;
  double omega;
  double epsabs, abserr, epsrel, result;
  Nag_TrigTransform wt_func;
  Nag_QuadProgress qp;
  Integer max_num_subint;
  static NagError fail;

  Vprintf("d01anc Example Program Results\n");
  epsrel = 0.0001;
  epsabs = 0.0;
  a = 0.0;
  b = 1.0;
  omega = X01AAC * 10.0;
  wt_func = Nag_Sine;
  max_num_subint = 200;
  d01anc(g, a, b, omega, wt_func, epsabs, epsrel, max_num_subint, &result,
         &abserr, &qp, &fail);
  Vprintf("a      - lower limit of integration = %10.4f\n", a);
  Vprintf("b      - upper limit of integration = %10.4f\n", b);
```

```
  Vprintf("epsabs - absolute accuracy requested = %9.2e\n", epsabs);
  Vprintf("epsrel - relative accuracy requested = %9.2e\n\n", epsrel);
  if (fail.code != NE_NOERROR)
    Vprintf("%s\n", fail.message);
  if (fail.code != NE_INT_ARG_LT)
    {
      Vprintf("result - approximation to the integral = %9.5f\n", result);
      Vprintf("abserr - estimate of the absolute error = %9.2e\n", abserr);
      Vprintf("qp.fun_count  - number of function evaluations = %4ld\n",
              qp.fun_count);
      Vprintf("qp.num_subint  - number of subintervals used = %4ld\n",
              qp.num_subint);
      exit(EXIT_SUCCESS);
    }
  exit(EXIT_FAILURE);
}

#ifdef NAG_PROTO
static double g(double x)
#else
     static double g(x)
     double x;
#endif
{
  return (x>0.0) ? log(x) : 0.0;
}
```

8.2. Program Data

None.

8.3. Program Results

```
d01anc Example Program Results
a      - lower limit of integration =     0.0000
b      - upper limit of integration =     1.0000
epsabs - absolute accuracy requested =  0.00e+00
epsrel - relative accuracy requested =  1.00e-04

result - approximation to the integral =  -0.12814
abserr - estimate of the absolute error =  3.58e-06
qp.fun_count  - number of function evaluations =  275
qp.num_subint  - number of subintervals used =    8
```