BAGNOLD FORMULA REVISITED: INCORPORATING PRESSURE GRADIENT INTO ENERGETICS MODELS

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Abstract: In this paper, we examine a reformulation of the Bagnold sediment transport formula, based on using friction velocity to express bottom shear stress. This modification allows the transport formulation to retain the effect of phase lag between free stream velocity and bottom stress, neglected in Bagnold's original formula and usually associated with flow acceleration (or, in other words, pressure gradient). Friction velocity is computed using a linearized 1-D bottom boundary layer model. The modified Bagnold model can predict net onshore sediment transport for asymmetric, zero skewness waves. Results for analytical waveforms are compared with previous work based on discrete particle simulations and two-phase flow methods and show qualitative consistency.

INTRODUCTION

The Bagnold Model

Bagnold (1963, 1966) derived a stream-based sediment transport model. In that model, Bagnold assumes the sediment is transported in two modes, i.e., the bedload transport and the suspended transport. The bedload sediment is transported by the flow via grain to grain interactions, the suspended sediment transport is supported by fluid flow through turbulent diffusion. The total load sediment transport rate i reads (Bagnold, 1966)

$$i = \left[\frac{\epsilon_b}{\tan\phi - \tan\beta} + \frac{\epsilon_s(1-\epsilon_b)}{(w/u_b) - \tan\beta}\right]\omega \tag{1}$$

where ω is the available fluid power, w is the fall velocity of sediment. ϵ_b and ϵ_s are the bedload and suspended load efficiencies, respectively. They both smaller than one. $\tan\beta$ is the bottom slope, and ϕ is the particle friction angle. The available fluid power ω is the work done by the bottom shear stress $\vec{\tau}_b$

$$\omega = \vec{\tau}_b \cdot \vec{u}_b \tag{2}$$

where $\vec{u}_b$ is the near bed free stream velocity. The bottom shear stress is parameterized using the quadratic drag law

$$\vec{\tau}_b = \rho c_f |\vec{u}_b| \vec{u}_b \tag{3}$$

with c_f the bottom friction coefficient.

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Wave-averaged Bagnold Model-The BBB Concept

Substituting (2)-(3) into (1), and considering the bottom slope effect, Bailard and Inman (1981) obtained the total sediment transport rate $\vec{i}$ written in terms of free stream velocity

$$\begin{aligned} \vec{i} &= \rho c_f \frac{\epsilon_b}{\tan\phi}\left[|\vec{u}_b|^2\vec{u}_b - \frac{\tan\beta}{\tan\phi}|\vec{u}_b|^3\right] \\ &+ \rho c_f \frac{\epsilon_s(1-\epsilon_b)}{w}\left[|\vec{u}_b|^3\vec{u}_b - \frac{\epsilon_s(1-\epsilon_b)}{w}\tan\beta|\vec{u}_b|^5\right] \end{aligned} \tag{4}$$

The first bracket in (4) represents the bedload sediment transport, and the second bracket represents the suspended sediment transport. Both contain the sediment transport in the direction of the instantaneous velocity(the first and third term), and the sediment transport down slope (the second and fourth term). In (4), the near bed free stream velocity $\vec{u}_b$ is taken to be the near bed orbital velocity.

Bailard (1981) took a time-average of (4) for the wave period T to obtain the total net sediment transport rate $<\vec{i}>$ due to waves and currents.

$$\begin{aligned} <\vec{i}> &= \rho c_f \frac{\epsilon_b}{\tan\phi}\left[<|\vec{u}_b|^2\vec{u}_b> - \frac{\tan\beta}{\tan\phi}<|\vec{u}_b|^3>\right] \\ &+ \rho c_f \frac{\epsilon_s(1-\epsilon_b)}{w}\left[<|\vec{u}_b|^3\vec{u}_b> - \frac{\epsilon_s(1-\epsilon_b)}{w}\tan\beta<|\vec{u}_b|^5>\right] \end{aligned} \tag{5}$$

Here $<f>$ denotes the wave-averaging of arbitrary variable f,

$$<f>=\frac{1}{T}\int_0^T f dt \tag{6}$$

Equation (5) is often referred to as the "BBB energetics model", with BBB representing Bagnold (1963,1966), Bowen(1980) and Bailard (1981).

Limitation of the Bagnold-type Models

The Bagnold model (1)-(3) and its wave-averaged version (5) relate the total sediment transport rate to the near bed free stream velocity. This model lacks information on flow acceleration or pressure gradient effects within the bottom boundary layer, and, in particular, predicts zero sediment transport in wave fields with zero velocity skewness.

Recent field observations (Gallagher *et al.*, 1998; Elgar *et al.*, 2001) have shown net onshore sediment transport between storms (manifested mainly by net onshore motion of shore-parallel bars) when unbroken waves are strongly skewed but undertow over the bar crest is weak. The BBB transport formula successfully predicts offshore sediment transport and bar crest movement during energetic wave events, buts fail to predict the onshore bar motion in lower energy conditions when calibrated with coefficients that are appropriate for offshore transport (Gallagher *et al.*, 1998). Based on field observations, Elgar *et al.* (2001) noticed that the onshore sediment transport (or bar crest motion) is strongly correlated to cross shore gradient in a wave-averaged acceleration skewness, indicating that the local transport rate is likely to be directly dependent on this statistic as well. Numerical computations based on a discrete particle method (Drake and Calantoni, 2001) have also shown that net transport can occur under zero-skewness waves, and is correlated with flow acceleration. Base on this as a starting point, Drake and Calantoni (2001) introduced an *ad hoc* extra term $a_{spike} =< (\partial u_b/\partial t)^3 > / < (\partial u_b/\partial t)^2 >$ into the BBB model to account for the difference in the magnitude of the acceleration under the front and back of the wave. The extra sediment transport rate to be added into the BBB formula (5) due to acceleration was given as

$$< \vec{i}_{spike} >= \begin{cases} K_a(a_{spike} - sgn[a_{spike}]a_{crit}) & |a_{spike}| >= a_{crit} \\ 0 & |a_{spike}| < a_{crit} \end{cases} \tag{7}$$

where a_{crit} is the critical value of a_{spike} that must be exceeded before acceleration enhances the sediment transport, and K_a is the model parameter. The total net sediment transport rate is then

$$\begin{aligned} < \vec{i} > &= \rho c_f \frac{\epsilon_b}{\tan\phi}\left[< |\vec{u}_b|^2\vec{u}_b > - \frac{\tan\beta}{\tan\phi} < |\vec{u}_b|^3 >\right] \\ &+ \rho c_f \frac{\epsilon_s(1-\epsilon_b)}{w}\left[< |\vec{u}_b|^3\vec{u}_b > - \frac{\epsilon_s(1-\epsilon_b)}{w}\tan\beta < |\vec{u}_b|^5 >\right] \\ &+ < \vec{i}_{spike} > \end{aligned} \tag{8}$$

The modified BBB formula (8) has been tested using measured near bed velocities (Hoefel and Elgar, 2003) and showed favorable results to the original BBB formula (5) when comparing with data. Long and Kirby (2003) have also used an extended Bagnold model, incorporating an instantaneous acceleration term, to model transport in a phase-resolving Boussinesq model calculation, and were able to predict qualitatively accurate onshore bar migration in a model prediction using only incident wave conditions measured offshore.

A REFORMULATED BAGNOLD MODEL

Despite the success of the extended Bagnold models discussed above, it is clear that the added acceleration (or pressure gradient) effects are incorporated in an ad hoc manner which does not have a clear theoretical foundation. More recently, attention has turned to a more direct examination of the bottom boundary layer mechanics and the relation between unsteady free-stream velocity and resulting bed shear stress. Hsu and Hanes (2004) have considered the calculation of bed stress using artificial skewed and asymmetric free stream velocities, and have shown that predictions of a detailed two-phase transport model can be recovered by a simple application of the Meyer-Peter Muller formula using the calculated bed stress. Henderson *et al.* (2004) have used a similar approach and have shown, using measured data to drive a 1-D vertical boundary layer model, that both erosional and accretionary behavior of sand bar crests can be modelled and that good agreement with field observation can be obtained. Long et al (2004) have used a similar approach but employ free stream data predicted by a Boussinesq model to similarly predict onshore bar migration using computed bed stress and the Meyer-Peter Muller transport formula.

The Bagnold model and its wave-averaged version, the BBB model, do not account for flow with strong acceleration or pressure gradient. This limitation is introduced when the bottom shear stress is parameterized using the near bed free stream velocity through the quadratic drag law (3) through Reynolds similarity. We recognize that for flows with a free surface, where gravitational forces (pressure gradient due to free surface) must be considered, the parameterization of the bottom shear stress using (3) is not valid (Schlichting, 1960). This is also pointed out by Nielsen (2002) and Nielsen and Callaghan (2003). They acknowledge that the Bagnold type formula written in terms of the odd moments of near bed free stream velocity "fail spectacularly under some real waves". Based on laminar bottom boundary layer theory, Nielsen (1992) obtains the bottom shear stress written in terms of near bed free stream velocity and a phase shift φ_τ, in which φ_τ is the phase lag between the bottom shear stress and the free stream velocity, and found to be around 40 degree based on the data they tested. Here, we employ a more brute force approach and compute the bottom shear stress and friction velocity $\vec{u}_*$ using a bottom boundary layer (BBL) model (see Appendix), in the spirit of Hsu and Hanes (2004).

In the following, we replace (3) with the expression

$$\vec{\tau}_b = \rho|\vec{u}_*|\vec{u}_* \tag{9}$$

which is the fundamental definition of friction velocity. However, the parameterization for the fluid power ω is not straightforward and needs further discussion. Here, we follow Bagnold (1963) and consider that the work done on the sediments is at the center of pressure, and the velocity at that point is parameterized as $\vec{u}' = c_r\vec{u}_*$. c_r needs to be determined experimentally. Then the work done by the bottom shear stress is

$$\omega = \vec{\tau}_b \cdot \vec{u}' = \rho c_r |\vec{u}_*|^2 \vec{u}_* \tag{10}$$

Therefore, the modified Bagnold formula is

$$\begin{aligned} \vec{i} &= \rho\frac{\epsilon_b c_r}{\tan\phi}\left[|\vec{u}_*|^2\vec{u}_* - \frac{\tan\beta}{\tan\phi}|\vec{u}_*|^3\right] \\ &+ \rho\frac{\epsilon_s(1-\epsilon_b)c_r}{w}\left[|\vec{u}_*|^3\vec{u}_* - \frac{\epsilon_s(1-\epsilon_b)}{w}\tan\beta|\vec{u}_*|^5\right] \end{aligned} \tag{11}$$

It is seen that (11) is very similar to (4), except we replace the free stream velocity $\vec{u}_b$ by the friction velocity $\vec{u}_*$. It is worthwhile to point out that the effect of pressure gradient is implicitly included in the friction velocity $\vec{u}_*$, but not in the free stream velocity $\vec{u}_b$.

By performing wave-averaging, we obtain the modified BBB model

$$\begin{aligned} <\vec{i}> &= \rho\frac{\epsilon_b c_r}{\tan\phi}\left[<|\vec{u}_*|^2\vec{u}_*> - \frac{\tan\beta}{\tan\phi}<|\vec{u}_*|^3>\right] \\ &+ \rho\frac{\epsilon_s(1-\epsilon_b)c_r}{w}\left[<|\vec{u}_*|^3\vec{u}_*> - \frac{\epsilon_s(1-\epsilon_b)}{w}\tan\beta<|\vec{u}_*|^5>\right] \end{aligned} \tag{12}$$

It will be shown in the next section that using $<|\vec{u}_*|^2\vec{u}_*>$ results in net onshore sediment transport for asymmetric, zero skewness waves, whereas using $<|\vec{u}_b|^2\vec{u}_b>$ produces no net sediment transport. Finally, the wave-averaged net bedload flux $<\vec{q}>$ is related to the immersed weight transport rate $<\vec{i}>$ (Drake and Calantoni, 2001),

$$<\vec{q}> = \frac{<\vec{i}>}{g(\rho_s-\rho)}\rho_s \tag{13}$$

RESULTS

In this section we examine the performance of the BBB type formula written in terms of the near bed free stream velocity and the friction velocity, respectively. We study the instantaneous and wave-averaged bedload sediment transport under zero-skewness saw-tooth shape waves. Saw-tooth shape waves are of interest because they are similar to broken waves, and the BBB formula fails to predict any onshore sediment transport due to zero skewness. However, both field observations and numerical modelings using discrete particle method (Drake and Calantoni, 2001) and two-phase flow approach (Hsu and Hanes, 2004) show extensive sediment transport due to flow acceleration (or pressure gradient).

In Drake and Calantoni (2001), the near bed free stream velocity $\vec{u}_b$ is driven by the pressure gradient $F(t)$, and $F(t)$ follows

$$F(t) \propto \sum_{n=0}^{4} \frac{1}{2^n} \sin\left[(n+1)\frac{2\pi}{T}t + n\phi\right] \tag{14}$$

and ϕ is defined as the "waveform parameter". The typical saw-tooth wave is that of $\phi = \frac{\pi}{2}$. In Hsu and Hanes (2004), the saw-tooth shaped free stream velocitiy is described by

$$\begin{aligned} u_b(t) &= U_{0s} \sum_{n=1}^{5} \frac{1}{2^{n-1}} \sin\left[n\frac{2\pi}{T}t + (n-1)\pi\right] \\ &= U_{0s} \sum_{n=0}^{4} \frac{1}{2^n} \sin\left[(n+1)\frac{2\pi}{T}t + n\pi\right] \end{aligned} \tag{15}$$

in their Equation (17), with U_{0s} the velocity amplitude. However, we found that the near bed free stream velocity in Drake and Calantoni (2001) do not response to the pressure gradient force $F(t)$ linearly, and the wave form discussed in Hsu and Hanes (2004) do not correspond to equation (15)(Equation (17) in their paper). In the present work, the saw-tooth wave is constructed using

$$u_b(t) = \sum_{n=0}^{4} \frac{1}{2^n} \sin\left[(n+1)\frac{2\pi}{T}t\right] \tag{16}$$

This result roughly corresponds to the saw-tooth wave with waveform parameter $\phi = \frac{\pi}{2}$ in Drake and Calantoni (2001).

Figure 1 shows the results for this saw-tooth wave. The upper panel of Figure 1 shows the free stream velocity (solid line) and the flow acceleration (dashed line). The lower panel of Figure 1 shows corresponding friction velocity $|\vec{u}_*|^2\vec{u}_*$ (solid line) and near bed velocity $|\vec{u}_b|^2\vec{u}_b$ (dashed line). It shows in Figure 1 that $|\vec{u}_b|^2\vec{u}_b$ is symmetric, and equals to zero upon wave-averaging. Whereas $|\vec{u}_*|^2\vec{u}_*$ is asymmetric and the wave-average is positive (onshore directed). If we consider the bed slope is very small, and bed-load sediment transport dominates, then $|\vec{u}_b|^2\vec{u}_b$ and $|\vec{u}_*|^2\vec{u}_*$ are proportional to the time-dependent bed load flux $\vec{q}$. And if we compare this result with that shown in Drake and Calantoni (2001), we see that the bed-load flux computed using the friction velocity $\vec{u}_*$ and (12) is very similar to the results of Drake and Calantoni (2001). Moreover, this result is also qualitatively similar to that presented by Hsu and Hanes (2004) in their Figure 2, if we integrate their results vertically. However, it is noticed that Figure 1 shows virtually no sediment transport where the

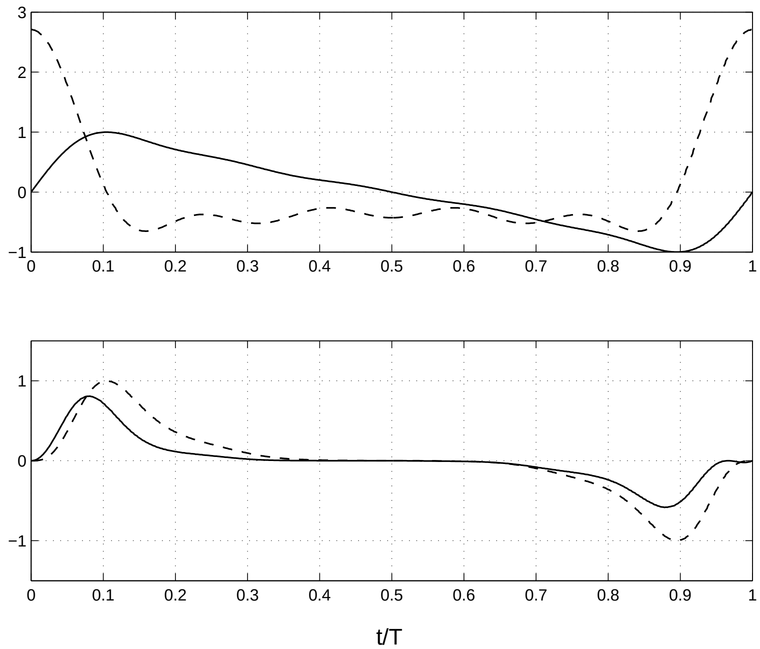

Fig. 1. Results corresponding to Drake and Calantoni (2001), $\phi = \pi/2$. Upper panel: Free stream velocity (solid line) and flow acceleration (dashed line). Lower panel: $|\vec{u}_*|^2\vec{u}_*$ (solid line) showing net on shore sediment transport when averaged over wave period T, and $|\vec{u}_b|^2\vec{u}_b$(dashed line) showing no net sediment transport due to zero skewness. The magnitudes of the variables are adjusted to fit the figure.

free stream velocity is zero, whereas Hsu and Hanes (2004) showed limited rate of sediment transport. We think this may be due to the fact that in the present work we neglected the nonlinear convective terms in the BBL equations, so that we may miss some phase information in the bottom shear stress. Nonetheless, the important feature is that the modified Bagnold formula can predict net onshore sediment transport under asymmetric zero skewness waveforms provided the bottom friction is parameterized using the friction velocity.

For comparison, a case of a symmetric, skewed wave form is shown in Figure 2. The near bed free stram velocity is calculated using

$$u_b(t) = \sum_{n=0}^{4} \frac{1}{2^n} \sin\left[(n+1)\frac{2\pi}{T}t + (n + \frac{3}{2}\pi)\right] \tag{17}$$

and it corresponds to the waveform parameter $\phi = 0$, Figure 5 in Drake and Calantoni (2001) and Figure 8-9 in Hsu and Hanes (2004). For this case, both $|\vec{u}_*|^2\vec{u}_*$ and $|\vec{u}_b|^2\vec{u}_b$ predict net onshore sediment transport due to wave skewness.

Next we compare the wave-averaged bedload flux $< \vec{q} >$ versus the third moments of free stream velocity $< u_b^3 >$ for different wave forms on a flat bottom. The results are computed using (12) and (13). The wave period is $T = 6.0$s and the maximum free stream velocities are $0.5, 0.75, 1.0, 1.25, 1.5$m/s. The wave forms roughly correspond to $\phi = 0, \frac{\pi}{4}, \frac{\pi}{2}$ in Drake and Calantoni (2001), though a precise match of the wave forms is found difficult to achieve.

Parameters generally suggested for the use of the BBB formula (Gallagher *et al.*, 1998)

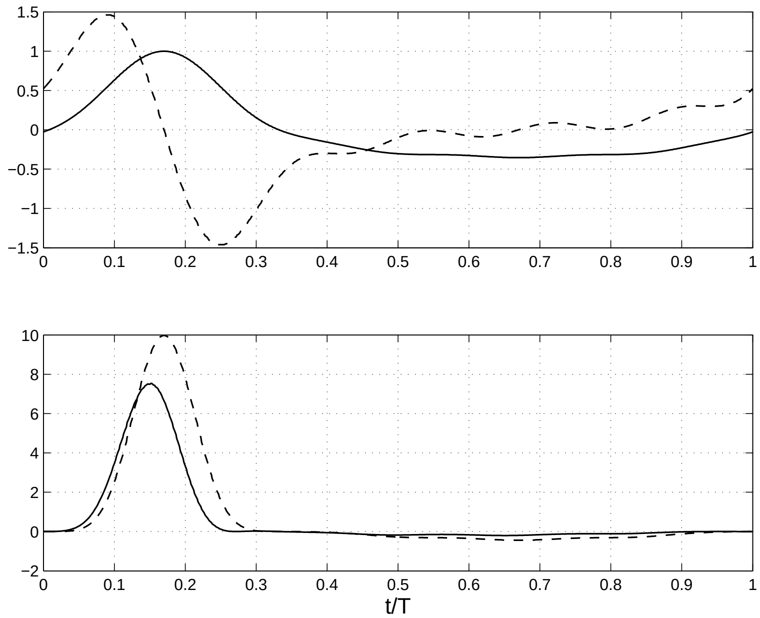

Fig. 2. Results corresponding to Drake and Calantoni (2001), $\phi = 0$. Both cases show net onshore sediment transport due to wave skewness. Legends are the same as the previous figure.

are used here with $\tan\phi = 0.63$ and $\epsilon_b = 0.12$. The parameter c_r in (10) is chosen to be 5 for the present study. We see that Figure (3) shows strikingly similar pattern to that shown in Drake and Calantoni (2001) for their Figure (7). However, our results are significantly smaller than those presented by Drake and Calantoni (2001). The reason is not clear, but we notice that the Drake and Calantoni (2001) results were obtained using a unusually large bedload efficiency, $\epsilon_b = 1.03$, whereas physics requests the bedload efficiency to be less than one. Besides, field comparisons using the Drake and Calantoni (2001) formula (14)-(8) showed that their K_a is orders larger than the calibrated value(see Hoefel and Elgar, 2003, Long and Kirby 2003).

CONCLUSIONS

In this paper we rederive the Bagnold formula and we found that the net onshore directed sediment transport is correlated to the bottom shear stress. The BBB-type formula written in terms of odd moments of free stream velocity was derived using the parameterized bottom shear stress for quasi-steady flow. Because this parameterization for bottom shear stress is not valid for flow with acceleration, the BBB-type formula was unable to predict net onshore sediment transport under zero skewed sawtooth waves. Using bottom shear stress computed from the BBL equations, the corrected BBB-type formula can predict net onshore sediment transport. Results for analytical waveforms are discussed and compared with previous works using discrete particle (Drake and Calantoni, 2001) and two-phase flow method (Hsu and Hanes, 2004) and showed qualitative consistency.

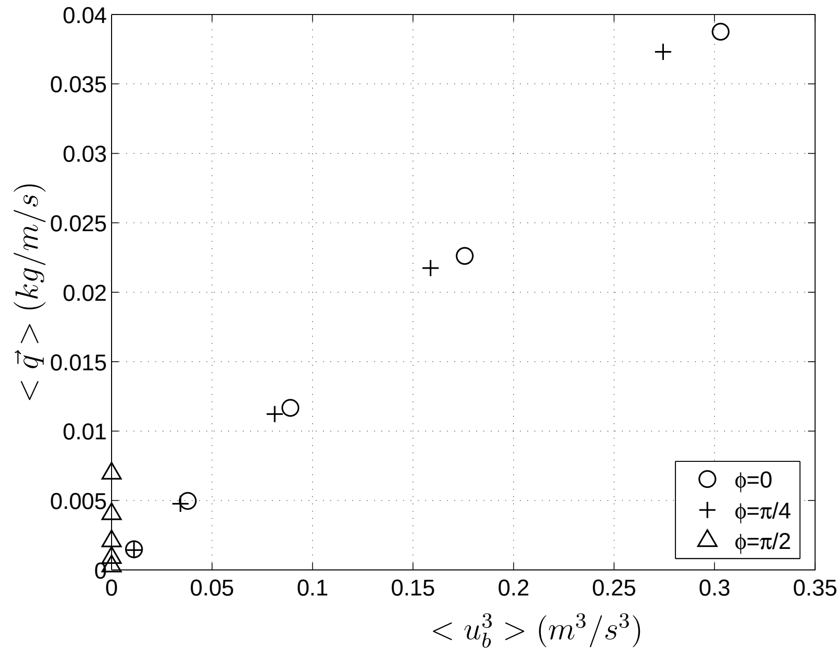

Fig. 3. Computed bedload flux versus odd moments of the free stream velocity $< u_b^3 >$ for different wave forms. The wave period is $T = 6.0$s and the maximum free stream velocities are $0.5, 0.75, 1.0, 1.25, 1.5$ m/s.

APPENDIX: SOLVING THE BOTTOM BOUNDARY LAYER EQUATIONS

The continuity and momentum equations inside the bottom boundary layer (BBL) read,

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \tag{18}$$

and

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} = \frac{\partial u_b}{\partial t} + u_b\frac{\partial u_b}{\partial x} + \frac{1}{\rho}\frac{\partial \tau_{zx}}{\partial z} \tag{19}$$

where u_b and u are the near bed free stream velocity and velocity inside the BBL, respectively. The boundary conditions for (18) and (19) are

$$u = 0; \quad w = 0; \quad \text{at} \quad z = z_0 \tag{20}$$

$$u = u_b; \quad w = 0; \quad \text{at} \quad z \to \infty \tag{21}$$

with z_0 the bed level.

Assuming the nonlinear convective terms are higher order terms, we obtain the linearized BBL equation (Trowbridge and Madsen, 1984),

$$\frac{\partial u}{\partial t} = \frac{\partial u_b}{\partial t} + \frac{1}{\rho}\frac{\partial \tau_{zx}}{\partial z} \tag{22}$$

The Boussinesq assumption relates the shear stress to the gradient of velocity through

$$\tau_{zx} = \rho(\nu + \nu_T)\frac{\partial u}{\partial z} \tag{23}$$

where ν and ν_T are the kinematic and turbulent eddy viscosity, respectively. And ν_T is computed using a turbulence model. In general, turbulence models that record flow history, such as the $k-\epsilon$ type models, are more accurate than those don't. However, we also notice that virtually almost all turbulence models impose the "log law" (also termed as the "wall function") at the wall. And previous researches (Wilcox, 1998 for example) show that the BBL results are not sensitive to turbulence models used near the bottom. Therefore, in this research, we will use the more efficient mixing length model. Then the eddy viscosity is computed through the friction velocity

$$\nu_T = \kappa |u_*| z \tag{24}$$

where $\kappa = 0.40$ is the Karman coefficient, and z is the distance from bed. $|\cdot|$ denotes the absolute value of a variable. Imposing the "log law", the friction velocity relates to the near bed velocity

$$u_* = \kappa u_1 \ln\left(\frac{z}{z_0}\right) \tag{25}$$

where u_1 is the BBL velocity at the first numerical grid point following Launder and Spadling (1974), and z_0 is the bed level and normally taken as $z_0 = k_N/30$. The bed roughness k_N relates to the grain diameter d through $k_N = Nd$. Here $d = 0.11$mm and $k_N = 15d$ is chosen for this study.

The above linearized BBL problem is a typical diffusion problem. Without loose of generality, we assume the the boundary layer thickness $\delta = 15$cm and solve it using the FTCS (forward time, center space) scheme. 60 grid points are solved in the vertical direction and the time step is computed so that the diffusion in one time step is smaller than one grid size.

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