Landslide Generated Waves
- Smoothed Particle Hydrodynamics (SPH) Model for Soil–Water Coupling

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Benchmark Problem #5
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"**Numerical simulation of landslide-generated waves using a soil–water coupling smoothed particle hydrodynamics model**" Advances in Water Resources 92 (2016) 130–141

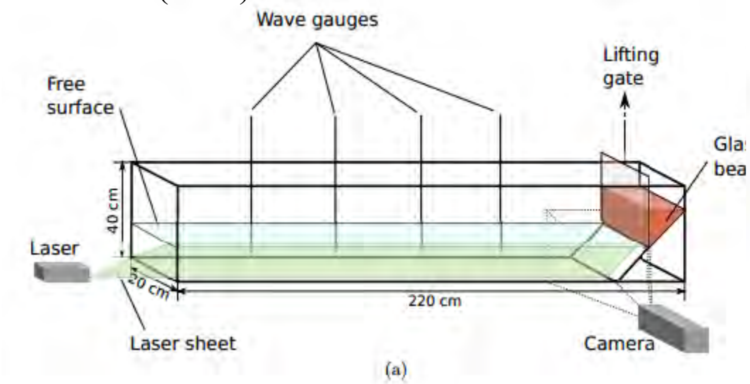
a b s t r a c t

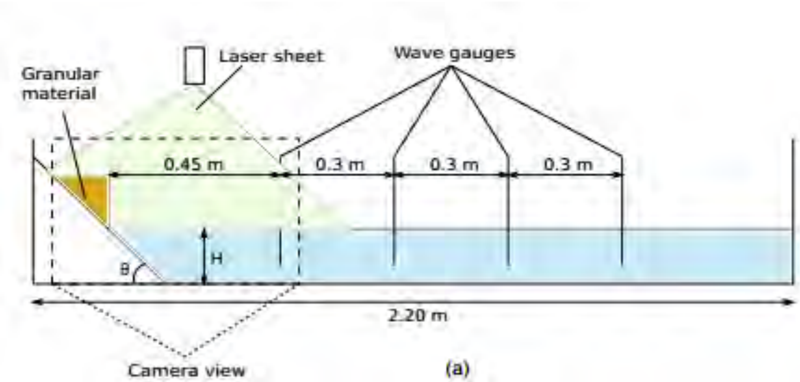
We simulate the generation of a landslide-induced impulse wave with a newly-developed soil–water coupling model in the smoothed particle hydrodynamics (SPH) framework. The model includes an elasto– plastic constitutive model for soil, a Navier–Stokes equation based model for water, and a bilateral coupling model at the interface. The model is tested with simulated waves induced by a slow and a fast landslide. Good agreement is obtained between simulation results and experimental data. The generated wave and the deformation of the landslide body can both be resolved satisfactorily. All parameters in our model have their physical meaning in soil mechanics and can be obtained from conventional soil mechanics experiments directly. The influence of the dilatancy angle of soil shows that the non-associated flow rule must be selected, and the value of the dilatancy angle should not be chosen arbitrarily, if it is not determined with relative experiments

1. Introduction

- **Experimental setup**

Viroulet et al (2014)





- **Test Cases:**

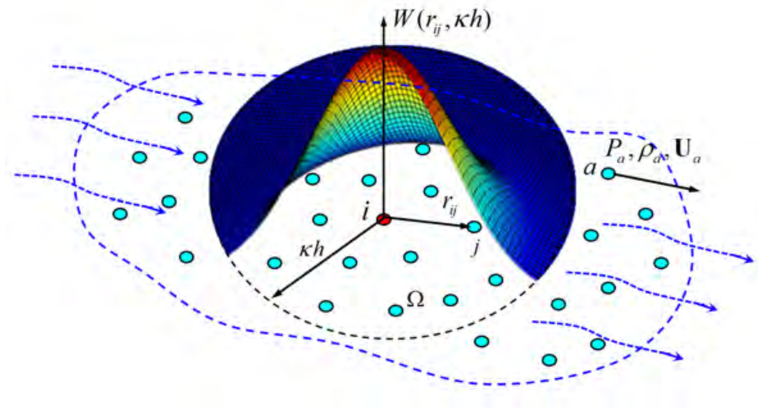
Case 1 : *D* = 1.5 mm, *H* = 14.8 cm, *L* = 11cm
Case 2 : *D* = 10 mm, *H* = 15 cm, *L* = 13.5 cm

granular material	diameter (mm)	θ_c
small glass beads	1.5	$25,7° \pm 0,9°$
medium glass beads	4	$23,3° \pm 0,8°$
large glass beads	10	$20,1° \pm 1,2°$
aquarium sand	$\simeq 4$	$37,3° \pm 0,6°$

Table 1: Mean particle diameter, d and critical angle of avalanche θ_c for the four different granular media.

2. Numerical Model

- **Model for Water**



➢ Governing equations:

$$\frac{d\rho}{dt} = -\rho \frac{\partial v^{\beta}}{\partial x^{\beta}} \qquad \sigma_i^{\alpha\beta} = -p\delta^{\alpha\beta} + \tau^{\alpha\beta}$$

$$\frac{d\boldsymbol{v}^{\alpha}}{dt} = \frac{1}{\rho}\frac{\partial \sigma^{\alpha\beta}}{\partial x^{\beta}} + g \qquad P = B[(\frac{\rho}{\rho_0})^{\gamma} - 1]$$

➢ SPH form:

$$\frac{D\rho_i}{Dt} = \sum_{j=1}^{N} m_j (\boldsymbol{v}_i - \boldsymbol{v}_j) \cdot \nabla_i W_{ij}$$

$$\frac{Dv_i^{\alpha}}{Dt} = -\sum_{j=1}^{N} m_j (\frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2} + \Pi_{ij}) \cdot \nabla_i W_{ij} + g^{\alpha}$$

Artificial viscosity term

$$\Pi_{ij} = \begin{cases} \dfrac{-\alpha \overline{c}_{ij} \mu_{ij}}{\overline{\rho}_{ij}}, & \boldsymbol{v}_{ij} \cdot \boldsymbol{r}_{ij} < 0 \\ 0, & \boldsymbol{v}_{ij} \cdot \boldsymbol{r}_{ij} \geq 0 \end{cases}$$

Weight function or kernel

$$W(r,h) = \alpha_D \begin{cases} 1 - \frac{3}{2}q^2 + \frac{3}{4}q^3 & 0 \leq q \leq 1 \\ \frac{1}{4}(2-q)^3 & 1 \leq q \leq 2 \\ 0 & q \geq 2 \end{cases}$$

2. Numerical Model

- **Model for Soil: elasto-plastic** model by Bui et al (2008)

Total strain rate tensor

$$\dot{\boldsymbol{\varepsilon}}^{\alpha\beta} = \frac{1}{2}\left(\frac{\partial \mathbf{v}^{\alpha}}{\partial x^{\beta}} + \frac{\partial \mathbf{v}^{\beta}}{\partial x^{\alpha}}\right)$$

Elastic strain rate tensor

Generalized Hooke's law

$$\dot{\varepsilon}_{\mathrm{e}} = \frac{\dot{s}^{\alpha\beta}}{2G} + \frac{1-2\upsilon}{3E}\dot{\sigma}^{\gamma\gamma}\delta^{\alpha\beta}$$

Plastic strain rate tensor

Plastic flow rule

$$\dot{\varepsilon}_{p} = \dot{\lambda}\frac{\partial g}{\partial \sigma^{\alpha\beta}}$$

Total strain rate tensor

$$\dot{\varepsilon}^{\alpha\beta} = \frac{\dot{S}^{\alpha\beta}}{2G} + \frac{1-2\upsilon}{3E}\dot{\sigma}^{\gamma\gamma}\delta^{\alpha\beta} + \dot{\lambda}\frac{\partial g}{\partial \sigma^{\alpha\beta}}$$

2. Numerical Model

- **Model for Soil**

Drucker–Prager yield criterion

Yield condition

$$f(I_1, J_2) = \sqrt{J_2} + \alpha_\varphi I_1 - k_c = 0$$

Plastic potential function

$$g(I_1, J_2) = \sqrt{J_2} + \alpha_\psi I_1 - \text{constant}$$

Constitutive equations

$$\frac{D\sigma_i^{\alpha\beta}}{Dt} = \sigma_i^{\alpha\gamma}\dot{\omega}^{\beta\gamma} + \sigma_i^{\gamma\beta}\dot{\omega}_i^{\alpha\gamma} + 2G\dot{e}_i^{\alpha\beta} + K\varepsilon_i^{\gamma\gamma}\delta_i^{\alpha\beta} - \dot{\lambda}_i\left[3\alpha_\psi K\delta^{\alpha\beta} + \frac{G}{\sqrt{J_2}}s_i^{\alpha\beta}\right]$$

Plastic multiplier

$$\dot{\lambda}_i = \begin{cases} \dfrac{3\alpha_\phi K\dot{\varepsilon}_i^{\gamma\gamma} + (G/\sqrt{J_2})s_i^{\alpha\beta}\dot{\varepsilon}_i^{\alpha\beta}}{9\alpha_\phi\alpha_\psi K + G} & f(I_1, J_2)=0 \\ 0 & f(I_1, J_2) < 0 \end{cases}$$

2. Numerical Model

- **Model for Soil**

➢ Constitutive equations in SPH form:

$$\frac{D\sigma_i^{\alpha\beta}}{Dt} = \sigma_i^{\alpha\gamma}\dot{\omega}^{\beta\gamma} + \sigma_i^{\gamma\beta}\dot{\omega}_i^{\alpha\gamma} + 2G\dot{e}_i^{\alpha\beta} + K\varepsilon_i^{\gamma\gamma}\delta_i^{\alpha\beta} - \dot{\lambda}_i\left[3\alpha_\psi K\delta^{\alpha\beta} + \frac{G}{\sqrt{J_2}}s_i^{\alpha\beta}\right]$$

$$\dot{\varepsilon}^{\alpha\beta} = \frac{1}{2}\left[\sum_{j=1}^{N}\frac{m_j}{\rho_j}(v_j^{\alpha} - v_i^{\alpha})\frac{\partial W_{ij}}{\partial x_i^{\beta}} + \sum_{j=1}^{N}\frac{m_j}{\rho_j}(v_j^{\beta} - v_i^{\beta})\frac{\partial W_{ij}}{\partial x_i^{\alpha}}\right]$$

$$\dot{\omega}^{\alpha\beta} = \frac{1}{2}\left[\sum_{j=1}^{N}\frac{m_j}{\rho_j}(v_j^{\alpha} - v_i^{\alpha})\frac{\partial W_{ij}}{\partial x_i^{\beta}} - \sum_{j=1}^{N}\frac{m_j}{\rho_j}(v_j^{\beta} - v_i^{\beta})\frac{\partial W_{ij}}{\partial x_i^{\alpha}}\right]$$

$$K = \frac{E}{3(1-2\upsilon)} \quad \text{and} \quad G = \frac{E}{2(1+\upsilon)}$$

For 2D simulations:

$$\alpha_\varphi = \frac{\tan\varphi}{\sqrt{9+12\tan^2\varphi}}$$

$$k_c = \frac{3c}{\sqrt{9+12\tan^2\varphi}}$$

$$\alpha_\psi = \frac{\tan\psi}{\sqrt{9+12\tan^2\psi}}$$

➢ Governing equations in SPH form:

$$\frac{D\rho_i}{Dt} = \sum_{j=1}^{N} m_j (v_i^{\alpha} - v_j^{\alpha})\frac{\partial W_{ij}}{\partial x_i^{\alpha}}$$

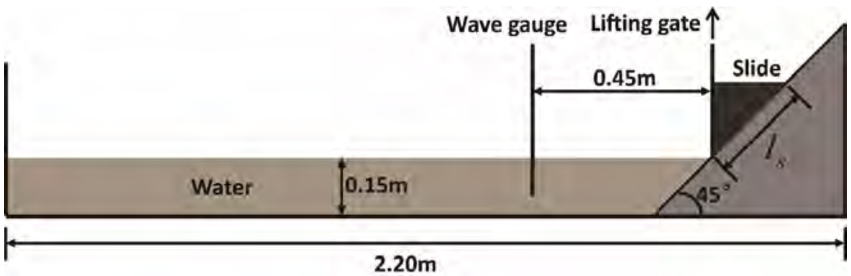
$$\frac{Dv_i^{\alpha}}{Dt} = \sum_{j=1}^{N} m_j\left(\frac{\sigma_i^{\alpha\beta} + \sigma_j^{\alpha\beta}}{\rho_i\rho_j} - \Pi_{ij}\delta^{\alpha\beta} + F_{ij}^{n}R_{ij}^{\alpha\beta}\right)\frac{\partial W_{ij}}{\partial x_i^{\beta}} + g^{\alpha}$$

Artificial stress tensor:

$$R^{\alpha\beta} = \begin{cases} -\varepsilon\dfrac{\sigma'^{\alpha\beta}}{\rho^2} & \sigma'^{\alpha\beta} > 0 \\ 0 & \sigma'^{\alpha\beta} \le 0 \end{cases}$$

3. Test cases and results

- **Numerical setup**



➢ Two treatments for landslide density

$$\rho_s = (1-n)\rho_g + n\rho_w = 1900kg\ m^{-3}$$
$$\rho_s = (1-n)\rho_g = 1500kg\ m^{-3}$$

➢ Values of Soil Parameters for Simulations

Cases	$\rho_g\left(kg.m^{-3}\right)$	$n(\%)$	$c(kPa)$	$\varphi(o)$	$\psi(o)$	$E(MPa)$	υ
Case1	2500	40	0	25.7	0	20	0.3
Case2	2500	40	0	20.1	0	20	0.3
Case3	2500	40	0	25.7	0	20	0.3

➢ Particles' amount for Simulations

Cases	*dp* (m)	Soil	Water	Bound	Total
Case1	0.002	1600	42561	9706	53867
Case2	0.002	2401	43089	9706	55196
Case3	0.002	2695	40227	9690	52612

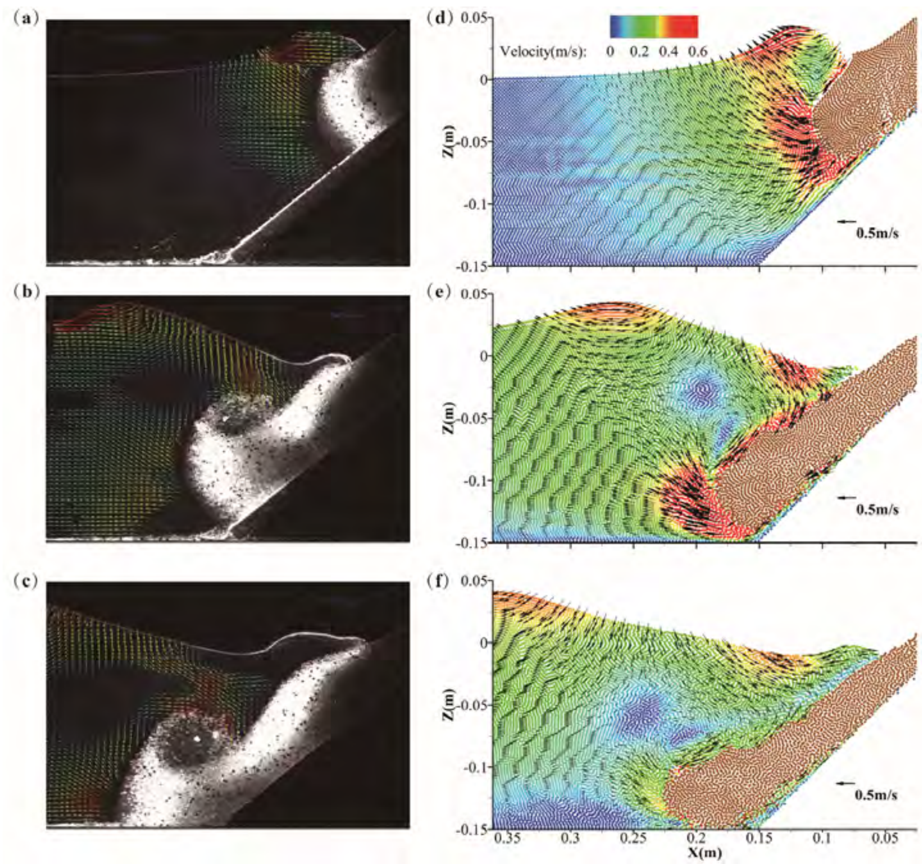
3. Test cases and results

- **Case 3**

  ➢ Left:
  snapshots of the experiment with m_s =3kg, reprinted from Viroulet et al (2013)

  ➢ Right: simulation results,
  It should be noticed that the figures at right side have larger zone than the left ones.

  ➢ Comparison
  flow field
  soil configuration



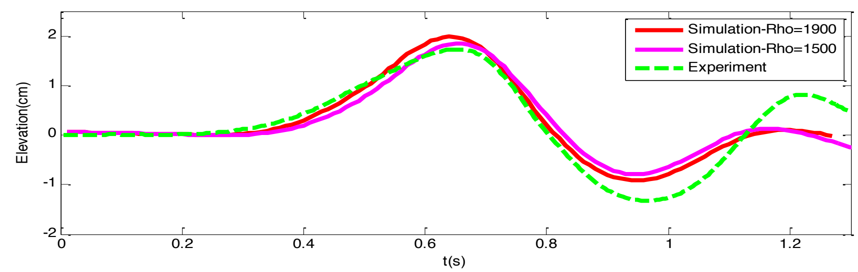
$t = 0.23s$

$t = 0.41s$

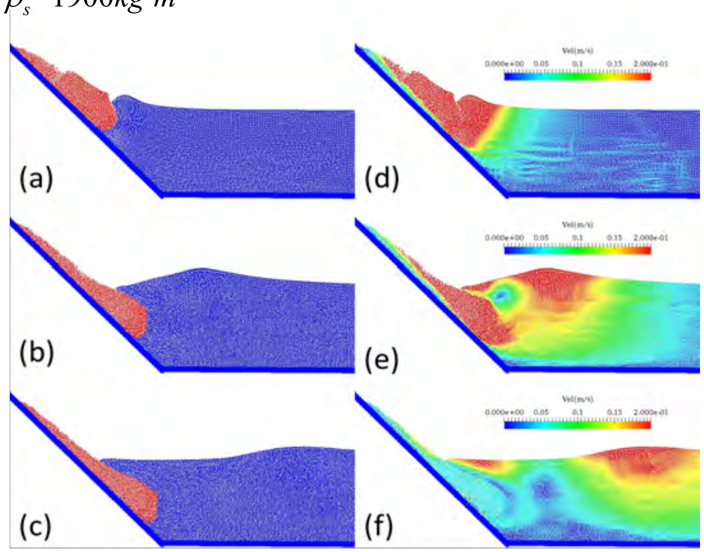
$t = 0.52s$

3. Test cases and results

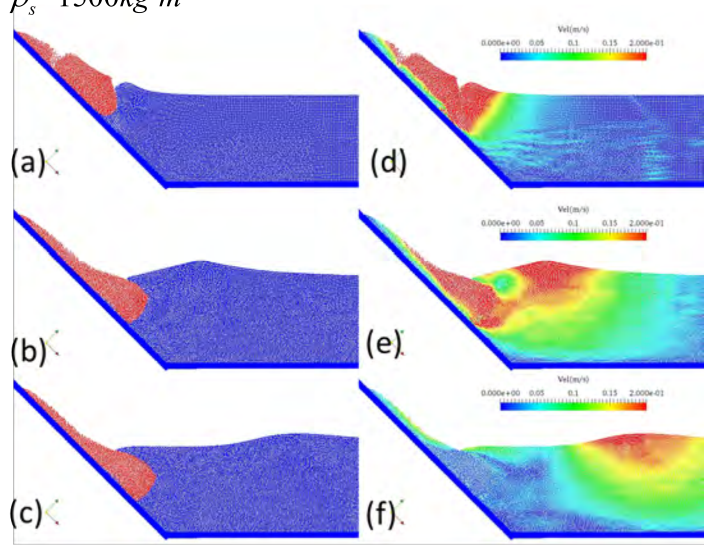
- **Case 1**



ρ_s=1900$kg\ m^{-3}$



ρ_s=1500$kg\ m^{-3}$



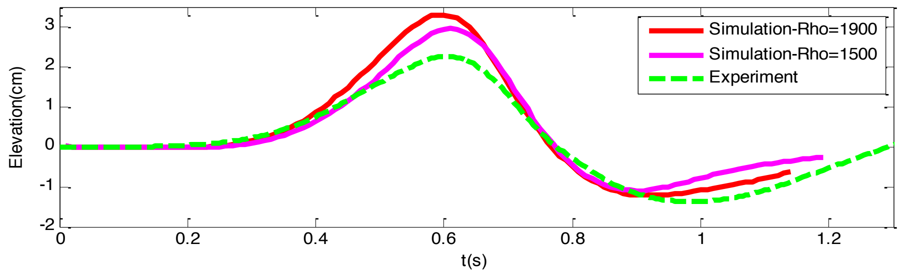
$t = 0.20s$

$t = 0.40s$

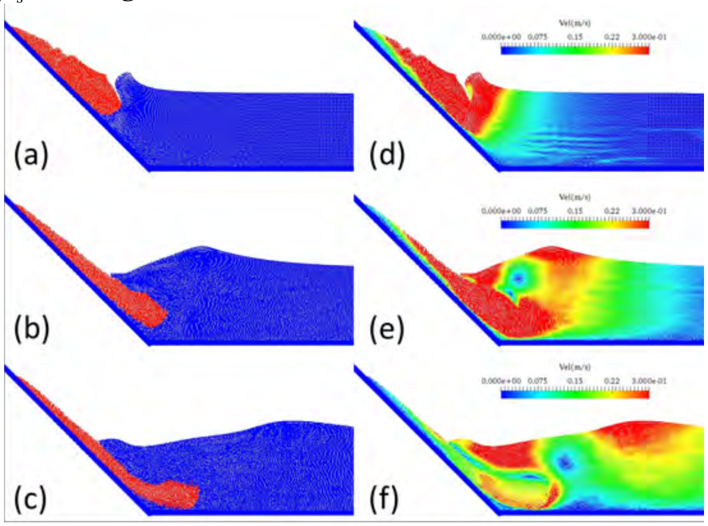
$t = 0.60s$

3. Test cases and results

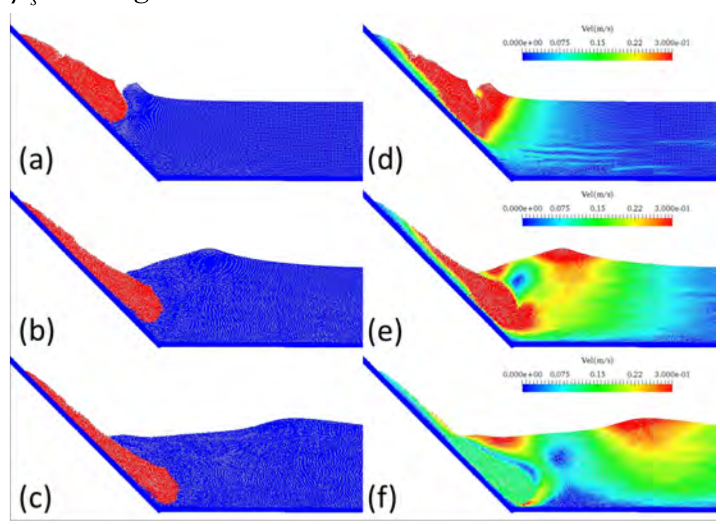
- **Case 2**



ρ_s=1900$kg\ m^{-3}$



ρ_s=1500$kg\ m^{-3}$



$t = 0.20s$

$t = 0.40s$

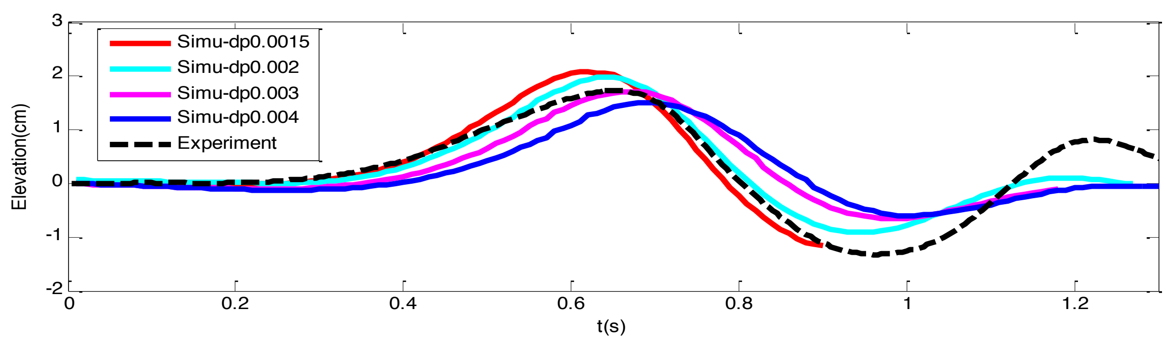
$t = 0.60s$

4. Discussion

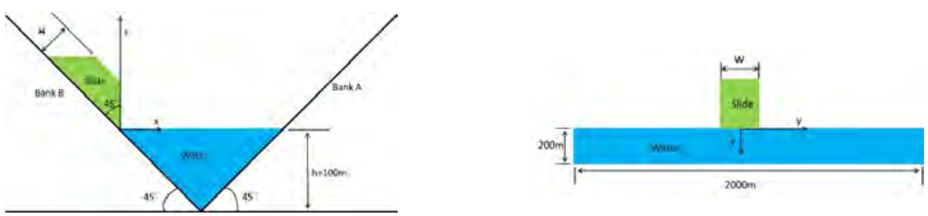
- **Convergence tests**

➢ Numbers of Particles used for simulations

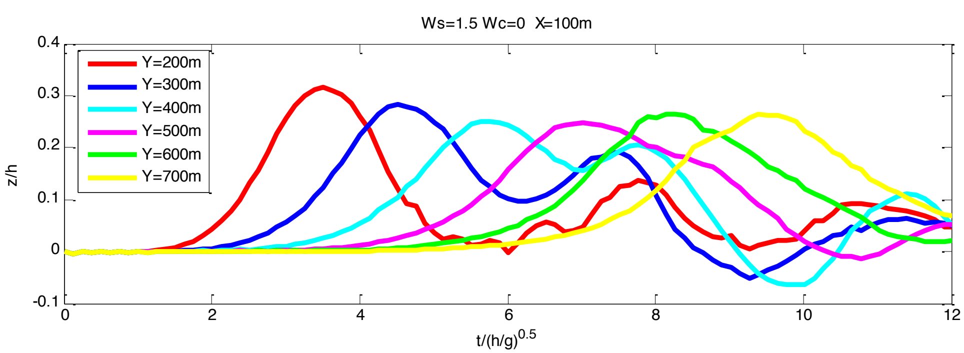
Cases	*dp* (m)	Soil	Water	Bound	Total	Amplitude(cm)
Case1	0.0015	2809	75578	13062	91449	2.0773
	0.002	1600	42561	9706	53867	1.9850
	0.003	729	18935	6488	26152	1.7142
	0.004	417	10724	4922	16063	1.5100



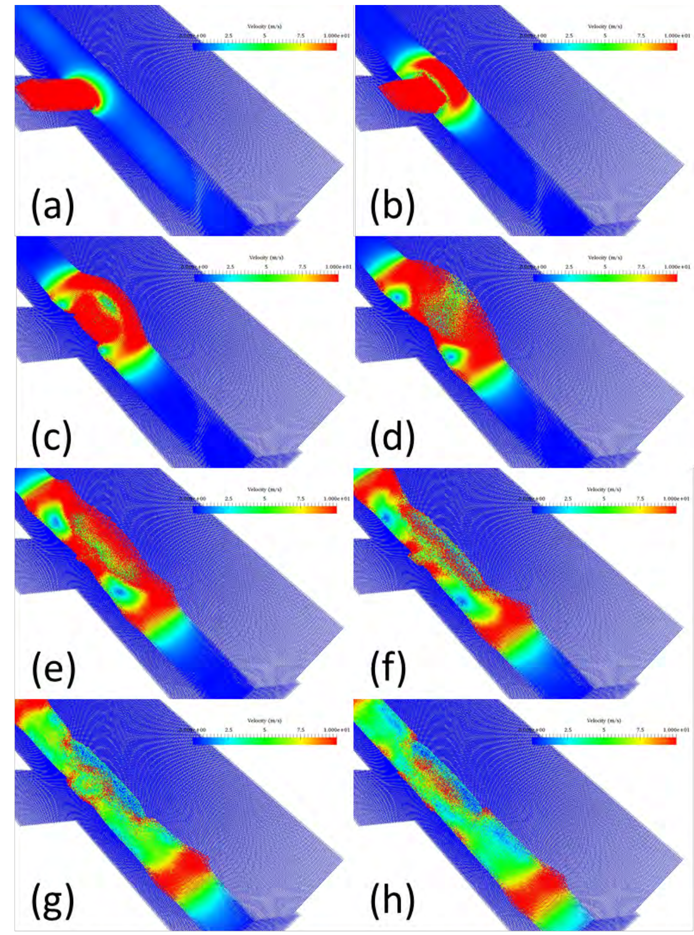
4. Discussion



Schematic diagram of fjord-like channel and the deformable landslide



Water surface displacement at different points along the channel



Wave generation process: (a) t=4s (b) t=8s (c) t=12s (d) t=16s (e) t=20s (f) t=24s (g) t=28s (h) t=32s.

4. Discussion

- **DEM method**
  - Case 2

DEM-SPH model by Canelas et al (2016)

Case 2:
D=1cm
L=13.5cm

Landslide ? → Continuum body