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A Vision for Mathematics

A common, coherent, and challenging curriculum can transform mathematics education in the United States.

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The No Child Left Behind (NCLB) Act has the potential to change U.S. education forever, but if we continue to hold some of the beliefs that underlie our current education policy and practice, then realizing the NCLB vision will be difficult, if not impossible. NCLB's vision is to provide rigorous and demanding subject-matter content for all students. As a crucial subject area, mathematics is vital to this effort. How can we change the curriculum of mathematics in the United States to make it rigorous and accessible to all students?

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The Importance of Curriculum

Because NCLB's vision emanates from the federal government, it challenges the long-standing U.S. tradition of local control of the curriculum. Individual districts or even schools can direct everything from curriculum to tracking policies—such as creating special programs for gifted students in middle school mathematics—and can even "adjust" the content standards for students who come to school disadvantaged by such factors as poverty or limited English language skills.

If we are to provide all U.S. students with a rigorous mathematics curriculum, we should examine effective curriculums that are already in place and consider what we have learned from the curriculum component of the Trends in International Mathematics and Science Study (TIMSS). The most striking feature of the school experiences of students in most other countries, especially those in countries whose test performance is very high, is that of a common, coherent, and challenging curriculum through 8th grade. Differences in mathematics achievement among countries are clearly related to these countries' different curriculums (Schmidt et al., 2001).

Although some argue that the teacher or pedagogy is more important than the curriculum in improving student achievement, the data do not determine conclusively which of these three factors has the greatest impact. The TIMSS international data relegate pedagogy to a lesser role than those of the curriculum and the teacher, but, again, the data are not definitive. My argument is simply that curriculum is crucial for improving mathematics achievement.

Curriculum is at the core of any education system because it defines what schooling should accomplish; it specifies in what content areas no child is to be left behind. Once the education system has delineated the mathematics curriculum, the teacher uses subject-matter background and pedagogical knowledge to accomplish the curriculum's goals. The teacher serves the intended curriculum and is, in fact, central to its successful implementation. In this sense, teacher and curriculum together facilitate student learning. The first priority, however, is to develop a common, coherent, and challenging curriculum.

A Curriculum in Common

If all students are to have equitable opportunities to learn rigorous mathematics, then each state must stipulate what such mathematics content should be for each grade level. The content standards must be the same for everyone—a standard curriculum for all students within a state.

In this case, curriculum does not refer to a particular textbook, learning activity, or pedagogical style, but rather to content expectations or standards. The totality of the learning experience need not be the same for all students. If memorizing the multiplication tables through 5s is the 3rd grade standard, for example, then all students should have an adequate opportunity to acquire this knowledge, although all might not have exactly the same learning experiences.

And if we really mean "all students," then why should the content expectations for 4th grade students in Illinois be any different from those for 4th graders in Alabama or Maryland? This question appears to challenge the notion of local school control, an almost sacred notion in U.S. public education. But by setting grade-by-grade content standards, a common curriculum challenges local control only in a limited way; states and districts still exercise control of textbook selection or preferred learning activities. Content standards currently vary across states and across districts (Schmidt, Wang, & McKnight, in press), which indicates that different students, whose only real differences might be where they live, often receive different learning opportunities and are held to different expectations.

Other countries offer some insights. The vast majority of the 40-plus countries participating in TIMSS had common standards for all students in 1st grade through at least 8th grade (Schmidt, McKnight, Cogan, Jakwerth, & Houang, 1999). The grade-level content expectations were usually the same, even in countries with different schools for different students of the same age. Most of the countries had common national standards for all K-8 students. To many people in the United States, common national standards are synonymous with federal standards. "National," however, does not have to mean "federally imposed." Looking at these same countries, TIMSS data showed that even a national—federal—curriculum did not necessarily translate into federal control of the schools. The data showed that the final decision regarding specific aspects of curriculum and its implementation varied greatly among countries and across different parts of the curriculum, even when a common set of national content standards guided education overall.

The TIMSS data made clear that even national or federal control of curricular goals does not imply a totally centralized system. Even in highly centralized systems, decisions about textbooks, examination content, and instructional practices are under local or even teacher control (Schmidt et al., 2001). Another analysis of the international data indicates that the most crucial element in the implementation of the curriculum is not the location of its institutional center (federal versus regional versus local) but rather the perceived authority of the institution that coordinates the curriculum, no matter where that institution is located (Schmidt, Prawat, & Houang, in press). The task in the United States is to identify or create such an institution for curricular standard setting. Given the U.S. political context, such an institution should be outside the official agencies of the federal government.

Tracking is another practice in the United States that changes the curriculum for different students. Practiced widely in the United States during the middle grades but especially in the 7th and 8th grades (Cogan, Schmidt, & Wiley, 2001), content tracking is clearly not consistent with the vision of NCLB. Tracking sorts students by various criteria into different courses so extensively that a given school can have more than five different variations of 8th grade mathematics. This practice is unique to the United States, at least during the middle grades.

One consequence of tracking in the United States was that in 1995, one-third of all U.S. 8th graders attended a middle school that did not even offer an algebra class. Most TIMSS countries have only one course in mathematics. If algebra in the 8th grade is standard for all students across most of the world, as is clear from international data, then one-third of U.S. students have not even had the opportunity to study what international standards have established as rigorous mathematics appropriate to the 8th grade.

A Coherent Curriculum

Mathematics is a formal body of knowledge defined by axioms and derived theorems. School mathematics should reflect that structure and the ways in which mathematical topics intertwine. A mathematics curriculum should identify a progression of topics that build on the structure of mathematics, with topics in one year depending on topics covered in a previous year (Schmidt, Wang, & McKnight, in press). Although there may not be a single correct sequence of topics, mathematics topics are not interchangeable pieces that we can place in an arbitrary sequence. When the sequence does not reflect the cumulative nature of mathematics, the resulting sequence of topics becomes nothing more than a meaningless list of items that students memorize but soon forget. Such arbitrariness makes learning mathematics even more difficult for disadvantaged and struggling students and, once again, makes realizing the vision of NCLB more difficult.

One version of a coherent set of mathematics standards for 1st through 8th grades appears in Figure 1 (p. 8), which represents the grade-level content expectations in a majority of the top-achieving countries in TIMSS (Schmidt, Houang, & Cogan, 2002; Schmidt, Wang, & McKnight, in press). The structure of the data makes its implied coherence self-evident. By contrast, Figure 2 shows the content standards for three U.S. states. The arbitrariness of these curriculums, which are representative of the great variances among the 21 states studied, contrasts sharply with the coherence of the curriculums depicted in Figure 1. The lack of coherence across states likely makes in-depth learning of demanding mathematics more difficult for all students, especially at-risk students. A coherent curriculum, therefore, is crucial for providing rigorous mathematics instruction for all students.

Figure 1. The Sequence of Mathematics Topics in Top-Achieving Countries

Topic	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8
Whole Number Meaning	*	*	*	*	*			
Whole Number Operations	*	*	*	*	*			
Measurement Units	*	*	*	*	*	*	*	
Common Fractions			*	*	*	*		
Equations and Formulas			*	*	*	*	*	*

Data Representation & Analysis			*	*	*	*		*
2-D Geometry: Basics			*	*	*	*	*	*
Polygons & Circles				*	*	*	*	*
Perimeter, Area, & Volume				*	*	*	*	*
Rounding & Significant Figures				*	*			
Estimating Computations				*	*	*		
Properties of Whole Number Operations				*	*			
Estimating Quantity & Size				*	*			
Relationship of Common & Decimal Fractions				*	*	*		
Properties of Common & Decimal Fractions					*	*		
Percentages					*	*		
Proportionality Concepts					*	*	*	*
Proportionality Problems					*	*	*	*
2-D Coordinate Geometry					*	*	*	*

Geometry: Transformations						*	*	*
Negative Numbers, Integers, & Their Properties						*	*	
Number Theory							*	*
Exponents, Roots, & Radicals							*	*
Exponents & Orders of Magnitude							*	*
Measurement Estimation & Errors							*	
Constructions w/ Straightedge & Compass							*	*
3-D Geometry							*	*
Congruence & Similarity								*
Rational Numbers & Their Properties								*
Patterns, Relations, & Functions							*	
Slope & Trigonometry								*
A majority of top-achieving countries in the Third International Mathematics and Science Study address mathematics topics in this order at each grade level.								

Figure 2. The Sequence of Mathematics Topics in Three Sample U.S. States

	State L								**State E**								
Topics	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**1**
Whole Number Meaning	*	*	*	*	*	*	*	*	*	*	*	*	*				*
Whole Number Operations	*	*	*	*	*	*	*	*	*	*	*	*	*	*			*
Measurement Units	*	*	*	*	*	*	*	*	*	*	*		*	*	*		*
Common Fractions	*	*	*	*	*	*	*	*		*	*	*	*	*	*		
Equations & Formulas	*	*	*	*	*	*	*	*			*	*	*	*	*	*	*
Data Representation & Analysis	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*		*
2-D Geometry: Basics	*	*	*	*	*	*	*	*			*	*	*	*	*	*	*
Polygons & Circles	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*		*
Perimeter, Area, & Volume	*	*	*	*	*	*	*	*			*	*	*	*	*		*
Rounding & Significant Figures	*	*	*	*	*	*	*	*	*		*	*	*				*
Estimating Computations	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*		*
Properties of Whole Number Operations	*	*	*	*						*	*	*				*	
Estimating Quantity & Size	*	*	*	*	*	*	*	*			*	*	*				

Relationship of Common & Decimal Fractions	*	*	*	*	*	*	*	*			*	*	*		*		*
Properties of Common & Decimal Fractions	*	*	*	*	*	*	*	*									
Percentages					*	*	*	*					*	*	*	*	*
Proportionality Concepts	*	*	*	*	*	*	*	*						*	*		*
Proportionality Problems	*	*	*	*	*	*	*	*			*			*	*	*	*
2-D Coordinate Geometry	*	*	*	*	*	*	*	*				*	*			*	*
Geometry: Transformations	*	*	*	*	*	*	*	*	*		*	*			*		*
Negative Numbers, Integers, & Their Properties	*	*	*	*	*	*	*	*				*	*		*		*
Number Theory	*	*	*	*	*	*	*	*				*	*	*			
Exponents, Roots, & Radicals	*	*	*	*	*	*	*	*					*	*	*	*	
Exponents & Orders of Magnitude	*	*	*	*	*	*	*	*							*		
Measurement Estimation & Errors	*	*	*	*	*	*	*	*	*	*	*				*		*
Constructions w/Straightedge & Compass	*	*	*	*	*	*	*	*									

3-D Geometry	*	*	*	*	*	*	*	*	*	*	*	*			*		*
Congruence & Similarity	*	*	*	*	*	*	*	*			*				*		
Rational Numbers & Their Properties	*	*	*	*	*	*	*	*							*		
Patterns, Relations, & Functions	*	*	*	*	*	*	*	*	*	*	*	*		*	*	*	*
Slope & Trigonometry	*	*	*	*	*	*	*	*							*	*	
Individual state standards address mathematics topics in this order at each grade level. The figure shows the great variance among states.																	

A Challenging Curriculum

One of the most important conclusions to emerge from the TIMSS results was that mathematics in the middle grades in most countries introduced topics in algebra and geometry; these were the grade-level expectations for *all* students. In contrast, the majority—80 percent—of U.S. 8th graders were mostly studying such arithmetic topics as fractions, decimals, percentages, and ratios, with very little coverage of algebra and virtually no coverage of geometry topics (Schmidt et al., 1999). The NCLB legislation requires that all students have opportunities to learn challenging and demanding mathematics. Given the international context revealed by TIMSS and the increasing level of mathematical sophistication demanded by the workplace, the United States must move closer to offering what most countries in the world consider demanding mathematics for their middle school students, including such topics as slope, functions, solving linear equations in two unknowns, congruence, similarity, and the properties of rational numbers.

The Role of Teachers

Only by providing a common, coherent, and challenging curriculum can we garner the resources necessary to make the NCLB vision a reality. Studies such as TIMSS show that delineating a clear curriculum does make a difference in terms of what students learn (Schmidt et al., 2001) and that national differences in student achievement correlate with differences in curricular coverage.

Having such a curriculum will not guarantee the realization of the NCLB vision, however. Such a curriculum must be effectively implemented, which brings us back to the crucial role of the teacher. I have learned from more than 10 years of international work that teachers' knowledge of subject matter combined with a challenging curriculum is what often distinguishes the level of student achievement in the United States from that of high-achieving countries. Informal data make it clear that U.S. middle school teachers' training, in general, does not come close to the level of formal mathematical training found among teachers in high-achieving countries. As a part of an international study currently under way, representatives from 10 high-achieving countries were interviewed about the preparation requirements for middle school teachers in their countries. The results were as startling as those that the TIMSS study found in curriculum differences. These middle school mathematics teachers had the equivalent of at least a formal degree in mathematics in addition to their pedagogical training.

I am not suggesting that pedagogical knowledge—especially pedagogy specific to mathematics—is unimportant. But the study points to a striking deficit in subject-matter preparation on the part of U.S. middle school teachers compared with the preparation of teachers in other countries. Both subject-matter and pedagogical knowledge are crucial and combine with the curriculum to provide the only real answer to the vision of leaving no child behind.

Curriculum is Paramount

My argument is simple. Even teachers with a strong mathematics background cannot teach well in a context defined by a fragmented and incoherent curriculum. Teachers feel the constraints of state and district standards that define their education world, the tests that hold them accountable, and the textbooks that they use.

Even the best of the best-trained teachers find it difficult to succeed with a fragmented curriculum. For all teachers, even those with the best preparation, the only real hope for success is a common, coherent, and challenging curriculum. Our teachers deserve it; our students need it; the laudable vision of NCLB demands it. Such a curriculum might be only a first step. It might not be sufficient alone to realize our hopes. But it is a necessary step for the good of all of our students. Can we afford to do less?

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