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LEARNING TO LEARN TO TEACH: AN "EXPERIMENT" MODEL FOR TEACHING AND TEACHER PREPARATION IN MATHEMATICS

ABSTRACT. This paper describes a model for generating and accumulating knowledge for both teaching and teacher education. The model is applied first to prepare prospective teachers to learn to teach mathematics when they enter the classroom. The concept of treating lessons as experiments is used to explicate the intentional, rigorous, and systematic process of learning to teach through studying one's own practice. The concept of planning teaching experiences so that *others* can learn from one's experience is used to put into practice the notion of contributing to a shared professional knowledge base for teaching mathematics. The same model is then applied to the work of improving teacher preparation programs in mathematics. Parallels are drawn between the concepts emphasized for prospective teachers and those that are employed by instructors who study and improve teacher preparation experiences. In this way, parallels also are seen in the processes used to generate an accumulating knowledge base for teaching and for teacher education.

KEY WORDS: knowledge for mathematics teaching, knowledge for mathematics teacher education, learning to teach, lesson study

An enduring problem in mathematics education is how to design preparation programs that influence the nature and quality of teachers' practice (Borko et al., 1992; Cooney, 1985, 1994; Ebby, 2000; Lortie, 1975). The absence of strong effects resulting from such programs is noted primarily when prospective mathematics teachers are asked to develop teaching practices different from those they have experienced. This can be explained, in part, by the observation that teaching is a cultural practice (Gallimore, 1996) and changing cultural practices is notoriously difficult. People learn to teach, in part, by growing up in a culture – by serving as passive apprentices for 12 years or more when they themselves were students. When they face the real challenges of the classroom, they often abandon new practices and revert to the teaching methods their teachers used.

The absence of strong effects of preparation programs also can be explained, in part, by the lack of a widely shared knowledge base for both teaching and teacher education (Grimmett & MacKinnon, 1992; Hiebert et al., 2002; Holmes Group, 1986; Huberman, 1985; Raths & McAninch, 1999; Yinger, 1999). Prospective teachers studying to enter the profes-

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sion cannot consult a common source of knowledge that allows them to begin where their predecessors left off. They often start anew, developing teaching methods that work for them. In a parallel way, teacher educators lack a shared knowledge base for building more effective teacher preparation programs. Teacher colleges and universities might learn from each other about program features and requirements, but little shared information is at the instructional level and even less is supported by research on effectiveness. Like schoolteachers, teacher educators mostly start anew, learning how to teach preparation courses more effectively.

If mathematics teaching showed signs of continuing improvement and if students were learning mathematics well, the concern about the effectiveness of teacher preparation programs would be less urgent. But the average classroom in the United States reveals the same methods of teaching mathematics today as in the past (Fey, 1979; Stigler & Hiebert, 1999; Welch, 1978). U.S. students continue to learn disappointingly little mathematics (Gonzales et al., 2000; Silver & Kenney, 2000) and are especially deficient in the competencies required to understand mathematics deeply and use it effectively (National Research Council [NRC], 2001).

Given these facts, is it possible to nurture, during a preparation program, the knowledge, competencies, and dispositions that teachers will need to become expert mathematics teachers when they enter the classroom? Probably not. The model we propose claims that it is both more realistic and more powerful to help prospective teachers learn how to learn to teach mathematics effectively when they begin teaching. In other words, preparation programs can be more effective by focusing on helping students acquire the tools they will need to learn to teach rather than the finished competencies of effective teaching.

The model for teacher preparation described in this article is built on two primary and over-arching learning goals. We believe that achieving these goals will provide prospective teachers with the tools they need to become increasingly effective mathematics teachers as they enter the classroom. The goals are:

- Become "mathematically proficient" (NRC, 2001).
- Develop the knowledge, competencies, and dispositions to learn to teach, with increasing effectiveness over time, in ways that help one's own students become mathematically proficient.

In this paper, we elaborate the two primary goals and then describe more fully how we interpret the concept of learning to teach – the concept that underlies the proposed model of teacher preparation. We then describe the kinds of environments that prospective teachers must create in order to sustain their own learning and that of the profession. Finally, we step back to describe the larger educational program in which preparation programs often are embedded and apply the same concepts of learning from experience to the task of building a knowledge base for teacher preparation and improving the effectiveness of preparation programs.

TWO PRIMARY LEARNING GOALS FOR PROSPECTIVE TEACHERS

Goals are expressions of values. The two goals described below provide complete statements of the values built into the proposed model. All decisions about teacher preparation programs that are aligned with the model, both in substance and in the processes used to develop them, are driven by the desire to help prospective teachers achieve the two goals. Likewise, the knowledge needed to improve the effectiveness of preparation programs is the knowledge of how to help prospective teachers achieve these goals. For the reader to understand the proposed model, it is essential to understand the nature of the goals and why they were selected.

Goal 1: Become Mathematically Proficient

The mathematics education community in the United States is in the midst of a debate about the future of mathematics education. The core of the debate is about what mathematical outcomes are of most value for school students (Hiebert, 1999; Kilpatrick, 1997). In other words, what learning goals should be set for students? In a deliberate attempt to address these "math wars" in the United States, the National Research Council issued a report that offers recommendations on appropriate mathematics learning goals for students in the 21st century (NRC, 2001). The recommendations are based on widely solicited expert advice and on a synthesis of research on mathematics teaching and learning.

The mathematics learning goal for school students proposed in the NRC report is to become "mathematically proficient". In brief, mathematical proficiency is the simultaneous and integrated acquisition of five kinds of mathematical competencies, or "*strands*:

- *conceptual understanding* – comprehension of mathematical concepts, operations, and relations
- *procedural fluency* – skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- *strategic competence* – ability to formulate, represent, and solve mathematical problems

- *adaptive reasoning* – capacity for logical thought, reflection, explanation, and justification
- *productive disposition* – habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy" (NRC, 2001, p. 116).

We endorse the goal of increasing mathematical proficiency for school students. This means, in turn, that we endorse the goal of increasing mathematical proficiency for prospective teachers.

The concept of mathematical proficiency carries with it the notion that success in mathematics is achieved by making progress along each of the five strands rather than by completely mastering any one individual strand. Furthermore, there is evidence that progress is made more easily along each strand when all five strands are interwoven and treated simultaneously than when one strand is singled out for prolonged attention (NRC, 2001). But the common teaching methods in the United States often have separated these strands and emphasized some at the expense of others (Fey, 1979; Stigler & Hiebert, 1999; Stodolsky, 1988). Indeed, this is one way of describing the impoverished nature of U.S. school mathematics teaching.

Because it is unrealistic to expect prospective teachers to learn to teach for mathematical proficiency without becoming proficient themselves, the proposed model focuses on the integrated development of the five strands of mathematical proficiency. The mathematical topics of school curricula, along with related mathematical ideas, must be studied in ways that encourage attention to all five of the strands. In addition, the study of school students' thinking and the ways in which it can reveal mathematical proficiency (or its absence), and how such proficiency develops, must be regular features of teacher education courses.

Goal 2: Prepare to Learn to Teach for Mathematical Proficiency

The second goal can be elaborated in two parts – preparing to learn to teach, and preparing to learn to teach for mathematical proficiency. Preparing to learn to teach is a relatively uncommon way of conceiving the goal of a teacher preparation program and requires some justification.

Learning to learn is not an easy task (Bereiter & Scardamalia, 1989) and brings its own set of challenges. Learning to learn to teach cannot be an easy goal to achieve. Why do we believe it is a more appropriate goal for prospective teachers than the more conventional goal of mastering some aspects of preferred teaching practices by graduation day? First, the complexity of teaching and the difficulty of mastering all aspects of effective teaching, especially as defined by the new and ambitious learning goal of mathematical proficiency, nearly ensure that prospective teachers cannot become experts, or even accomplished novices, during a relatively brief program. Even if the current knowledge base identified the complete set of skills and dispositions for effective teachers, it is unlikely that prospective teachers could acquire these competencies in a relatively brief preparation program. Without such a knowledge base, it becomes essential for beginning teachers to know how to learn to teach with increasing effectiveness over time, taking advantage of new knowledge generated by themselves and others.

A second reason for targeting the goal of learning to teach is that the richest environments for learning to teach effectively are school classrooms (Ball & Cohen, 1999; Clark, 2001; Jaworski, 1998; Schön, 1991). Teachers who are equipped with tools for learning from their experiences are in a strong position to learn more effective methods over the full course of their careers.

A third reason for focusing on tools for learning to teach is that teacher preparation programs are suited better for developing the knowledge, dispositions, and competencies prospective teachers need to take advantage of their experiences when they become teachers than for simulating the daily experience of teaching. Formal study of teaching mathematics, in structured courses and field experiences, allows novices to slow down the classroom and examine its apparent chaos. Course experiences, such as studying cases or interviewing students, which pose problematic teaching situations in meaningful but digestible chunks permit prospective teachers to consider the various elements of classrooms – subject matter, students' thinking, teacher-student interactions – and to develop tools for monitoring and examining these elements as they enter the classroom and begin experiencing life as a teacher (Masingila & Doerr, 2002; Moyer & Milewicz, 2002).

A final reason for selecting learning to teach as a goal for preparation programs is that many schools in the United States today do not have organizational structures that provide novice teachers with the support they need in order to take advantage of the rich potential of classrooms as learning sites (Darling-Hammond, 1997). Learning to teach, in planned and systematic ways, is not a process into which beginning teachers will be inducted as they enter the average school. It is a process that they will need to create. This makes it even more essential that beginning teachers are equipped with the tools and are encouraged to develop the dispositions that will enable them to learn from their experience.

Preparing to learn to teach *for mathematical proficiency* requires, in addition to one's own proficiency and in addition to knowing how to learn to teach, the appreciation of the importance of setting mathematical proficiency as the learning goal for one's own students. This appreciation comes from two sources: the recognition of the long-term benefits for young students of becoming mathematically proficient, and of the importance of measuring teaching effectiveness against a well-defined goal, in this case mathematical proficiency. To know whether changes are improvements in practice, or merely changes, one needs to measure their effects against a clear, consistent standard. The second learning goal for prospective teachers includes appreciating the importance of using mathematical proficiency as the standard for measuring one's own teaching effectiveness.

LEARNING TO TEACH BY TREATING LESSONS AS EXPERIMENTS

We believe that a teacher preparation program aligned with the proposed model – a program self-consciously committed to helping students achieve the two goals identified above – would be designed quite differently from conventional teacher preparation programs. Because the concept of preparing to learn to teach is a distinguishing feature of the model, we consider the concept further and describe how it might look in practice.

Preparing to learn to teach means knowing how to learn from classroom teaching experiences. It means planning these experiences in a way that affords learning and then reflecting on the outcomes in order to maximize the benefits that can be gained from the experiences (Artzt, 1999). As C. Roland Christensen phrased it, "Every good teaching plan has an experiment in it" (J. Simon, 1995). In our re-phrasing: *Prospective teachers should be inclined and able to treat the lessons they teach as experiments*. Treating lessons as experiments is, in our view, precisely what is needed to learn to teach. Phrased in this way, the goal has a distinct and clear focus.

The notion of treating lessons as experiments carries the recognition that experience, by itself, does not ensure better knowledge or improved performance (Sullivan, 2002). In order to take advantage of their experience, teachers need to design lessons with clear goals in mind, monitor their implementation, collect feedback, and interpret the feedback in order to revise and improve future practice.

An additional explanation is needed for our use of the word "experiment". We can clarify our intended meaning by comparing our use with both the increasingly popular phrase "design experiment" in education and the social sciences and the more orthodox use of "experiment" in the natural sciences. First, our use of experiment shares many features with those highlighted in descriptions of design experiments (Brown, 1992;

Cobb, Confrey, diSessa, Lehrer & Schauble, 2003; Design-Based Research Collective, 2003; Kelly & Lesh, 2000). When teachers treat lessons as experiments, they engage in many of the practices critical for conducting design-based research or design experiments. For example, the goals include both the actual improvement of classroom environments and the generation of shareable knowledge about such environments. The process plays out through continuing cycles of planning, enactment, analysis, and revision, and hypotheses about connections between teaching and learning are used to drive each cycle of the process. However, the model we propose does not treat the process of experimenting with lessons as a particular research method, applied with researchers' expertise and resources and designed to collect data for a defined period of time and to generate a specific product. Rather, we use the treating of lessons as experiments as a way of making some aspects of teachers' routine, natural activity more systematic and intensive. Teachers routinely plan lessons and then wonder about their effectiveness. Treating lessons as experiments provides a more systematic way to engage in these activities by focusing attention on, and making more explicit, the process of forming and testing hypotheses, a process that is contained in most definitions of "experiment".

We also note some similarities and differences with the way in which we use the word experiment and its more orthodox connotations. The primary difference is that, by experiment, we do not mean the traditional form that involves a controlled study with random assignment of participants. The methods we have in mind are not randomized assignment of participants to comparison groups but rather replications and observations of individual classroom experiments over multiple trials (Hiebert et al., 2002). But our use of the word does share the traditional connotation of intentional learning from carefully planned experiences. Experiment is used to emphasize the systematic and rigorous way in which teachers can study and reason about their practice in order to improve their teaching. Experiment also is used to emphasize the open and public process needed to grow a shared knowledge base for teaching.

The term experiment is useful, in addition, because it helps to identify the knowledge, dispositions, and competencies that prospective teachers must develop in order to design, implement, and learn from instructional experiences. In fact, one way to identify the requirements is to ask what is needed to conduct more traditional forms of experiments.[1]

Clarifying the Research Question

When teaching lessons, clarifying the research question means articulating two related but distinct statements about the lesson: the learning goals for the lesson, and the hypotheses that link planned instructional activities with expected learning outcomes. Both are needed in order to make decisions about the lesson design. Learning goals set parameters on the pool of potential learning activities and establish the criteria for judging the lesson's effectiveness. Hypotheses about how the learning activities will support students' achievement of the goals direct the selection and sequencing of activities. Explicit statements of learning goals and instructional hypotheses allow the lesson to be a learning opportunity for both the students and the teacher.

In what form should learning goals be expressed? This is a nontrivial question because, just as in other scientific experiments, the research question (learning goal) shapes everything that follows. First, the learning goal for a lesson should fit within the over-arching goal of developing mathematical proficiency. This means that making progress along the multiple strands for mathematical proficiency should be part of the goal. Second, the goal should be precise enough to guide lesson design decisions. This means, among other things, that the goal should be measurable. To learn from teaching the lesson, a teacher needs to know whether the lesson was effective in helping students reach the goal. But goals do not need to take the form of performance objectives, popular in the past (Gagné, 1985). Students' progress toward the goal can be measured in ways other than written performance on narrowly defined tasks. We believe that it is more useful to define learning goals for lessons in terms of students' thinking (Wittrock, 1986; M. Simon et al., 1999): How does the teacher expect students to be thinking about a concept or procedure before the lesson begins and how is their thinking expected to change during the lesson? Expressing learning goals in terms of students' thinking has the advantage of providing a rich source of information that the teacher can use to assess progress along multiple strands of mathematical proficiency.

To set appropriate learning goals for lessons, prospective teachers need to be making progress toward the first of the twin goals – becoming mathematically proficient. Part of mathematical proficiency is the construction of a mental map of the curriculum, with key concepts, skills, reasoning forms, and dispositions as landmarks, along with possible routes for traversing the territory. Knowing how to select and express appropriate goals for individual lessons means knowing how the goal for a single lesson fits within the larger sequence of learning goals.

Formulating hypotheses for a lesson that predicts changes in students' thinking due to instructional activities moves the lesson from a planned learning experience solely for the school students to a planned learning experience for the teacher as well. It transforms the lesson into an experiment that yields an empirical test of a teacher's local theory of how students learn and how instruction facilitates learning. It allows the teacher to generate knowledge of teaching that can be recorded, preserved, and applied in order to improve teaching in the future. It is, in a very real sense, the heart of the process of treating lessons as experiments and improving teaching in a gradual but steady and continuing way.

Designing the Experiment

As the lesson is designed, decisions are guided by the learning goals and the hypotheses about which instructional activities will help students achieve them. The hypotheses might describe why particular activities will help students change their thinking in particular ways, or how students will be thinking at a particular point in the lesson and why a particular instructional task will trigger a desired change in thinking. In this way, the planned lesson becomes a series of researchable questions about students' thinking and instructional moves. This kind of cause-effect analysis is at the heart of lesson design. The more explicit teachers can be about why they selected particular activities, and what they expect will happen at specific points of the lesson and why, the more they can learn from the feedback they receive from the students.

Treating lessons as experiments means that lesson designs are more than sequences of activities to keep students occupied during a 45-minute period. Lessons-as-experiments require, on one hand, constructing local theories regarding the relationships between teaching and learning and, on the other hand, the tying of the theories to *this* learning goal in *this* context. The knowledge is concrete but the local theories search for patterns and find connections between this knowledge and more general principles of teaching and learning.

When lessons are treated as experiments, the usual emphasis on making appropriate spontaneous decisions while implementing a lesson shifts to making appropriate predictions and decisions while planning a lesson. Greater emphasis is placed on clarifying the learning goal(s), on specifying the activities used to help students achieve them, on providing an accompanying rationale for each activity in the form of teaching/learning hypotheses, and on justifying every facet of the lesson before it is implemented in real time. To plan with this level of explicitness and detail, the teacher must consider the content, the students, the information collected from prior implementations, and so on.

Designing the mathematics lesson as an experiment requires considerable mathematical proficiency. Guided by the learning goal for the lesson, prospective teachers must be able to create or select mathematics problems that afford their students the opportunity to engage the various strands of proficiency.[2] They must predict how students are likely to solve the problems in order to build on students' thinking and plan discussions about solution methods that help students improve their thinking. They must think about how students' contributions to the lesson can be maximized and expanded. They must anticipate what ideas students can construct while working independently and what information students will need to move forward in their thinking. They must also be able to evaluate students' mathematical proficiency and the validity of their solution methods.

Gathering Data

The success of conventional scientific experiments can be judged by the quality of the data collected. Do the data help to answer the research question, do they help the researcher understand the phenomena being studied, and do they help the researcher formulate a follow-up experiment that might be even more useful? In a parallel way, does the information the teacher collects during the lesson indicate that students are moving toward the learning goal, does it help the teacher understand why and how the lesson worked well or not, and does it help the teacher plan an even more effective lesson?

Explicitly stated learning goals and hypotheses about how students' thinking will change during the lesson suggest what kinds of information the teacher needs to collect and when to collect it. With this approach, the evaluation of student learning and thinking is not something that is tacked onto the end of the lesson; instead, it becomes an essential and ongoing part of the lesson. Student data can be collected systematically during the lesson to evaluate students' learning, to help the teacher think about the lesson from the perspective of the learners who took part in it, to help the teacher interpret the results of the experiment, and to inform the lesson revision process.

The kind of information the teacher collects during the lesson is crucial to the success of the lesson. As noted earlier, students' thinking provides a rich source of information. But there is more to this than is at first apparent. The lesson must be designed so that students' thinking is revealed across multiple strands of mathematical proficiency. A natural way in which this can occur is by centering the lesson on solving significant mathematical problems and then listening to students discuss methods that can be used to solve the problems (Chazan, 2000; Lampert, 2001). As noted earlier, this approach fits well the goal of helping students acquire mathematical proficiency. Now one can see that it also provides an environment in which teachers can assess changes in students' thinking.[3]

Gathering useful data depends both on a lesson designed to reveal students' thinking and on a teacher competent and disposed to solicit and hear key ideas. The second of these requirements makes additional demands on the mathematical proficiency of prospective teachers. It provides another reason for the widely held belief that teachers themselves must be mathematically proficient. To hear emerging expressions of proficiency and to know how to support and improve them, teachers must be familiar with the domain.

Interpreting Data and Drawing Conclusions

Experiments are considered to be useful to the extent that something is learned about the research questions. Of course, there might be unintended learning as well. But the planned learning occurs as the researcher reflects on the data and thinks about how the data address the research questions. So, too, it is with teaching. Teachers learn whether and how well the lesson supported the learning goals for students as they reflect on the information they collected.

The cause-effect analysis used to construct the lesson comes under special scrutiny. Are the hypotheses – that changes in students' thinking will be prompted by particular lesson activities – supported by the evidence? Did the lesson activities have their intended consequences? Why or why not? Did the lesson facilitate students' achievement of the learning goal(s)? To the extent that the evidence collected can address these questions, the particular lesson can be revised. But, more important, the answers to these questions also produce more refined hypotheses about teaching and learning that can be tested further in future lessons. The experimentation is used as the basis for making more informed decisions later. This refinement of hypotheses and of local theories creates the kind of knowledge base for teaching that sustains continuing improvements (Stigler & Hiebert, 1999).

CREATING ENVIRONMENTS FOR GENERATING KNOWLEDGE FOR TEACHING

Acquiring the Identity and Dispositions of a Professional Teacher

Up to this point, we have treated prospective teachers as individuals, working to prepare themselves to become effective teachers. The previous discussion of treating lessons as experiments allows for an individual teacher to engage in this process alone, gradually improving his/her practice. Preparing to learn to teach, the second goal for prospective teachers, means more than this. It means becoming part of a profession (see also Darling-Hammond & Sykes, 1999; Shulman, 2000).

Becoming a professional teacher, in our view, means drawing from, and contributing to, a shared knowledge base for teaching. It means shifting the focus from improving as a teach*er* to improving teach*ing*. This requires moving outside the individual classroom, surmounting the insularity of the usual school environment, and working with colleagues with the intent of improving the professional standard for daily practice. This also requires redirecting attention from the teacher to the methods of teaching. It is not the personality or style of the teacher that is being examined but rather the elements of classroom practice.

Shifting from a vision of effective teachers to effective teaching requires a major shift in mindset. It requires a change in the culture identified in the opening paragraphs of this article. When teachers work together to experiment with lessons, with the intent of sharing what they learn with their professional colleagues, they are engaged in something more than becoming a better teacher; they are contributing directly to the knowledge base upon which a true profession is built. They are doing what members of many other professions do, but what teachers seldom have had the chance to do.

Shifting from a vision of effective teachers to effective teaching also requires a new set of obligations. Rather than considering only what one is learning from one's own experience, teachers must ensure that others can learn from their experience, and that they are disposed to learn from others' experiences. Planning teaching so that others can learn is different from planning teaching so that you can learn.

Considering what others can learn from your experience requires collaboration with other teachers who share the same learning goals for students. Such collaborations are characterized by a number of features that increase the chances of generating knowledge of teaching that is useful for the profession (Darling-Hammond & Sykes, 1999; Hiebert et al., 2002; Loucks-Horsley et al., 1998). Working with colleagues ensures that the learning goals, lesson designs, and data interpretation become explicit and public so they are accessible to others. Collaboration allows teachers to assist each other in collecting the kinds of data that can inform efforts to improve teaching and learning. Making these elements public also means that they can be examined, critiqued, and replicated in other contexts. This, in turn, yields further information about effective lessons, and about the hypotheses that shaped the lessons.

Mechanisms already have been developed through which this kind of collaborative teacher experimentation can occur. One form of such a mechanism – often called lesson study – has a 50-year history in some Asian countries and currently is being adapted and developed in local sites around the United States (Fernandez et al., 2001; Lewis, 2002; Lewis & Tsuchida, 1998; NRC, 2002). Lesson study is an especially promising mechanism for our model because it fits well the learning goals proposed for prospective teachers and, as will be seen shortly, it fits well the knowledge generation and continuing improvement processes proposed for teacher preparation.

Acquiring the Dispositions and Skills Needed to Create Learning Environments in Schools

Learning to teach effectively is a long-term enterprise. If prospective teachers are going to use the knowledge, dispositions, and competencies they develop for learning to teach, they need to work in schools that allow them to engage in this work. As noted earlier, many schools in the United States today do not offer such environments (Darling-Hammond, 1997). Often, beginning teachers are expected to know how to teach, even to be quite skillful. They are expected to fit into the on-going culture, working independently and projecting confidence in their performance as a teacher from the very beginning. Not surprisingly, in these cultures there are few provisions for learning to teach. For example, there is little time in the workweek for teachers to collaborate on designing and improving lessons.

Consequently, to achieve the goal of preparing to learn to teach for mathematics proficiency, prospective teachers need to acquire skills to create environments for themselves that support their learning as teachers. They must prepare to be agents for change. In our view, this does not mean that they enter the school armed with the answers for teaching effectively, but rather that they recognize the value of working with colleagues to improve their practice and that they possess the skills needed to create such environments. Of course, they cannot be expected to transform the culture of every school they enter, but they need to know how to connect with other teachers and form collaborative groups aimed toward improving teaching (Clark, 2001; Britt et al., 2001). They also need to learn to value this kind of work enough to invest the time and energy in order to create the environments that afford it. This can be achieved, in part, by providing supported practice in teacher-directed study groups during the teacher preparation program.

CREATING ENVIRONMENTS FOR GENERATING KNOWLEDGE FOR TEACHER EDUCATION

Goals for Teacher Educators

In the model we propose, the principles and processes that are proposed for the generation of the knowledge needed to improve teaching are the same as those that are proposed for the generation of the knowledge needed to improve teacher preparation. Assuming that mathematics teacher educators are proficient mathematically, we identify one major learning goal for teacher educators that parallels the learning-to-teach goal for prospective teachers: to learn to teach prospective teachers in ways that support the achievement of their learning goals and to do this in ways that generate a shared knowledge base for teacher education. In our model, the same process of experimenting with lessons is used as the means to achieve this goal. Treating lessons (for prospective teachers) as experiments becomes the routine, on-going activity for course instructors. The requirements to do this well are the same ones elaborated earlier when describing the components of treating lessons as (scientific) experiments. Prospective teachers now are the students and university faculty and doctoral students now are the teachers.

A Sample Learning Environment for Improving Teacher Preparation Programs

The best way to expand on the proposed model in the context of teacher preparation programs is to describe an example of how the process of treating lessons as experiments can be put into operation at this level. A key feature of this example is that the process is designed intentionally so that it will be sustainable over generations of teacher educators with the products always viewed as unfinished.

Imagine a teacher preparation program at a typical university in which multiple sections of courses are offered in mathematics and/or methods of teaching mathematics for prospective teachers. Now imagine the group of instructors for each course (e.g., doctoral students and faculty) meeting, at least weekly, to jointly plan the course and study its effectiveness. In lesson study fashion, the group sets clear goals for the course (specific sub-goals of the two primary goals identified earlier), identifies particular lessons that are key sites for helping students achieve these goals, plans these lessons together, implements them with careful monitoring of students' thinking, and revises the lessons for use the following term. Each term, the group of instructors for a particular course inherits a set of lesson plans (detailed in a special way that emphasizes the cause-effect analysis of the lesson) that provides the current knowledge base for effective instruction in that course. Each semester, the group of instructors takes up the challenge of increasing the knowledge base by improving the effectiveness of a targeted (perhaps different) small subset of lessons.

The lesson plans for each session of the course are the repository for the collective knowledge for teaching the course effectively. In addition to detailing the sequence of activities for the lesson and the role for the teacher in presenting tasks and leading discussions, the plans predict the responses of the students (based, increasingly, on past experience), suggest how to use these responses to further the goal of the lesson, provide rationales for the instructional decisions specified in the plans and hypothesize how and why particular instructional activities will facilitate particular learning. In fact, the plans are local theories of teaching and learning with the planned lesson serving as an example. The theories offer targeted, micro-hypotheses about the way in which teaching promotes learning. The implemented lesson serves as a test of the hypotheses proposed in the lesson plan, and the feedback received from the students is used, not only to revise the plan, but to revise the hypotheses and theories as well.

As noted earlier, local theories, with examples, are a useful form in which to package the knowledge generated about teaching a particular course effectively. Knowledge expressed in this form retains its connection to the context so it remains immediately useful for future instructors of the course, but it also rises above the details that vary unpredictably from class section to class section and from term to term and thereby moves toward a more principled knowledge base for teaching and teacher education.

Because teaching and learning are too complex and variable to presume that one could learn all there is to know from a single implementation of a course, the proposed model allows for continued authorship of the courses over time. A final version of a course is not expected. Rather, it is expected that instructors will learn to reason about teaching in an increasingly useful way and to accumulate knowledge for effective teacher preparation. Each term, an implemented course can be thought of as the accumulated wisdom of the previous instructors/authors. The process produces a single text, but the reading of the text and the accumulated knowledge always leads to remolding, reworking, rewriting, reevaluating, and reinterpreting over time. The text is both a repository of the wisdom and knowledge of the time and the locus of change as knowledge increases.

A Learning Environment for Program Improvement Provides an Image for Prospective Teachers

Central to the proposed model is the goal of treating lessons as experiments. As described earlier, this includes the ability to set clear learning goals, design lessons that support students' achievement of the goals (with a rationale for why the lesson might do so), collect data to evaluate the lesson's effectiveness, and interpret the data to revise the lesson accordingly. These are ambitious goals. One might wonder how prospective teachers can achieve them in a relatively short preparation program.

Two course-design strategies can help prospective teachers learn to treat lessons as experiments. The first is for instructors to include some of the component skills needed to treat lessons as experiments in each course. For example, treating lessons as experiments depends on listening carefully to students' thinking and assessing its apparent mathematics potential (validity of reasoning, connection to later ideas, etc.). Listening to (and really understanding) students can be set, early in the program, as a goal for courses. Techniques for supporting these goals include, for example, using videos of interviews with school students to generate the mathematical ideas that then are explored more deeply. Skills such as creating tasks that elicit students' thinking, inferring changes in students' thinking from changes in their solution methods, and expressing lesson goals in terms of changes in students' thinking, can be addressed in later courses as prospective teachers proceed through the program.

A second strategy that can help prospective teachers treat lessons as experiments is based on the fact that the knowledge, dispositions, and competencies that enable prospective teachers to treat lessons as experiments parallel, quite closely, the knowledge, dispositions, and competencies that instructors must develop collaboratively as the courses themselves are improved. The process of course improvement in which the instructors are engaged can be made transparent for prospective teachers so that they can see how the courses they are taking are being planned, evaluated, and revised. This provides an image of how the process can work to generate knowledge for, and improve, teaching.

One way in which the process of course improvement can be revealed to the prospective teachers in the program is through a gradually intensified experience, first as observers and informants, then as apprentices, and finally as full participants. In other words, prospective teachers first could be observers and informants in studying the improvement of courses they are taking. As observers, they might view a videotape of their instructors planning the lesson they just completed. As informants, they might view a videotape of a lesson in which they just participated as a student and provide feedback about critical learning moments during the lesson and about their own thinking during these points. As teachers, they might design lessons for school students which they treat as planned learning experiences – for the students, for themselves, and for others in their class. By playing different roles within the system of teaching improvement, the opportunities increase for prospective teachers to internalize the notions of learning to teach and, over time, improve their own teaching and enable others to learn from their experiences.

WHAT ABOUT CURRENT THEORIES OF LEARNING AND TEACHING?

Noticeably absent from the entire previous discussion of learning to teach is an endorsement, or at least an analysis, of grand theories of learning (e.g., behaviorism, constructivism, social-constructivism) and their corollary theories of teaching. Our model for teacher preparation is silent about these theories because we believe no a priori endorsement of particular theories is necessary. What is necessary, in our view, is that learning goals for students are precisely and explicitly articulated, and that hypotheses are formulated and tested for how the instructional activities will help students achieve the learning goals.

The role for current theories of learning and teaching is to provide resources that can help predict what kinds of instructional activities will best support students' efforts to achieve the learning goals. In this sense, they provide shortcuts for what otherwise would be a rather lengthy and chaotic process of trial-and-error. They suggest instructional approaches that can be translated into lesson designs and then tested for effectiveness. But no particular learning or teaching theories are privileged at the outset. Only the two learning goals of mathematical proficiency and preparing to learn to teach are privileged. How these can best be accomplished is the continuing task for those engaged in building the knowledge base for mathematics teaching and teacher education.

LESSON AS THE UNIT OF ANALYSIS AND IMPROVEMENT

A danger of building a model for teaching on the planning and analysis of lessons is the possible (mis)perception that individual lessons contain all of the information needed to construct a knowledge base for teaching. School learning occurs over sequences of lessons, and an adequate knowledge base will include information on students' learning trajectories and how these can be supported over time (M. Simon, 1995). We expect that extensions of the model proposed here, which devote explicit attention to students' learning over time, can and should be made. But this early version of the model focuses on individual lessons in order to anchor it to a nearly universal unit of teaching (the daily lesson) and because there are some good reasons to focus on a lesson, at least initially, in order to study and improve teaching. The individual lesson is a big enough unit of teaching to contain all of the complex classroom interactions that influence the nature of learning opportunities for students. At the same time, the individual lesson is the smallest natural unit for teachers that retains such interactions. The benefit of defining small units is that they allow the detailed analyses of teaching/learning relationships that make up the core of a knowledge base for teaching.

CONCLUSION

Fractals are curious geometric objects in which each piece of the object is identical to the larger object. A distant view of the whole object or a close-up view of a particular piece reveals the same basic design. The model we propose has some fractal characteristics. From a distance, the model as a whole looks like a learning system with an emphasis on continual study and incremental improvement. Zooming in reveals teacher educators engaged in learning how to study their teaching of prospective teachers. They are valuing incremental improvement and contributing to the professional knowledge base for teacher education. Zooming in further reveals the prospective teachers themselves engaged in learning how to study their own teaching, to value incremental improvement, and to contribute to the professional knowledge base for teaching.

We believe the model is promising, in part, because of the similarity of intellectual activity at all levels and sites. Research, teaching, and learning are tightly intertwined and are actively engaged, albeit in somewhat different ways, at all levels. As with fractals, it is possible to see similar structures and mechanisms at work in each level, and in the system as a whole.

We also believe the model is promising because it addresses the two problems identified at the outset – the culture of teaching which passes along, in a relatively unexamined way, the teaching methods of the past, and the absence of a knowledge base for teaching and for teacher preparation. The model outlines a system designed to achieve particular learning goals, for school students and for prospective teachers, using a process of continuing improvement through learning from planned instructional experiences. Such a process involves changing the culture of teaching in ways that afford building a professional knowledge base.

Is the model realistic? Can preparation programs be designed to help prospective teachers accomplish the twin goals of mathematics proficiency and learning to learn to teach for mathematics proficiency? Not in the near future. Among other obstacles, a sufficient knowledge base for mathematics teaching or for preparing mathematics teachers does not yet exist. The model we propose,however, is intended to guide *long-term* growth of knowledge in a gradual and incremental yet steady and lasting way. We believe that this goal can be achieved but achievement depends on building local cultures that value this kind of gradual and continuing progress.

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NOTES

[1] Thanks to Stephen Hwang for suggesting this parallel.

[2] Theoretical and empirical work suggest that instructional methods that ask students to solve challenging mathematical problems help students integrate, rather than separate, the five strands of mathematical proficiency (Hiebert et al., 1996; NRC, 2001; Schoenfeld, 1985; Silver, 1985). If the problems are appropriate, students will call on most or all of these strands while constructing and examining solution methods, thereby integrating them and becoming increasingly proficient.

[3] A single lesson will not provide a precise measure of every student's progress. This means that, in order to chart students' progress toward mathematical proficiency, many lessons will need to be planned and implemented in ways that make students' thinking transparent. This kind of a lesson cannot be a one-time event. It also means that an individual student's progress will be measured over time; any single snapshot will be incomplete. In this sense, students' progress is better conceived as a movie than a snapshot.

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