

Physical Chemistry

Lecture 1
Distributions and averages

Characteristics of macroscopic materials

- ◆ Large numbers of particles, N
- ◆ Distribution of properties: $F(v) = n(v)/N$
 - Example: speed
 - Describe with a distribution function, $F(v)$
 - Normalized: $\int_0^{\infty} F(v)dv = 1$
 - Average of a function of speed

$$\langle f(v) \rangle = \int_0^{\infty} F(v)f(v)dv$$

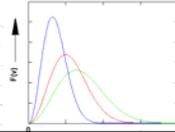
Boltzmann's distribution

- ◆ Speed distribution function at dynamic equilibrium
- ◆ Consider only kinetic energy

$$F(v) = 4\pi \left(\frac{m}{2\pi kT} \right)^{3/2} v^2 \exp\left(-\frac{mv^2}{2kT}\right)$$

- ◆ This is normalized

$$\int_0^{\infty} F(v)dv = 1$$



Calculating average molecular properties

- ◆ Use the distribution function

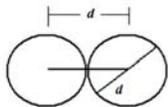
$$\langle f(v) \rangle = \int_0^{\infty} f(v)F(v)dv$$

- ◆ Example: average speed

$$\langle v \rangle = \int_0^{\infty} vF(v)dv = \sqrt{\frac{8kT}{\pi m}}$$

Molecular collisions

- ◆ Collisions dominate effects that depend on close proximity of molecules
- ◆ "Hard-sphere" model

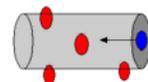


Collisions of a molecule

- ◆ Determined by the number of molecules in the space ($N = n^* V$)

- ◆ Collision frequency

$$\begin{aligned} \langle z_{AA} \rangle &= n^* \pi d^2 \langle v_{rel} \rangle \\ &= \sqrt{2} n^* \pi d^2 \langle v \rangle \end{aligned}$$



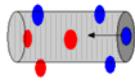
- ◆ Mean-free path

$$\lambda = \frac{\langle v \rangle}{\langle z_{AA} \rangle} = \frac{1}{\sqrt{2} \pi d^2 n^*} = \frac{kT}{\sqrt{2} P \pi d^2}$$

Collisions of unlike molecules

- ◆ Three different kinds of collision
- ◆ Collision frequencies per molecule

$$\begin{aligned} \langle z_{AA} \rangle &= n_A^* \pi d^2 \langle v_{rel} \rangle \\ &= \sqrt{2} n_A^* \pi d^2 \langle v \rangle \end{aligned}$$



$$\langle z_{A:B} \rangle = \pi d_{AB}^2 \langle v_{AB} \rangle n_B^*$$

$$\langle z_{B:A} \rangle = \pi d_{AB}^2 \langle v_{AB} \rangle n_A^* \quad d_{AB} = \frac{d_A + d_B}{2}$$

Total collisions in a mixture

- ◆ Often must know the total number of collisions per unit time

$$Z_{AA} = \langle z_{AA} \rangle \frac{n_A^*}{2} = \frac{\pi d_A^2 \langle v_A \rangle}{\sqrt{2}} n_A^{*2}$$

$$Z_{AB} = \langle z_{A:B} \rangle n_A^* = \pi d_{AB}^2 \langle v_{AB} \rangle n_A^* n_B^*$$

$$Z_{AB} = \langle z_{B:A} \rangle n_B^* = \pi d_{AB}^2 \langle v_{AB} \rangle n_A^* n_B^*$$

Summary

- ◆ Simple kinetic theory allows calculation of various average properties
- ◆ Calculate average properties with the distribution function
- ◆ Collision frequency important for properties that depend on the molecules being in close proximity
- ◆ Mixtures have multiple kinds of collisions