3

A Number Sense Assessment Tool for Identifying Children at Risk for Mathematical Difficulties

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Poor achievement in math can have serious educational and vocational consequences. Students with weak math skills at the end of middle school are less likely to graduate from college than are students who are strong in math (National Mathematics Advisory Panel, 2007). Competence in advanced math is important for success in college-level science courses and a wide range of vocations in the sciences (Sadler & Tai, 2007). Many students in US elementary schools do not develop foundations for success in algebra, and low-income learners lag far behind their middle-income peers (NAEP, 2007).

Fluency with math operations is a hallmark of mathematical learning in the early grades. Fluency is associated with foundational knowledge of key calculation principles (e.g., reciprocal relations among operations; Jordan et al., 2003a). Calculation fluency is necessary for math achievement at all levels – from solving simple whole number problems, to calculating with fractions, decimals and percentages, to solving algebraic equations. Even success with basic geometry depends on facility with calculation (e.g., calculating the angles of a triangle to add up to 180 degrees).

Mathematical Difficulties: Psychology and Intervention

Dysfluent calculation is a signature characteristic of students with learning disabilities in math (Jordan, 2007). Recent research suggests that calculation deficiencies, from the first year of formal schooling onward, can be traced to fundamental weaknesses in understanding the meaning of numbers and number operations, or *number sense* (Gersten et al., 2005; Malofeeva et al., 2004). Weak number sense can result in poorly developed counting procedures, slow fact retrieval, and inaccurate computation, all characteristics of math learning disabilities (Geary et al., 2000; Jordan et al., 2003a, b). It is difficult to memorize arithmetic 'facts' by rote, without understanding how combinations relate to one another on a mental number line (Booth & Siegler, in press). Accurate and efficient counting procedures can lead to strong connections between a problem and its solution (Siegler & Shrager, 1984). It has been suggested that basic number sense is a circumscribed cognitive function and relatively independent from general memory, language and spatial knowledge (Gelman & Butterworth, 2005; Landerl et al., 2004). Although there is a high rate of co-occurrence between math and reading/language difficulties, specific math difficulties with normal development in other cognitive and academic areas are well documented (Jordan, 2007; Butterworth & Reigosa, 2007).

WHAT IS NUMBER SENSE?

Although number sense has been defined differently and sometimes is used loosely in connection with math (Gersten et al., 2005), researchers generally agree that number sense in the 3- to 6-year-old period involves interrelated abilities involving numbers and operations, such as subitizing (derived from the Latin word subitus, for sudden) quantities of 3 or less quickly, without counting; counting items in a set to at least five with knowledge that the final count word indicates how many are in the set; discriminating between small quantities (e.g., 4 is greater than 3 or 2 is less than 5); comparing numerical magnitudes (e.g., 5 is 2 more than 3) and transforming sets with totals of 5 or less by adding or taking away items. Arguably, number and associated operational knowledge is the most important area of mathematical learning in early childhood (Clements & Sarama, 2007). Weak number sense prevents children from benefiting from formal instruction in math (Baroody & Rosu, 2006; Griffin et al., 1994).

Most children bring considerable number sense to school, although there are clear individual differences often associated with social class and learning abilities (e.g., Dowker, 2005; Dowker, in press; Ginsburg & Russell, 1981; Ginsburg & Golbeck, 2004; Jordan et al., 1992). Nonverbal number sense is present in infancy (Mix et al., 2002). For example, preverbal infants can discriminate between two and three objects and are sensitive to ordinal relations (i.e., more vs. less; Starkey & Cooper, 1980). Infants also can keep track of the results of adding of removing objects from an array, suggesting sensitivity to number operations (Wynn, 1992). Early sensitivities to number have a neurological basis in the intraparietal sulcus (IPS) regions of the brain and are the roots for learning symbol systems that represent number in preschool (e.g., number names, number symbols, counting; Berger et al., 2006). Atypicalities in IPS brain regions have been associated with specific impairments in mathematical operations (Isaacs et al., 2001).

Knowledge of the verbal number system is heavily influenced by experience or instruction (Geary, 1995; Levine et al., 1992). Case and Griffin (1990) report that number sense development is closely associated with children's home experiences with number concepts. Efforts to teach number sense to high-risk children in early childhood have resulted in significant gains on first-grade math outcomes compared to control groups (Griffin et al., 1994). Engaging young children in number activities (e.g., a mother asking her child to give her 3 spoons) and simple games (e.g., board games that emphasize one-to-one correspondences, counting and number lines) develop foundations and build number knowledge (Gersten et al., 2005).

COMPONENTS OF NUMBER SENSE

Counting: Counting is a crucial tool for learning about numbers and arithmetic operations (Baroody, 1987), and counting weaknesses have consistently been linked to mathematics difficulties (Geary, 2003). Most children develop knowledge of 'how to count' principles before they enter kindergarten (Gelman & Gallistel, 1978), including one-to-one correspondence (each item is counted only once), stable order (count words always proceed in the same order) and cardinality (the final word of a count indicates the number of items in a set). Typically, children learn the count sequence by rote and then map counting principles onto the sequence through their experiences with counting objects (Briars & Siegler, 1984). As children move through early childhood, they learn that items can be counted in any order (e.g., right to left or left to right), that sets do not have to be homogeneous, and that anything can be counted (e.g., the number of promises broken or the number of days in a week). They become increasingly flexible with counting, counting backward and by twos and fives. Children eventually acquire words for decades and learn rules for combining number words (e.g., combining 20 with 5 to make 25) (Ginsburg, 1989). Early difficulties in counting are precursors for later problems with math operations (Geary et al., 1999).

Number knowledge: Children as young as 4 years of age recognize and describe global differences in small quantities (Case & Griffin, 1990; Griffin, 2002, 2004). For example, they can tell which of two sets of objects has more or less. Although younger children rely on visual perception rather than on counting to make these judgments (Xu & Spelke, 2000), by 6 years of age most children incorporate their global quantity and counting schemas into a mental number line (Siegler & Booth, 2004). This overarching structure allows children to make better sense of their quantitative worlds (Griffin, 2002). Children gradually learn that numbers later in the counting sequence have larger quantities than earlier numbers. They come to see that numbers have magnitudes, such that 7 is bigger than 6 or that 5 is smaller than 8. Children use these skills in multiple contexts and eventually construct a linear representation of numerical magnitude, which allows them to learn place value and perform mental calculations. Number knowledge helps children think about mathematical problems, and its development reflects children's early experiences with number (Griffin et al., 1994; Saxe et al., 1987; Siegler & Booth, 2004). Middle-income children enter kindergarten with better-developed number knowledge than low-income children (Griffin et al., 1994; Jordan et al., 2006), and number knowledge is a strong predictor of math achievement in the early school years (Baker et al., 2002; Jordan et al., 2007).

Number Operations: The number knowledge and counting abilities that children acquire in early childhood are relevant to learning conventional math operations involving exact rather than approximate representations. Although preschoolers have limited success in performing verbally presented calculation problems, such as story problems ('Bob had 3 marbles. Jill gave him 2 more marbles. How many pennies does he have now?') and number combinations ('How much is 2 and 3?'), they are successful on nonverbal calculation tasks, which provide physical referents but do not require understanding of words and syntactic structures (Ginsburg & Russell, 1981; Hughes, 1986; Levine et al., 1992; Huttenlocher et al., 1994). Young children's success in solving nonverbal calculations requires children to hold and manipulate mental representations of numbers in working memory (Klein & Bisanz, 2000). Nonverbal calculation ability varies less across social classes than does the ability to solve verbal calculations (which clearly favors middle- over low-income children) (Jordan et al., 1992, 1994, 2006).

PREDICTABILITY OF NUMBER SENSE

Number sense predicts math outcomes in elementary school. A number knowledge test, first developed by Okamoto and Case (1996) and later tested by Baker et al. (2002), revealed strong predictability. The test, which assessed children's understanding of the magnitude concepts of 'smaller than' or 'bigger than' as well as knowledge of math operations, was given in the spring of kindergarten. The correlation with a math achievement criterion at the end of first grade was strong and significant. Clarke and Shinn (2004) also found that the ability to name numbers, to identify a missing number from a sequence of numbers, and to identify which of two numbers is larger predicts math outcomes at the end of first grade. Booth and Siegler (in press) report that the linearity of children's estimates on a number line is highly related to general math outcomes and that visual presentation of the magnitudes of addends and sums improves learning of number operations.

Although much of the research on number sense predictability is concerned with relatively near outcomes (e.g., kindergarten to first grade), we have found in our research lab (see more detailed discussion in the next section of this chapter) that number sense – even at the very beginning kindergarten – retains its predictive validity at least through the end of third grade (Jordan et al., 2007; Jordan et al., under review; Locuniak & Jordan, in press). Moreover, recent work (Duncan et al., 2007) suggests that the connection continues throughout the school years.

DEVELOPING A NUMBER SENSE BATTERY

Many math disabilities are not identified until middle school or later. In US schools, math interventions are much less common for early learners than are reading interventions (Jordan et al., 2002). To improve early identification of math problems, our research group (Jordan et al., 2006, 2007) developed a number sense battery for children from the beginning of US kindergarten to the middle of first grade (from approximately 5 to 6 years of age). The battery, developed as a part of a large longitudinal study of children's math, is based on the premise that number sense is of central importance to math learning and is guided by theoretically valid components of number sense. These components include counting, number knowledge and number operations. As noted earlier in this chapter, the components are closely linked to the skills children will need to acquire in formal math in elementary school. Reliability (alpha coefficient) of our core number sense battery ranged from 0.82 to 0.89 across the six time points. Our battery included the following tasks.

COUNTING AND NUMBER RECOGNITION

Children are assessed on counting skills and principles as well as their ability to recognize numbers. Children are asked to count to at least ten and allowed to restart counting only once but can self-correct at any time. Counting principles were adapted from Geary et al. (1999). For each item (after ruling out color blindness), children are shown a set of alternating yellow and blue dots. Then a finger puppet tells them he is learning how to count. The child is asked to indicate whether the puppet count is 'OK' or 'not OK.' Correct counting involves counting from left to right and counting from right to left. Unusual but correct counts involve counting the yellow dots first and then counting the blue dots or vice versa. For incorrect counts, the puppet counts from left to right but counts the first dot twice. For number recognition, children are asked to name a visually presented Arabic number.

NUMBER KNOWLEDGE

Number knowledge tasks were adapted from Griffin (2002). Given a number (e.g., 7), children are asked what number comes after that number and what number comes two numbers after that number. Given two numbers (e.g., 5 and 4), children are asked, which number is bigger or which number is smaller. Children also are shown visual arrays of three Arabic numbers (e.g., 6, 2 and 5), each placed on a point of an equilateral triangle and asked to identify which number is closer to the target number at the apex (e.g., 5).

NUMBER OPERATIONS

To assess number operations, we used three related tasks: Nonverbal calculation, number combinations and story problems.

On the nonverbal calculation task, adapted from Levine et al. (1992), the tester and child face each other with a 45 × 30 cm white mat in front of each and a box of 20 chips placed off to the side. The tester also has a box lid with an opening on the side. Three warm-up trials are given in which we engage the children in a matching task by placing a certain number of chips on the mat in a horizontal line, in view of the child and then covering the chips with the box lid. The child is asked to indicate how many chips are hidden, either with chips or by saying the number. After the warm-up, addition problems and subtraction problems are presented. The tester places a set of chips on her mat (in a horizontal line) and tells the child how many chips are on the mat. The chips are then covered with the box lid. Chips are either added or removed (through the side opening) one at a time. For each item, the children is asked to indicate how many chips are left hiding under the box, either by displaying the appropriate number of chips or giving a number word.

Addition and subtraction story problems and number combinations are presented orally, one at a time. The addition problems are phrased as follows: 'Jill has *m* pennies. Jim gives her *n* more pennies. How many pennies does Jill have now?,' while the subtraction problems are phrased: 'Mark has *n* cookies. Colleen takes away *m* of his cookies. How many cookies does Mark have now?' Number combinations are phrased as: 'How much is *m* and *n*?' and 'How much is *n* take away *m*?'

We used the battery to assess 400 US students in kindergarten (mean age = 5 years, 6 months at the beginning of the year). We followed more than 300 students through first grade and roughly 200 through third grade. About a third of the children were from low-income families. Children's number sense was assessed on six occasions, from the beginning of kindergarten through the middle of first grade. Math achievement was subsequently measured with the *Woodcock–Johnson Tests (WJ) of Achievement* (McGrew et al., 2007) on five occasions, from the end of first grade through the end of third grade.

KEY FINDINGS FROM THE CHILDREN'S MATH LONGITUDINAL STUDY

Between the beginning of US kindergarten and the middle of first grade, we found three statistically distinct number sense growth trajectories (Jordan et al., 2006, 2007): Children who start kindergarten at a high level and remain there; children who start kindergarten at a low to moderate level but start showing steep growth in the middle of the year; and children who start kindergarten at a low level and experience little growth. Low-income children were over represented in the latter low performing, low growth group and under represented in the other two groups.

Longitudinal analyses also revealed that number sense performance in US kindergarten, as well as number sense *growth*, accounts for 66% of the variance in first-grade math achievement (Jordan et al., 2007). Background characteristics of income status, gender, age and reading ability did not add explain variance in math achievement over and above number sense. Even at the beginning of kindergarten, number sense is highly related to end first-grade math achievement ($r = 0.70$). Moreover, the predictive value of kindergarten and first-grade number sense holds until at least the end of third grade or 8 to 9 years of age (Jordan et al., under review). Number sense predicted *rate* of math achievement between first and third grades as well as level of math performance at the end of third grade. We also found that least 80% of the time, children who meet state-defined math standards (as required by the US No Child Left Behind Law) have a higher-kindergarten number sense score than children who do not meet the standards.

We were particularly interested in how well early number sense predicts calculation fluency (Locuniak & Jordan, in press). Kindergarten number sense was a significant predictor of calculation fluency in elementary school, over and above age, reading and general cognitive competencies. Although facility with number combinations, number knowledge, and working memory capacity all were uniquely predictive, early facility with number combinations was the strongest single predictor of later calculation fluency.

STREAMLINING THE NUMBER SENSE SCREENING TOOL

As noted above, early number sense is highly predictive of important math outcomes in school. However, the instrument we used for the studies just described was a research tool with a relatively long administration time (more than 30 minutes in most cases). In 2007, we began work on the development of a shortened screening measure of number sense for 5 and 6 year olds. In the initial research battery, a third as many items were developed as the final number in the streamlined version. The final items were selected using Rasch item analyses, as well as a more subjective review of issues related to item bias. The exact number sense test items are presented in the Appendix.

Reliability: Several statistics are useful to describe a test's reliability. Among them are person and item separation indices culled from the Rasch model of Item Response Theory (Hambleton et al., 1991; Thissen & Wainer, 2001). In addition to the Rasch indices, the provision of internal-consistency reliabilities is recommended by the Standards for Educational and Psychological Testing (American Educational Research Association, 1999).

The person and item separation statistics in the Rasch measurement model provide statistical tools by which to evaluate the successful development of a variable and by which to assess the precision of the measurement (Harvey & Hammer, 1999; Wright & Stone, 1979). Respectively, person and item separation reliabilities were 0.80 and 0.99. The person separation statistic gives information on the test's capacity to distinguish among a sample of children on the basis of the total number of items answered correctly. Person separation reliabilities are equivalent to other measures of internal consistency and, accordingly, estimate the amount of error in measurement. Alternatively, item separation reliabilities indicate how well items define the variables being measured. The obtained estimate of 0.99 indicates that the items in the test are sufficiently separated from easy to hard to form variable lines that are complete and well spaced.

Cronbach's (1951) coefficient alpha was used to calculate internal-consistency reliability. Coefficient alpha provides a lower bound value of internal consistency and is considered to be a conservative estimate of a test's reliability (Gregory, 2007). The alpha coefficient was 0.84. As expected, this value exceeds the minimum, accepted internal-consistency level of 0.80 endorsed by leading measurement textbooks (Gregory, 2007; Reynolds et al., 2006; Salvia et al., 2007).

Validity: Strong construct validity is suggested whenever there is an appropriate pattern of convergent and divergent associations between an instrument and an external measure (Campbell, 1960; Reynolds et al., 2006; Thorndike, 1982). Because the number sense screen and the Woodcock–Johnson Achievement Test in math both purport to measure similar qualities, high correlations should occur between the measures. Likewise, scales measuring less similar qualities (e.g., reading achievement) would show lower correlations.

Table 3.1 shows correlations between scores on our number sense screen and the WJ-Math and reading (as measured by the TOWRE, Test of Word Reading Efficiency; Torgesen et al., 1999). Number sense scores were obtained on three occasions in US schools: (1) at the beginning of kindergarten, (2) the end of kindergarten and (3) at the middle of first grade. Criterion scores from the WJ-Math and TOWRE were obtained at the end of third grade.

TABLE 3.1 Correlations Between Scores on the Number Sense Screen and Wj-math (Math Achievement) and TOWRE (Reading Achievement)

Number sense	End of Grade 3 math achievement (WJ-Math)	End of Grade 3 reading achievement (TOWRE)
Beginning of kindergarten	0.65	0.33
End of kindergarten	0.63	0.29
Middle of first grade	0.62	0.40

All of the correlations were statistically significant ($p < 0.01$). With respect to the pattern of associations, the convergent correlations between the number sense screen and the WJ-Math were high, ranging from 0.62 to 0.65. The divergent associations between the number sense screen and the TOWRE were lower than the convergent associations, ranging from 0.29 to 0.40. All three convergent associations (number sense and math achievement) were much higher than the three divergent associations (number sense and reading achievement). Consequently, the pattern of convergent and divergent associations was appropriate and strongly support inferences of construct validity for the number sense screen.

Although the shortened number sense screening tool is still in development and we are collecting normative data in the US, it has good potential for identifying young children who are at risk for developing learning difficulties in math. At present, the measure is reliable, predicts math achievement in the early school years, and can be administered in less than 30 minutes.

INSTRUCTIONAL IMPLICATIONS

Although early interventions in math have received relatively little attention in the literature (Gersten et al., 2005), the available research offers insights for children who may be at risk for struggling in math. Number sense appears to be malleable and young children are likely to benefit from explicit help in representing, comparing and ordering small numbers as well as in joining and separating sets of 5 or less (National Council for Teachers of Mathematics, 2006; Fuson, 1992). Children should manipulate small quantities with sets of objects (e.g., as in the nonverbal calculation task) or their fingers (Jordan et al., in press) and then encouraged to imagine set transformations in their heads and to extract calculation principles (e.g., $2 + 3$ is the same as $3 + 2$ or $4 + 1$). Recent work by Siegler and colleagues (e.g., Booth and Siegler, in press) shows that activities as simple as board games that require children to move up and down a number list help children develop meaningful knowledge of quantities and number magnitudes and increase math achievement. Helping children build number sense early should give them the background they need to achieve in math during the school years. However, this assertion needs to be tested through randomized-controlled studies of number sense interventions. Our number sense instrument should be useful for reliably monitoring progress and responses to targeted interventions.

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APPENDIX

The streamlined number sense screening tool (N = 33 items).

(Write 1 for correct; 0 for incorrect)

Give the child a picture with 5 stars in a line. Say: '*Here are some stars. I want you to count each star. Touch each star as you count.*' When the child is finished counting, ask, '*How many stars are on the paper?*'

1. Enumerated 5 ___
2. Indicated there were 5 stars were on the paper ___

Say: '*I want you to count as high as you can. But I bet you're a very good counter, so I'll stop you after you've counted high enough, OK?*'

Allow children to count up to 10. If they don't make any mistakes, record '10.' Record the highest-correct number they counted up to without error.

3. Write in last correct number spoken ___
   Child counted up to 10 without error ___

Show the child a line of 5 alternating blue and yellow dots printed on a paper. Say: '*Here are some yellow and blue dots. This is Dino* (show a finger puppet), *and he would like you to help him play a game. Dino is going to count the dots on the paper, but he is just learning how to count and sometimes he makes mistakes. Sometimes he counts in ways that are OK but sometimes he counts in ways that are not OK and that are wrong. It is your job to tell him after he finishes if it was OK to count the way he did or not OK. So, remember you have to tell him if he counts in a way that is OK or in a way that is not OK and wrong. Do you have any questions?*'

Trial type	Response		
4. Left to right	*OK*	Not OK	___
5. Right to left	*OK*	Not OK	___
6. Yellow then blue	*OK*	Not OK	___
7. Double First	*OK*	Not OK	___

For items 8 through 11, point to each number that is printed on a separate card and say: '*What number is this?*'

8. 13 ___
9. 37 ___
10. 82 ___

11. 124 ___
12. What number comes right after 7? ___
13. What number comes two numbers after 7? ___
14. Which is bigger: 5 or 4? ___
15. Which is bigger: 7 or 9? ___
16. Which is smaller: 8 or 6?___
17. Which is smaller: 5 or 7?___
18. Which number is closer to 5: 6 or 2? ___

Say: '*We are going to play a game with these chips. Watch carefully.*' Place two chips on your mat. '*See these, there are 2 chips.*' Cover the chips and put out another chip. '*Here is one more chip.*' Before the transformation say, '*Watch what I do. Now make yours just like mine or just tell me how many chips are hiding under the box.*' Add/remove chips one at a time. Items 19 to 22 are the nonverbal calculations.

19. 2 + 1 ___
20. 4 + 3 ___
21. 3 + 2 ___
22. 3 − 1 ___

Say: '*I'm going to read you some number questions and you can do anything you want to help you find the answer. Some questions might be easy for you and others might be hard. Don't worry if you don't get them all right. Listen carefully to the question before you answer.*'

23. Jill has 2 pennies. Jim gives her 1 more penny. How many pennies does Jill have now? ___
24. Sally has 4 crayons. Stan gives her 3 more crayons. How many crayons does Sally have now? ___
25. Jose has 3 cookies. Sarah gives him 2 more cookies. How many cookies does Jose have now? ___
26. Kisha has 6 pennies. Peter takes away 4 of her pennnies. How many pennies does Kisha have now? ___
27. Paul has 5 oranges. Maria takes away 2 of his oranges. How many oranges does Paul have now? ___
28. How much is 2 and 1? ___
29. How much is 3 and 2? ___
30. How much is 4 and 3? ___
31. How much is 2 and 4? ___
32. How much is 7 take away 3? ___
33. How much is 6 take away 4? ___