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Characteristics of Children with Moderate Mathematics Deficiencies: A Longitudinal Perspective

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The reading and mathematics achievement and specific mathematical competencies of 74 children were followed over four time points during second and third grades. At the beginning of the study, children were classified into one of four groups: moderate mathematics deficiencies but normal reading (MMD-only); moderate mathematics and reading deficiencies (MMD/MRD); moderate reading deficiencies but normal mathematics (MRD-only); and normal achievement in reading and mathematics (NA). Although the MMD-only and the MMD/MRD groups started out at the same level in mathematics, the MMD-only group surpassed the MMD/MRD group over time. A parallel pattern in reading was not observed for the MRD-only and MMD/MRD groups, with children in both groups performing at consistently low levels. Weaknesses in fact retrieval and estimation characterized children with MMD, with or without RD. The MMD-only group showed an advantage over the MMD/MRD group in problem solving. Reading and language strengths help children compensate for deficiencies in selected areas of mathematics.

Learning disabilities in mathematics receive far less attention than do learning disabilities in reading, in part because mathematics is complex and draws on many different cognitive abilities (Dowker, 1998; Geary, 2000). Nevertheless, there is a substantial and growing body of research in mathematics difficulties. Of particular interest has been mathematical cognition in young children who have or who are at risk for learning disabilities. Longitudinal studies have begun to provide information about the developmental course of mathematics difficulties.

Our research group recently completed a two-year longitudinal investigation of second- and third-grade children with mathematics difficulties (MD). To date, we have found that some types of mathematics difficulties are more stable than others during primary school (Jordan, Kaplan, & Hanich, 2002). In particular, children with mathematics difficulties who are good readers (MD-only) make greater gains in mathematics achievement than do children with both mathematics and reading difficulties (MD/RD), even when IQ and SES are taken into account. The data suggest that reading skills are important to learning mathematics.

Children with MD-only show a different pattern of performance on cognitive variables related to mathematics competence than do children with MD/RD (Hanich, Jordan, Kaplan, & Dick, 2001; Jordan, Hanich, & Kaplan, 2003). That is, children with MD-only have an advantage over children with MD/RD in areas that are mediated by language (e.g., counting and word problems) but not in areas associated with estimation, base-10 concepts, and fact retrieval. Reading abilities reflect, and in some cases determine, language levels. Children with MD-only appear to use their strengths in reading and language to compensate for their weaknesses in the other areas. Deficits in number-fact retrieval are shared by children in both MD groups, suggesting that underlying strengths in linguistic representations (e.g., phonological or semantic representation of number) are not as important as aspects of mathematical thinking (e.g., estimation) for mastering number facts.

Our findings, combined with those from other investigations (Geary, Hamson, & Hoard, 2000; Geary & Hoard, 2001; Geary, Hoard, & Hamson, 1999; Rourke, 1993), point to the clinical value of differentiating between mathematics difficulties with normal reading and mathematics difficulties with co-morbid reading difficulties. The potential for achievement growth in children with MD-only is greater than for children with MD/RD. However, it should be noted that our achievement criterion for determining mathematics and reading difficulties was liberal (e.g., a cutoff of the 35th percentile or below in our studies). We purposely cast a wide net to catch all children with potential difficulties, including those with milder or borderline problems. Many of our participants were not eligible for special education services. It is possible that

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reading abilities make less of a difference in the growth of children with more serious mathematics deficiencies.

In the present study, we targeted children (in the fall of second grade) who exhibited extremely low performance in mathematics—with and without deficiencies in reading. Our criterion for "moderate" deficiency was performance at or below the 15th percentile on an individual diagnostic achievement test. Typically, the 25th percentile is used in studies of learning disabilities (Fletcher et al., 1994). For contrast, we included a group of children with moderate deficiencies in reading but not in mathematics and a group of normally achieving children. Moderate mathematics deficiencies correspond to the clinical category of dyscalculia and moderate reading deficiencies to the clinical category of dyslexia. To examine achievement growth as well as the stability of our original classifications, children were given the diagnostic achievement test at three additional time points (i.e., once in second grade and twice in third grade). We also assessed children's skills in exact calculation of number combinations, fact retrieval, estimation, problem solving, place value, and written computation at four time points spanning second and third grades. These areas tap different aspects of mathematical cognition and are relevant to mathematics instruction in elementary school (Hanich et al., 2001). Children's verbal and nonverbal cognitive abilities were measured with an IQ test.

We were especially interested in whether reading level made a difference in the mathematics achievement growth of children who exhibited extremely low performance in mathematics in second grade. We also examined whether children with general deficiencies in mathematics and reading showed lower IQ scores overall than children with specific deficiencies in mathematics or in reading and whether children with specific mathematics or reading deficiencies showed a greater discrepancy between verbal and performance IQ than children with deficiencies in both math and reading. If facility in mathematics reflects nonverbal reasoning ability more generally and facility in reading reflects verbal ability, then the discrepancy of children with moderate mathematics deficiencies only should favor performance IQ over verbal IQ. However, Dowker (1995) found that the discrepancies for children with specific mathematics deficiencies could go either direction, suggesting cognitive heterogeneity within this group.

METHOD

Participants

In the fall of second grade, 643 children from a public school district in New Castle County, Delaware were given the Broad Reading (Letter-Word Identification and Passage Comprehension) and Broad Mathematics (Calculation and Applied Problems) portions of the Woodcock-Johnson Psycho-Educational Battery-Revised, Form A (WJ; Woodcock & Johnson, 1990). The children were screened as a part of a large-scale longitudinal study of children with or at risk for learning difficulties. The WJ is a standardized, individually administered diagnostic test with a national norm group. Of this population, 52 (8 percent) children scored at or below the 15th percentile in Broad Reading and in Letter-Word Identification and 51 (8 percent) scored at or below the 15th percentile in Broad Mathematics. Of the 52 children with moderate RD, 11 performed above the 35th percentile in mathematics (MRD-only; 2 percent of the entire sample), and of the 51 children with moderate MD, 14 performed above the 35th percentile in reading (MMD-only; 2 percent of the entire sample). Twenty-five children performed at or below the 15th percentile in both reading and mathematics (MMD/MRD; 4 percent of the entire sample). The remaining children with moderate MD or moderate RD had borderline achievement in the other area (i.e., above the 15th percentile but at or below the 35th percentile). Because achievement levels depend on grade more than age, we used grade-level norms to derive percentile scores. The participants were a subset of those in earlier reports (Hanich et al., 2001; Jordan et al., 2002; Jordan et al., 2003).

The present study sample included children with MMD-only, MRD-only, or MMD/MRD who participated in all four phases of the longitudinal project and who were not retained in second grade during Year 2 of the study. Thus, the final group of participants included 11 children with MMD-only, eight children with MRD-only, and eight children with MMD/MRD. We compared the deficiency groups to 47 children with NA (children with achievement above the 35th percentile in both reading and mathematics). Data from the NA group were previously reported in Hanich et al. (2001) and Jordan et al. (2002). During the course of the study, all the children were in general education classrooms, but some were receiving special educational support services. The mean WJ reading and mathematics scores, along with demographic information, for the various groups are presented in Table 1.

Materials and Procedures

Children were given the tasks over a two-year period. One of several female research assistants (RA), who had extensive training in testing procedures, individually assessed each child at each time point. All the testing took place in the children's school during the school day.

Reading and Mathematics Achievement

In addition to taking the WJ (Broad Reading and Broad Mathematics tests) in the fall of second grade (as part of the screening measure described in the previous section), children took the WJ on three other occasions: spring of second grade (Form B), fall of third grade (Form A), and spring of third grade (Form B). Children's raw scores were transformed into Rasch-scaled scores, referred to as W-scores. W-scores were used for the longitudinal analyses in the study to provide a constant metric with equal interval properties. A W-score of 500 corresponds to children's average performance at the beginning of fifth grade.

Intellectual Abilities

The Weschler Abbreviated Scale of Intelligence (WASI) (Weschler, 1999) was administered to children in January

TABLE 1
Descriptive Information about Participants, by Group

Achievement Group	*n*	*M/F*	*Mean Age (In Months)*	*% Minority*[1]	*% Low SES*[2]	*% Special Education*[3]	*Reading Composite Percentile Scores*	*Letter-Word Ident. Percentile Scores*	*Mathematics Composite Percentile Scores*
MMD-only	11	5/6	93	55	36	9	65	56	9
			(6)				(14)	(16)	(5)
MMD/MRD	8	5/3	94	37	37	100	9	8	5
			(4)				(8)	(5)	(4)
MRD-only	8	5/3	96	50	75	63	12	11	65
			(7)				(8)	(5)	(20)
NA[4]	47	23/24	92	43	40	0	72	64	69
			(5)				(13)	(16)	(12)

[1]Within each achievement group, children identified as ethnic minority were African American.
[2]Children participating in the subsidized school lunch program were considered low SES.
[3]Children were receiving special services during second and third grade.
[4]Data for NA children also are reported in Jordan et al. (2002).
Note. Standard deviations are shown in parentheses.

of third grade. The verbal portion is composed of Vocabulary and Similarities subtests and the performance (nonverbal) portion of Block Design and Matrix Reasoning subtests. Full-scale IQ combined verbal and performance scores. For all scales, the mean score is 100 with a standard deviation of 15.

Specific Mathematics Competencies

Children were administered a battery of mathematics tasks, all of which were directly related to what children were learning in school. Tasks on the battery included: (1) exact calculation of arithmetic combinations; (2) story problems; (3) approximate arithmetic; (4) place value; (5) calculation principles; (6) forced retrieval of number facts; and (7) written computation. These tasks are also described in Hanich et al. (2001).

Children were administered the mathematics battery at four time points: winter of second grade, spring of second grade, fall of third grade, and spring of third grade.

Exact Calculation of Number Combinations. Eight arithmetic combinations were presented to children (i.e., $9 + 8$; $3 + 6$; $5 + 6$; $8 + 7$; $9 - 3$; $17 - 9$; $11 - 5$; $15 - 8$). The order of the problems varied across testing sessions, although the addition problems were always presented before the subtraction problems. Each item was printed on a separate 8.5×11 horizontal sheet of paper. The problem was shown to the child as the RA read it aloud. Children were told they could do anything they wanted to help them solve the problem. On each problem, the RA noted children's calculation strategies. We were particularly interested in whether children used their fingers for calculation support.

Story Problems. Children were given 10 story problems. The problems ranged from simple (e.g., Nina had 9 pennies. Then she gave 3 pennies to Anthony. How many pennies does Nina have now?) to complex (e.g., Claire has 4 pennies. Ben has 9 pennies. How many pennies does Claire need to get to have as many as Ben?) (Riley & Greeno, 1998; Riley, Greeno, & Heller, 1983). The entire set of story problems is presented in Hanich et al. (2001). As the RA read each problem aloud, the child also was shown a written version. The child was told to wait until the RA finished reading the problem before giving an answer.

Approximate Arithmetic. There were 20 approximate arithmetic problems (10 addition and 10 subtraction). Each item consisted of a number combination that was accompanied by two proposed solutions (e.g., $4 + 2 = 5$ or 12) (Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999). Both answers were incorrect, although one answer was closer to the correct answer than the other. The RA told the child to respond right away and not to calculate answers. To prevent children from calculating, we imposed a five-second time limit. The addition approximate items were administered first. Children were given two practice items, on which the RA provided feedback. As the RA read each problem, she also displayed it visually.

Place Value. Three different place value tasks were given to children: counting and number identification, positional knowledge, and digit correspondence. The tasks were based on empirical work by Hiebert and Wearne (1996), Kamii (1989), and Ross (1989). The total number of items was 12.

For counting and number identification, children were given 16 plastic chips and asked to count the chips. If the child miscounted, the RA counted the chips aloud in front of the child to ensure that he or she understood there were 16 chips. Next, the child was shown a series of multidigit numbers (i.e., 16, 37, 415) and asked to read each number aloud. Each number was printed on a separate page.

In the positional knowledge task, children were queried on digit position using the problems from the counting and number identification task. As children were shown the number, they were asked to identify which digit was in the ones place, the tens place, and the hundreds place (in mixed order). For each item, children had to correctly identify all positions to receive a correct score.

In the digit correspondence activity, the RA first presented a card with the number "16" printed on it, along with 16 plastic chips. The RA circled the 6 in the number 16 on the card and asked the child to use the plastic chips to show what that part stood for in the number 16 (i.e., 6 chips). Next, the RA circled the 1 and asked the child to represent that part of the number (i.e., 10 chips). The child had to answer both items correctly to receive credit.

A second digit correspondence activity assessed children's understanding of place value with standard and nonstandard partitioning tasks. In the standard partitioning task, children were presented with a horizontal sheet of paper that had a number printed in the bottom right-hand corner (i.e., 43). On the paper was a graphic display of 43 squares. Four columns of 10 squares grouped together graphically represented 40 squares. The remaining 3 squares were graphically represented as individual unit squares. The RA said to the child, "There are 43 squares on this sheet of paper." She circled the 3 in the number 43 and said to the child, "Draw a circle around the squares that this part of the number 43 stands for." She then circled the 4 and said to the child, "Draw a circle around the squares that this part of the number 43 stands for." To receive credit, the child had to answer both parts of the item correctly.

The nonstandard partitioning task forced the child to demonstrate understanding of place value in a more challenging context, one that goes beyond rote learning. As in standard partitioning, the child was shown a sheet of paper with 43 squares. However, on this paper, there were 3 tens groupings and 13 individual unit squares. The directions and scoring procedures were the same as those used for the standard partitioning task. The standard and nonstandard partitioning tasks were repeated with the number 52.

In a final digit correspondence activity (also a nonstandard partitioning task), the child was shown a graphic representation of 26 stars, with the number 26 printed at the bottom of the page. The stars were arranged in 6 groups of 4 stars and 1 group of 2 stars. The RA said to the child, "There are 26 stars on this sheet of paper." She then circled the 6 in the number 26 and said to the child, "Draw a circle around the stars that this part of the number 26 stands for." She then circled the 2 and said, "Draw a circle around the stars that this part of the number 26 stands for."

Calculation Principles. Calculation principles, a problem-solving task, required children to use the first part of the problem to solve the second part (Baroody, 1999; Russell & Ginsburg, 1984), with a total of six items. Two items assessed children's understanding of the commutative principle—that the order of the addends does not affect the sum (i.e., $47 + 86 = 133$, so $86 + 47 = ?$, and $94 + 68 = 162$, so $68 + 94 = ?$); two items assessed the inversion principle—that addition is the inverse of subtraction (i.e., $27 + 69 = 96$, so $96 - 69 = ?$, and $36 + 98 = 134$, so $134 - 36 = ?$); and two items assessed the doubles plus one pattern (i.e., $37 + 37 = 74$, so $37 + 38 = ?$, and $64 + 64 = 128$, so $65 + 64 = ?$). Each problem was printed on a separate sheet of paper and read aloud by the RA. To prevent calculation, children were required to respond within five seconds.

Forced Retrieval of Number Facts. This task assessed automatic retrieval of number facts. Children were presented with eight number combinations (i.e., $4 + 2$; $9 + 4$; $7 + 9$; $3 + 8$; $6 - 4$; $13 - 9$; $16 - 7$; and $11 - 8$). Children were told to answer each problem as quickly as possible, without calculating. Problems were individually presented on a horizontal sheet of paper, with the addition problems presented first. As the RA showed each problem, she also said it aloud. As soon as the RA finished reading the problem, she started timing the children. When children began to state their answer, the RA stopped timing. Only problems that were answered correctly within three seconds or less were given credit.

Written Computation. Children were presented with eight multidigit written computation problems in a paper-and-pencil format (i.e., $45 + 23$; $38 + 29$; $624 + 312$; $475 + 189$; $67 - 31$; $42 - 27$; $849 - 524$; and $701 - 397$). Four addition problems were printed vertically on one sheet of paper, and four subtraction problems were printed on a second one. Carrying was necessary on half the addition problems and borrowing on half the subtraction problems.

RESULTS

Our main data analytic techniques were analysis of variance (ANOVA) and analysis of covariance (ANCOVA), with group as a between-subjects factor (MRD-only, MMD-only, MMD/MRD, and NA). We used Tukey tests ($p < 0.05$) to follow up on significant results.

IQ Data

The mean verbal, performance, and full-scale IQ scores by group are shown in Table 2. For comparison, Table 2 also shows each group's mean WJ standard scores. ANOVAs showed significant group effects for verbal IQ, $F(3, 70) = 16.92$, $p < 0.0001$, $\eta^2 = 0.42$; performance IQ, $F(3, 70) = 6.71$, $p < 0.000$, $\eta^2 = 0.22$; and full-scale IQ, $F(3, 70) = 14.97$, $p < 0.0001$, $\eta^2 = 0.39$. For verbal and full-scale IQ, the NA group performed significantly better than the other three groups (which did not differ from each other) and the MMD-only group performed significantly better than the MMD/MRD group. For performance IQ, the only significant group difference was between the NA and MMD/MRD groups, favoring the NA group.

We calculated the percentage of children in each group who showed significant discrepancies between verbal and performance IQ. A difference of at least 15 points was considered significant, which is in line with other studies (D'Anguilli & Siegel, 2003). In the NA group, six of 47 children (14 percent) showed a significant discrepancy, with five favoring verbal IQ and one favoring performance IQ. In the MMD-only group, four of 11 children (36 percent) showed a significant discrepancy, with three favoring verbal IQ and one favoring performance IQ. None of the children in the MRD-only or in the MMD/MRD group showed a significant discrepancy between verbal and performance IQ.

Because achievement-IQ discrepancies are frequently used to determine eligibility for learning disabilities services

TABLE 2
Mean WJ Standard Scores and WASI IQ Standard Scores, by Group

Achievement Group	*WJ Mathematics Composite Standard Score*	*WJ Reading Composite Standard Score*	*Verbal IQ*	*Performance IQ*	*Full-Scale IQ*
MMD-only	77	107	93	88	90
	(8)	(9)	(11)	(11)	(8)
MMD/MRD	74	76	77	79	76
	(6)	(10)	(10)	(6)	(8)
MRD-only	107	82	88	92	89
	(11)	(6)	(6)	(9)	(8)
NA	108	110	103	97	100
	(7)	(7)	(11)	(12)	(11)

Note. Standard deviations are shown in parentheses.

(Siegel, 2003), we calculated the number of children in each group who had IQ scores that were significantly higher (i.e., ≥15 points) than their achievement scores. For WJ reading, the MMD-only group ($n = 11$) had three (38 percent) children who showed a significantly higher verbal IQ and none with a higher performance IQ; the MMD/MRD group ($n = 8$) had two (25 percent) children who showed a higher verbal IQ and two (25 percent) who showed a higher performance IQ; the MRD-only group ($n = 8$) had one (13 percent) child who showed a higher verbal IQ and three (38 percent) who showed a higher performance IQ; and the NA group ($n = 47$) had no children with a higher verbal IQ and one (2 percent) child who showed a higher performance IQ. Not surprisingly, significant achievement-IQ discrepancies were more common in the specific deficiency groups (i.e., MMD-only and MRD-only), but the majority of children in all groups failed to show such discrepancies.

Achievement Data

The mean WJ W-scores by group and time point (1 through 4) in Broad Mathematics and Broad Reading are shown in Figure 1. For Broad Mathematics, there were significant effects of group, $F(3, 70) = 95.48$, $p < 0.0001$, $\eta^2 = 0.80$, and time, $F(3, 68) = 137.92$, $p < 0.0001$, $\eta^2 = 0.86$, as well as a time-by-group interaction, $F(9, 166) = 7.68$, $p < 0.0001$, $\eta^2 = 0.25$. At Time 1, the MMD-only and MMD/MRD groups did not differ. However, the MMD-only performed better than the MMD/MRD group at the other three time points, suggesting that children with moderate mathematics deficiencies (at the beginning of second grade) who are good readers grow faster in mathematics achievement than those who are poor readers. The MRD-only group did not differ from the NA group at Times 1 or 3 but performed significantly more poorly at Times 2 and 4. When we adjusted for IQ in an ANCOVA, the group effects and the group × time interaction held. (It should be noted that in this and subsequent ANCOVAs, full-scale IQ was used.)

In Broad Reading, there were significant effects of group, $F(3, 70) = 76.39$, $p < 0.0001$, $\eta^2 = 0.77$, and time, $F(3, 68) = 174.52$, $p < 0.0001$, $\eta^2 = 0.89$, as well as a time-by-group interaction, $F(9, 166) = 5.57$, $p < 0.0001$, $\eta^2 = 0.19$. Group contrasts showed that the MMD/MRD and MRD-only groups did not differ at Times 1 through 3, but the MRD-only group performed better than the MMD/MRD group at Time

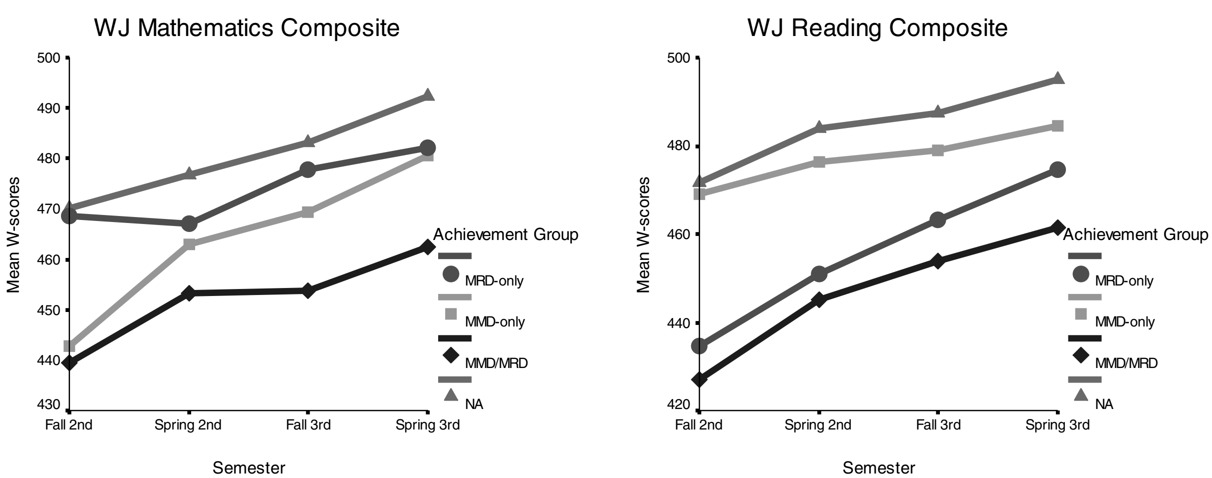

FIGURE 1 Mean scores on the WJ Mathematics Composite and WJ Reading Composite, by achievement group and time.

TABLE 3
Reading and Mathematics Achievement, by Group, at the End of Third Grade

Achievement Group	*Reading Composite Percentile Scores*	*Letter-Word Ident. Percentile Scores*	*Mathematics Composite Percentile Scores*
MMD-only	49	48	39
	(23)	(24)	(25)
MMD/MRD	10	10	6
	(10)	(10)	(5)
MRD-only	28	21	42
	(16)	(14)	(18)
NA	74	67	73
	(16)	(20)	(22)

Note. Standard deviations are shown in parentheses.

4, most likely due to the better comprehension skills of the former group. The MMD-only group did not differ from the NA group at Times 1 or 2 but performed more poorly than the NA group at Times 3 and 4. The group effects and the group × time interaction held when we adjusted for IQ.

How many children still met our a priori classifications of moderate deficiencies (at or below the 15th percentile) or normal achievement (above the 35th percentile) in reading or in mathematics or in both at end of third grade? The mean WJ percentile scores for each group at the end of third grade are presented in Table 3. For the NA and the MMD/MRD groups, the mean percentile scores remained about the same between the beginning of second grade and the end of third grade. For the MMD-only group, the mean reading score decreased by 16 points, while the mean math score increased by 30 points. Conversely, for the MRD-only group, the mean reading score increased by 16 points, while the mean math score decreased by 23 points. The percentage of children who retained their classification from Time 1 to Time 4 were 100 percent for the NA group, 88 percent (seven of eight) for the MMD/MRD group, 25 percent (two of eight) for the MRD-only group, and 18 percent (two of 11) for the MMD-only group.

Specific Mathematics Competencies

A multivariate analysis of variance (MANOVA) revealed a significant main effect of group, $F(21,184) = 5.74$, $p < 0.0001$, $\eta^2 = 0.38$, and time, $F(21, 50) = 13.29$, $p < 0.0001$, $\eta^2 = 0.85$, as well as a time-by-group interaction, $F(63, 150) = 1.63$, $p < 0.01$, $\eta^2 = 0.40$. We followed up with split plot ANOVAs for individual tasks assessing mathematics competencies.

The mean scores for each mathematics task, by time and group, are graphically displayed in Figure 2. We were especially interested in comparisons between the two MMD groups (i.e., MMD-only and MMD/MRD).

For *exact calculation of number combinations*, there were significant main effects of time, $F(3, 68) = 5.88$, $p < 0.001$, $\eta^2 = 0.21$, and group, $F(3, 70) = 32.90$, $p < 0.0001$, $\eta^2 = 0.59$, but no time-by-group interaction, $F(9,166) = 1.87$, $p < 0.06$. Multiple comparisons showed that the NA and MRD-only groups performed better than the MMD-only and MMD/MRD groups at all time points. The MMD-only group performed better than the MMD/MRD group. The group effects held when we adjusted for IQ.

The mean number of trials on which children used their fingers on exact calculation of arithmetic combinations and the mean percentage of times finger counting produced a correct answer are presented in Table 4. At Time 1, children in all groups used finger counting on more than half the trials. However, children in the MMD/MRD group were much less accurate than children in the MMD-only group (23 percent vs. 74 percent of trials). An analysis of frequency of finger use showed a significant effect of group, $F(3, 70) = 4.57$, $p < 0.01$, $\eta^2 = 0.16$, as well as a group-by-time interaction, $F(9, 166) = 2.22$, $p < 0.05$, $\eta^2 = 0.09$. When we covaried for IQ, however, the effect of group was no longer significant. There was not a significant effect of time, $F(3, 68) = 0.23$, $p < 0.88$.

For *story problems,* there were significant main effects of time, $F(3, 68) = 13.82$, $p < 0.0001$, $\eta^2 = 0.38$, and group, $F(3, 70) = 24.85$, $p < 0.0001$, $\eta^2 = 0.52$, as well as a time-by-group interaction, $F(9, 166) = 3.36$, $p < 0.001$, $\eta^2 = 0.13$. Although the MMD-only and the MMD/MRD groups did not differ from each other at Times 1 and 2, there were significant differences between these groups at Times 3 and 4. The NA and MRD-only groups did not differ at Times 1 and 4 but did at Times 2 and 3. The group and time-by-group effects held when we adjusted for IQ.

Analyses on the *approximate arithmetic* task showed significant main effects of time, $F(3, 68) = 4.98$, $p < 0.005$, $\eta^2 = 0.18$, and group, $F(3, 70) = 9.51$, $p < 0.0001$, $\eta^2 = 0.29$, but no time-by-group interaction, $F(9, 166) = 0.93$, $p < 0.50$. At all time points, both the MMD/MRD and MMD-only groups did worse than the NA group. The two MMD groups did not differ from each other. The group effect held when we adjusted for IQ.

For *place value*, there were significant main effects of time, $F(3, 68) = 7.78$, $p < 0.0001$, $\eta^2 = 0.27$, and group, $F(3, 70) = 10.62$, $p < 0.0001$, $\eta^2 = 0.31$, with no interaction between time and group, $F(9, 166) = 0.74$, $p < 0.69$. The NA group performed better than each of the other three groups, which did not differ from each other. When we covaried for IQ, the group effects held.

There were significant main effects of time, $F(3, 68) = 19.31$, $p < 0.0001$, $\eta^2 = 0.46$, and group, $F(3, 70) = 10.47$, $p < 0.0001$, $\eta^2 = 0.31$, for *calculation principles* but no time-by-group interaction, $F(9, 166) = 0.51$, $p < 0.87$. When we covaried for IQ, however, the group effects became nonsignificant.

For *forced retrieval*, there were significant main effects of time, $F(3, 68) = 9.49$, $p < 0.0001$, $\eta^2 = 0.30$, and group, $F(3, 70) = 15.12$, $p < 0.0001$, $\eta^2 = 0.39$, but no time-by-group interaction, $F(9,166) = 1.33$, $p < 0.23$. Regardless of time, both the MMD/MRD and the MMD-only groups did more poorly than the NA group. The two MMD groups did not differ from each other, performing at almost identical levels. The MRD-only group did not differ from the other three groups. The group effects held when we covaried for IQ.

Finally, analyses on the *written computation* task revealed significant main effects of time, $F(3, 68) = 40.89$, $p < 0.0001$,

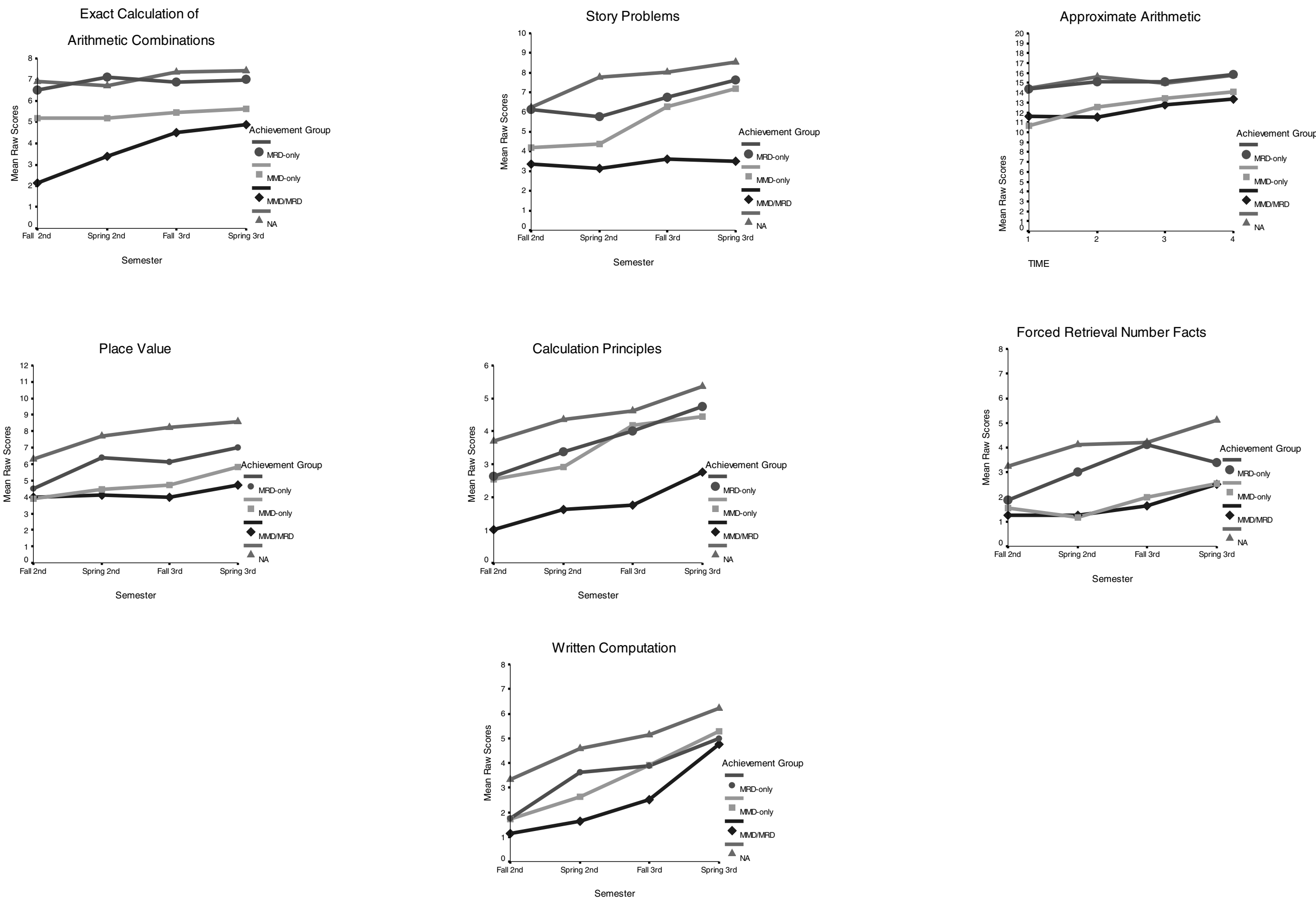

FIGURE 2 Mean raw scores on tasks assessing mathematics competencies, by achievement group and time.

$\eta^2 = 0.64$, and group, $F(3, 70) = 14.44$, $p < 0.0001$, $\eta^2 = 0.38$, but no time-by-group interaction, $F(9,166)$, $p < 0.44$. An ANCOVA adjusting for IQ showed that the NA group performed better than the two MMD groups but not better than the MRD-only group.

In summary, there were significant effects of group and time on all tasks. However, group effects on frequency of finger use and on performance accuracy on calculation principles did not reach significance when we co-varied for IQ. Time-by-group interactions were observed only on story problems, where the MMD-only group gained an advantage over the MMD/MRD group with time. The MMD-only group also performed consistently better than the MMD/MRD group on exact calculation of number combinations. The

TABLE 4
Mean Number of Times Finger Counting Was Used on Exact Calculation of Arithmetic Combinations and Mean Percent of Times it Yielded a Correct Answer Across Four Time Points (Eight Trials)

Achievement Group	*Time 1*	*Percent Correct*	*Time 2*	*Percent Correct*	*Time 3*	*Percent Correct*	*Time 4*	*Percent Correct*
MMD-only	5.0	74	5.0	81	5.7	73	6.0	72
	(2.9)		(2.4)		(2.6)		(2.1)	
MMD/MRD	5.1	23	5.4	51	6.0	66	6.5	49
	(2.4)		(2.1)		(2.4)		(1.8)	
MRD-only	4.1	71	4.6	94	4.3	78	3.5	75
	(2.4)		(2.7)		(1.6)		(2.0)	
NA	4.1	84	3.6	79	3.3	88	2.8	85
	(2.7)		(2.5)		(2.5)		(2.6)	

Note. Time 1 = Winter 2000; Time 2 = Spring 2000; Time 3 = Fall 2000; Time 4 = Spring 2001.

MMD-only and the MMD/MRD groups did not differ from each other in approximate arithmetic, forced retrieval of number facts, place value, and written computation.

DISCUSSION

About 2 percent of children in our second-grade screening pool had moderate mathematics deficiencies with normal reading, 2 percent had moderate reading deficiencies with normal mathematics, and 4 percent had moderate deficiencies in both reading and mathematics. Although moderate learning deficiencies represent a small segment of the school population, they are a serious educational problem, one that may result in school dropout, delinquency, and lifelong underachievement (Stillington & Frank, 1993).

A main concern of the present study was whether reading level makes a difference in the mathematics achievement growth of children with a restricted range of low performance in mathematics. Previous work with a wider population showed that children with MD alone grow at a faster rate in mathematics achievement than children with both MD and RD (Jordan et al., 2002). In the present study, we found this pattern holds for a subset of children identified with serious deficiencies in mathematics achievement. Although children with MMD-only and MMD/MRD did not differ in mathematics achievement early in second grade, a performance gap became apparent later in second grade and got wider in third grade. The low rate of classification retention (i.e., moderate deficiency, as defined by performance at or below the 15th percentile on a standardized achievement test) among the MMD-only group at the end of third grade versus the high rate among the MMD/MRD group suggests that the presence or absence of a reading deficiency is a powerful predictor of the stability of a mathematics deficiency. A child with MMD/MRD at the beginning of second grade is likely to have intractable problems in mathematics throughout primary school, despite special educational support. (One-hundred percent of children in the MMD/MRD group were receiving special education.) However, our previous work (Jordan et al., 2002) suggests that reading difficulties are addressed more often than are mathematics difficulties in primary school special education. Thus, it is possible that the MMD/MRD children were not receiving a significant amount of special educational support in mathematics over the course of the study.

What factors contribute to the differential gains in mathematics achievement between our two groups of children with MMD? As expected, children in the MMD-only group had higher verbal abilities (as measured by an IQ test) than children with MMD/MRD. The relatively strong reading performance of the MMD-only group also reflects underlying linguistic capacity. On our mathematics tasks, which examined cognitive variables associated with mathematical competencies, children with MMD-only showed a gradual advantage over children with MMD/MRD on story problems involving simple computations, problems that rely heavily on language comprehension. For example, on the problem, "Claire has 4 pennies. Ben has 9 pennies. How many pennies does Claire need to get to have as many as Ben?" the child must understand the meaning of the problem in order to use the correct arithmetic operation. The MMD-only group also had a steady advantage over the MMD/MRD group on calculation principles, another area of problem solving (Russell & Ginsburg, 1984), although these differences were not reliable when we adjusted for IQ.

It could be argued that children in the MMD-only group were false positives. Indeed, some of the problems of children identified with MMD-only in second grade were resolved by the end of third grade. However, their stable weaknesses in fact retrieval and estimation, even when we adjust for IQ, suggest real difficulties in these aspects of mathematics. Although MMD-only children used finger-counting strategies much more accurately on number combinations than MMD/MRD children, the groups both showed low accuracy on a forced-retrieval task, which required rapid answers to number facts without any calculation aids. It has been suggested that fact-retrieval deficits are associated with weaknesses in phonological and semantic representations in long-term memory (Geary, 1993). However, the strong reading skills of the MMD-only group and the relatively intact fact-retrieval skills of the MRD-only group suggest that language weaknesses, at least at the phonological level, do not play a major role in fact-retrieval deficits. Robinson, Menchetti, and Torgeson (2002) argue that fact-retrieval deficits in children with specific mathematics deficiencies are associated with poorer number sense. However, the relationship between particular aspects of number sense and the ability to retrieve number facts requires further investigation. For example, weak estimation skill may underlie fact-retrieval deficits. The ability to use a mental number line to estimate answers might increase children's speed and accuracy with addition and subtraction facts. However, the development of number-fact skill may also improve estimation capacity.

Despite large cognitive differences between the MRD-only and the MMD/MRD groups, they do not differ at any time point in reading. Moderate reading deficiencies, regardless of whether they are accompanied by mathematics deficiencies, are extremely stable over time, although their developmental course may be altered by instruction (Foorman & Torgeson, 2001).

The majority of children with learning deficiencies in our study, specific as well as garden-variety deficiencies, did not show large discrepancies between achievement and IQ. This observation also has been reported elsewhere (D'Anguilli & Siegel, 2003).

Our findings demonstrate the importance of a longitudinal perspective in the study of learning disabilities in mathematics. Although children with specific and general deficiencies in mathematics may look similar in mathematics achievement in early elementary school, their developmental trajectories and ultimate prognoses are likely to be different. Whether curriculum approaches or specialized interventions can alter the course of math development for children with MMD-only and MMD/MRD remains an unanswered question. It is possible that some children with early number deficiencies need only small scaffolds to boost development, while others require more intensive remedial approaches. Children with MMD-only may benefit from compensatory verbal strategies to improve math performance. It would also be important to know whether subgroups of MMD (i.e., specific and general deficiencies) can be reliably identified earlier than second grade and whether we can uncover precursors to mathematics deficiencies before formal instruction in school.

From a practical perspective, school psychologists and educational diagnosticians should focus on patterns of achievement, rather than patterns of IQ scores or IQ-achievement patterns, in their assessments of mathematics disabilities. Achievement patterns predict progress and provide a relevant context for providing educational interventions.

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