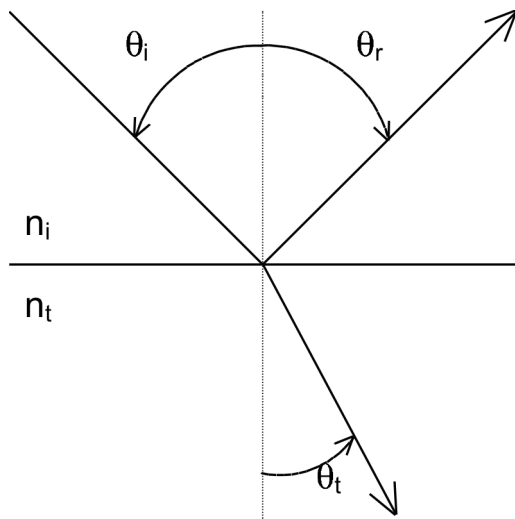


## 5. Reflection, refraction and polarization

Figure 5.1 illustrates what happens when electromagnetic radiation encounters a smooth interface between dielectric media. We see two phenomena: reflection and refraction (a specific type of transmission). The angle of reflection,  $\theta_r$ , equals the angle of incidence,  $\theta_i$ , where each is defined with respect to the surface normal. The angle of refraction,  $\theta_t$ , (t for transmitted) is described by Snell's law (of Refraction):

$$n_i \sin \theta_i = n_t \sin \theta_t .$$

Refraction is used to guide light in optical instruments. Lenses for imaging are the most common application of refraction, and will be discussed in detail in a later section. Snell's law also predicts the phenomenon of total internal reflection. When  $n_i > n_t$ ,  $\theta_t$  appears to exceed  $90^\circ$ . In reality, at incident angles



**Figure 5.1: Reflection & Refraction at interface of 2 media**

above the critical angle,  $\theta_c$ , the transmitted beam becomes vanishingly small and propagates parallel to the interface. The incident beam is totally internally reflected. A small electric field extends beyond the boundary of the interface by a distance approximately equal to the wavelength of light. This electric field beyond the interface is called the evanescent field, and it can be absorbed by molecules on the other side ( $n_t$ ) of the interface, provided they are within a distance on the order of the wavelength of light. The evanescent field strength is dependent upon the polarization (the orientation of the electric field), the angle of the incident beam and the refractive indices of the two media.

The technique of “attenuated total internal reflectance”, or ATR, is commonly used in infrared spectroscopy to make measurements of films or solids by launching an IR evanescent wave into a thin layer of the material adsorbed on or clamped to a crystal in which an IR beam undergoes multiple internal reflections.

The radiant power of the reflected and refracted light depend on the angle of incidence, the refractive indices of the two media and, in some cases, the polarization of the incident beam. The Fresnel equations describe the dependence of the reflected light on these quantities. In the case of normal incidence, the incident beam polarization doesn't appear. The reflectance ( $\rho = E_r/E_0$ ) is

$$\rho = \left[ \frac{n_i - n_t}{n_i + n_t} \right]^2 .$$

The reflectance is thus related to the difference in refractive indices between the two media. For glass and air, which have refractive indices of 1.50 and 1.00, respectively, the reflection coefficient is 0.04. For water and diamond, which has a refractive index of 2.4, the reflection coefficient is 0.08. It is therefore easier to see diamonds in water than it is to see glass in air.

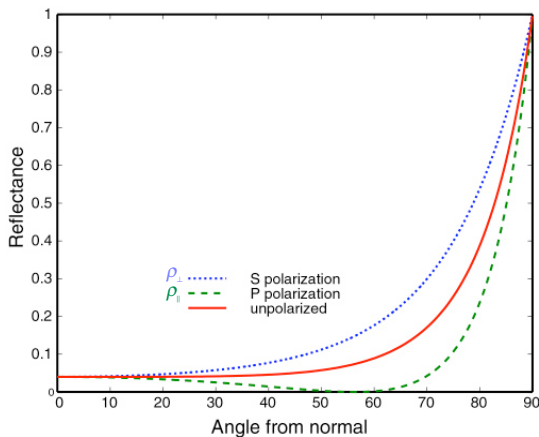
When polarized light encounters a dielectric interface, the polarization of the reflected and or refracted beams may be impacted. This is described at arbitrary angles of incidence, using Fresnel equations that refer to the reflection coefficient (related to beam amplitude) of the EMR beams rather than the reflectance (related to beam power) and include the polarization of the beam. If the electric vector oscillates in the plane of incidence, the reflection coefficient is denoted  $r_{\parallel}$ .

$$r_{\parallel} = \left[ \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)} \right]$$

If the electric vector vibrates in the direction perpendicular to the plane of incidence, the reflection coefficient is  $\rho_{\perp}$ .

$$r_{\perp} = - \left[ \frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)} \right]$$

The reflectances related to these two reflection coefficients ( $\rho = r^2$ ) plotted as



<http://www.rendermanacademy.com/docs/Fresnel.htm>

**Figure 5.2: Dependence of polarized reflection coefficients on incidence angle**

functions of the angle of incidence are shown to the left for the case of the air/glass interface, where  $n_{\text{glass}} \sim 1.5$ . For both cases, the reflection coefficient is approximately 0.04 at normal incidence ( $\theta_i = 0^\circ$ ) and approaches one at grazing incidence ( $\theta_i = 90^\circ$ ). In between, there is a considerable change in the reflection coefficients, with  $r_{\perp} > r_{\parallel}$ . This is why polarized (as opposed to just tinted) sunglasses are useful. They are made to absorb the horizontally polarized light reflected from horizontal surfaces, i.e., lakes and highways. Notice that at a point just under  $60^\circ$  in Figure 5.2 (air/glass interface),  $\rho_{\parallel} \rightarrow 0$ ; this is Brewster's

angle. Brewster's angle is calculated using the relation  $\theta_B = \text{atan}(n_t/n_i)$ , which is derived using Snell's law and imposing the constraint that  $(\theta_i + \theta_t) = 90^\circ$ . Brewster angle reflection is frequently used in laser optics to minimize stray reflections of the intense laser beam. In spectroscopy the polarization of the incident beam (on the sample) matters because waves polarized parallel to the absorption dipole are preferentially absorbed. This phenomenon is called photoselection.

The amount of light refracted at arbitrary incidence angles is most easily determined using the conservation law, which is

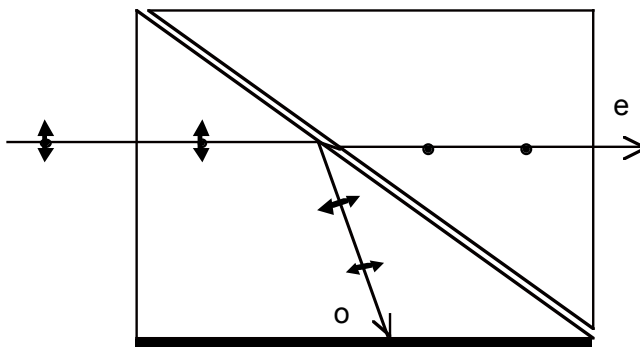
$$\rho + T = 1$$

as long as the absorbance is negligible.

## Polarizers

The simplest means of controlling light polarization is to use a Polaroid film (not photographic film), which transmits one polarization and absorbs the other. Polaroid films are based on the concept that molecules absorb light along one molecular axis. Strongly absorbing dyes are incorporated into a polymer e.g., polyethylene, film, which is then stretched to align the molecular axes to the stretch axis. The transmission axis of the film is perpendicular to the stretch axis. The polarization extinction coefficient,  $\mathcal{E}_{\parallel}/\mathcal{E}_{\perp}$ , of a Polaroid can be as high as  $10^3$ . Even high quality Polaroid film is inexpensive. The disadvantages of Polaroid films are two-fold. First, even the transmitted polarization is strongly attenuated by the film. This can be appreciated by noticing that Polaroid sunglasses are dark. In a low intensity fluorescence or Raman experiment, a Polaroid film would reduce the signal dramatically. Second, Polaroid film is easily damaged by intense, e.g., laser, beams because of strong absorbance. More sophisticated means of controlling polarization are required for such demanding applications.

A more transmissive device that is used to create highly polarized light is the Glan-Thompson prism. Its geometry is illustrated below. It is actually constructed using two birefringent prisms.



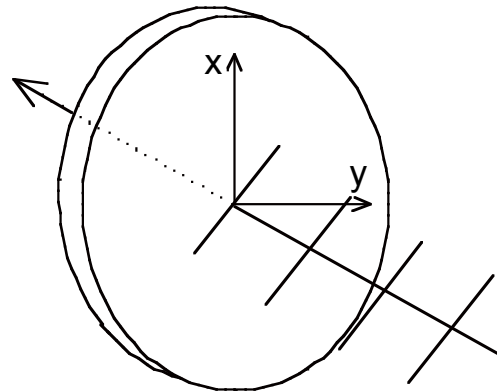
A birefringent material is asymmetric on the molecular level and thus optically anisotropic. Birefringent crystals such as calcite ( $\text{CaCO}_3$ ) and quartz can divide monochromatic incident beams into two beams of orthogonal polarization because the components of incident light traveling along most crystal directions experience two refractive indices. The refractive index for one polarization (ordinary rays),  $n_o$ , obeys Snell's law and the other polarization (extraordinary rays),  $n_e$ , doesn't.

**Figure 5.3: Glan-Thompson Prism**

However, these crystals have an optic axis along which components of the electric field travel at the same speed regardless of the direction of their polarization. The optic axis of the crystals in Figure 5.2 is perpendicular to the plane of the page. The Glan-Thompson prisms are cut so that the light component experiencing  $n_o$  (1.6584 for calcite) is totally internally reflected, while the other component ( $n_e=1.4864$ ) is transmitted. Cementing the first crystal

section to a second allows the  $n_e$  component to propagate essentially undeviated. (Remember the light bends on entering a new medium, so this enables the device to accept light from a wider field of view.) Polarization extinctions on the order of  $10^4$  to  $10^5$  are achieved with high transmission. Most vendors use a glue that matches the index of refraction between the two crystals so that there is even less deviation of the beam. For applications to high-powered lasers, air is left in the space between the two crystals, (the polarizer is technically called Glan-Foucault in this case) and the crystals are cemented into a tubular shell to hold them together. These devices have a narrower field of view than the Glan-Thompson configuration.

The plane of polarization of a source can be rotated to orient the electric field at an arbitrary angle. One way to do this is to use a randomly polarized (unpolarized) source and rotate a Polaroid (film) in the beam. The disadvantages of Polaroids have already been discussed, and often the light, e.g. from lasers, is already polarized. To rotate the plane of polarization of polarized sources, birefringent crystals are usually used. A device called a “half-wave” plate or retarder is used to rotate plane-polarized light through a  $180^\circ$  phase angle; this device can maintain very high polarization extinction. Half-wave plates utilize three concepts. First, the superposition principle (more on this in the section on gratings) states that electric fields are additive. Second, for a birefringent crystal, the two polarization components of the incident beam will experience different refractive indices (provided that the beam is not propagating along the crystal’s optic axis) and different phase changes as they go through the crystal. Third, the crystal faces are cut parallel to the optic axis so the fast axis (lower  $n$ ,  $n_o$  in positive uniaxial crystals such as quartz) and slow axis (higher  $n$ ) are perpendicular and the crystal width can impose the largest possible phase shift for the crystal material.



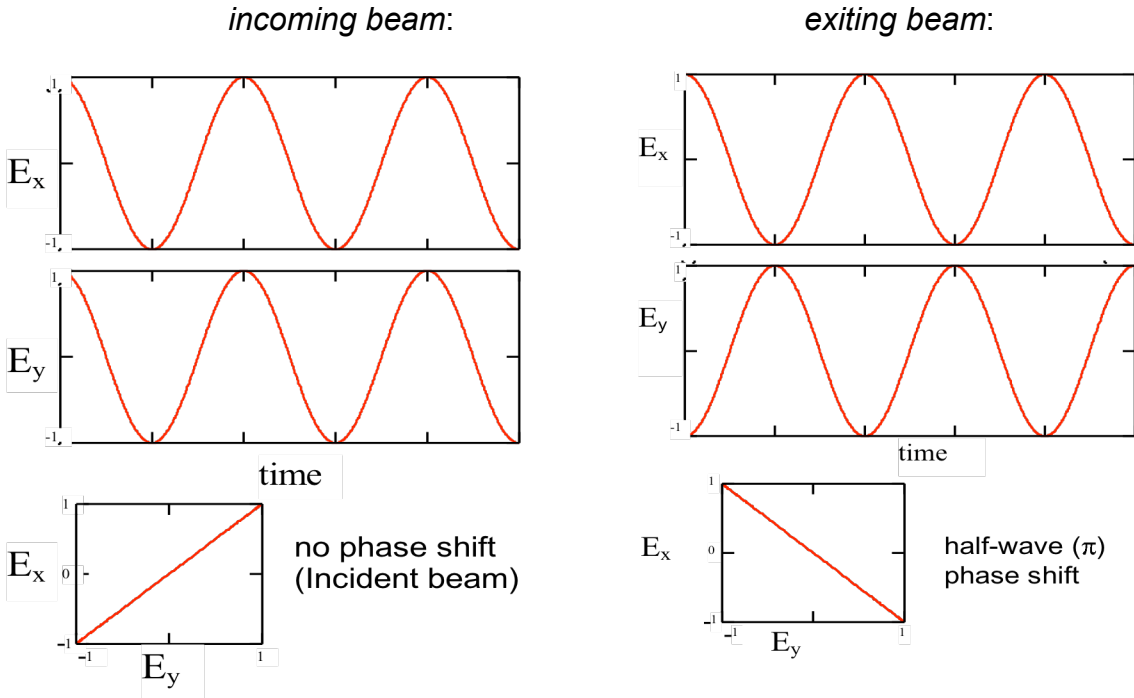
**Figure 5.4: Polarized beam incident on birefringent crystal**

Consider a beam with its polarization axis exactly halfway between the two (x and y) crystal axes incident upon a birefringent crystal, as illustrated in Figure 5.4. For the incident beam, the x- and y-components of the electric field have the same amplitude:  $E_0/2 \cdot \cos(\omega(t - n_{air}z/c))$ . Spatially, the x- and y-components are in-phase with one another (go through zero and crest at the same points along z). The graph at the bottom left of Figure 5.5 shows the electric vector resulting from the two components, which is the initial plane of polarization of  $45^\circ$ . The total electric field oscillates at a  $45^\circ$  angle between the x and y axes.

When the beam transits the birefringent crystal (not along optic axis), the x- and y-components are phase-shifted relative to one another. A half-wave plate is a crystal whose thickness is chosen, based on its refractive indices, to

phase-shift one component relative to the other by  $180^\circ$  ( $1/2 \cdot 2\pi$  radians, hence the name half-wave plate). The field components become

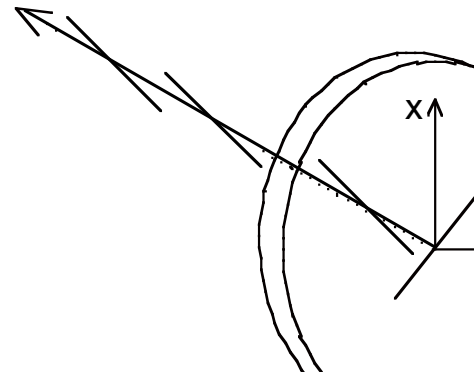
$$E_x = \frac{E_0}{2} \cos\left(\omega\left(t - n_{air} \frac{z}{c}\right)\right); \quad E_y = \frac{E_0}{2} \cos\left(\omega\left(t - n_{air} \frac{z}{c} \pm \pi\right)\right)$$



**Figure 5.5: EMR beam before entering and after exiting half-wave plate**

The graph at the bottom right of Figure 5.5 shows how the orientation of the electric field has rotated to the perpendicular direction. A view of the emerging beam, in three dimensions is shown on the right (Figure 5.6).

In summary, the key to rotating the plane of polarization to the perpendicular direction is that the half-wave plate 1) imparts exactly a  $180^\circ$  phase shift and 2) preserves the amplitudes of the x- and y-components ( $E_0/2$ ). If either factor is missing, the resulting polarization will not be perpendicular to the original polarization. It is also clear that retarders are cut for a particular wavelength because  $n_x$  and  $n_y$  are wavelength-dependent. A retarder can be made to polarize wavelengths close to the one for which it was cut by rotating the plate around its vertical axis to increase the width of the class, but this solution has limited applicability. Mica is an inexpensive birefringent material that works well but is somewhat fragile. Quartz is more long-lived but is more expensive. Quartz

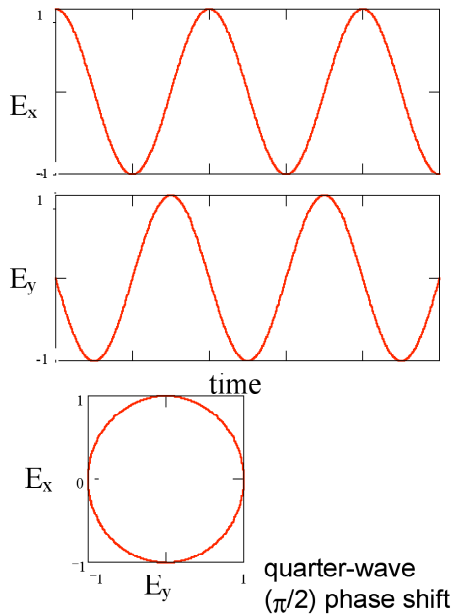


**Figure 5.6: Rotated beam emerging from birefringent crystal**

also has a stronger wavelength dependence than mica. So mica is the choice if the wavelength is to be varied. Later, we will see that a Soliel-Babinet compensator allows indefinitely broadband operation.

Circularly polarized light also can be generated from linearly polarized light by phase-shifting the x- and y-components relative to one another. Here a “quarter-wave” ( $90^\circ$ ,  $\pi/2$  shift) plate is used. After passage through the material, the electric fields are the following.

$$E_x = \frac{E_0}{2} \cos\left(\omega\left(t - n_{air} \frac{z}{c}\right)\right); \quad E_y = \frac{E_0}{2} \cos\left(\omega\left(t - n_{air} \frac{z}{c}\right) \pm \frac{\pi}{2}\right)$$

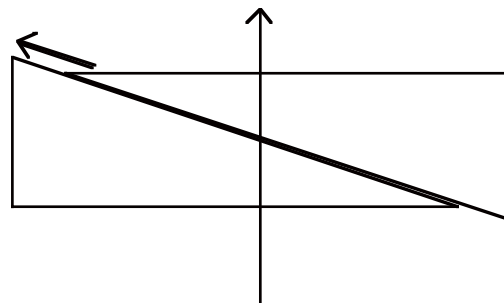


**Figure 5.7: Quarter Wave Plate**

Elliptically polarized light traces out an ellipse rather than a circle or a line. Elliptically polarized light can also be generated (accidentally) if the incoming electric field does not have equal x and y components even if the phase shift is exactly  $90^\circ$ . To generate circularly polarized light in the lab, quartz or mica quarter-wave plates are typically used, with the thickness cut for a particular wavelength as in the case of half-wave plates. For a laboratory in which broadband control of polarization is required, a Soliel-Babinet compensator can be used. This device consists of a pair of wedges that allows the thickness of the retarder to be controlled, as illustrated in Figure 5.8. (Consider this a top view.) The compensator has a fast and slow axis as any retarder would. The phase-shift can be arbitrarily varied to

Plots of  $E_x$  vs.  $E_y$  given below illustrate that circularly polarized light has a maximum amplitude electric field, but a plane of polarization that rotates in a circular fashion about the axis of propagation as a function of time. If the y-component is shifted by  $+\pi/2$ , then right-handed circularly polarized light is generated, if the phase shift is  $-\pi/2$ , then left-handed circularly polarized light is generated.

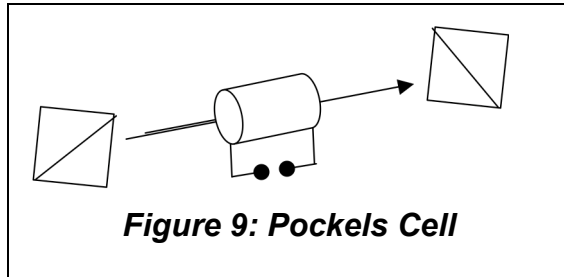
The right-hand rule is used to assign a handedness to circularly polarized light. Pointing the thumb in the direction of propagation, the fingers point to in direction of rotation of the electric field. If the phase shift of the wave-plate is not exactly  $\pm 90^\circ$ , the magnitude of the electric field changes with angle.



**Figure 5.8: Soliel-Babinet compensator**

generate either linearly or circularly polarized light over the range of the compensator.

Another way of imposing broadband polarization control is to use a device called a Pockels cell. This device is based on a birefringent crystal that is



isotropic until a voltage is applied across it. When a voltage is applied across the crystal, a birefringence, that is difference in  $n_e$  and  $n_o$  develops. The relative values of  $n_e$  and  $n_o$  can be controlled electronically in the lab with high accuracy. The additional advantage is that the Pockels cell allows easy control of polarization by a computer. When this

device is mounted between crossed polarizers, it becomes a useful source modulator: no light is observed when  $V_{\text{applied}} = 0$ , but is transmitted when a voltage is applied.

Polarized light can be analyzed (as opposed to generated) using a second polarizer, e.g., Polaroid film or Glan-Thompson prism, to select  $E_x$  or  $E_y$  for measurement. For linearly polarized light, the irradiance passed by the analyzer oscillates as the analyzer is rotated as the transmission axis of the analyzer moves toward and then away from the polarization axis of the beam. In fact,  $\mathcal{E}(\theta) = \mathcal{E}(0) \cos^2 \theta$  where  $\theta$  is the angle between the analyzer transmission axis and the polarization axis of the radiation. This procedure is not applicable to circularly polarized light because the analyzer allows the investigator to measure the amplitude of polarization components in various directions. Circularly polarized light is indistinguishable from randomly polarized light because the amplitude of the transmitted component doesn't change as the analyzer is rotated. However, in the context of an experiment, when  $\mathcal{E}_x = \mathcal{E}_y$ , after linearly polarized light is passed through a quarter-wave plate, it can be surmised that the light is circularly rather than randomly polarized. If the phase-shift must be known, then the Soliel-Babinet compensator or Pockels cell can be used to determine the phase shift needed to bring the polarization back to linear. This is sometimes necessary for experiments where the phase shift must be determined experimentally. For most purposes, the Polaroid or Glan-Thompson analyzer is sufficient.