6. Imaging: Lenses & Curved Mirrors

The basic laws of lenses allow considerable versatility in optical instrument design. The focal length of a lens, f, is the distance at which collimated (parallel) rays converge to a single point after passing through the lens. This is illustrated in Fig. 6.1. A collimated laser beam can thus be brought to a focus at a known position simply by selecting a lens of the desired focal length. Likewise, placing a lens at a distance f can collimate light emanating from a single spot.

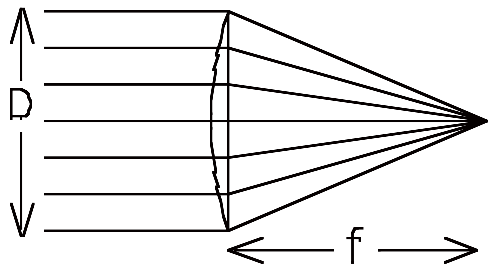

Figure 6.1: Parallel beams focused by a lens

This simple law is widely used in experimental spectroscopy. For example, this configuration allows collection of the greatest amount of light from a spot, an important consideration in maximizing sensitivity in fluorescence or Raman measurements. The collection efficiency of a lens is the ratio $\Omega/4\pi$, where Ω is the solid angle of the light collected and 4π is the solid angle over all space. The collection efficiency is related to the F-number of the lens, also abbreviated as F/# or f/n. F/# is defined as the ratio of the distance of the object from the lens to the lens diameter (or limiting aperture diameter). In Fig. 6.1 above, the f-number of lens is f/D -

$$\frac{\Omega}{4\pi} = \frac{1}{\left[4\left(F/\#\right)\right]^2}.$$

The collection efficiency increases with decreased focal length and increased lens diameter. Collection efficiency is typically low in optical spectroscopy, such as fluorescence or Raman.

On the other hand, two-dimensional imaging (as opposed to light collection) is limited when the object is at a distance of f from the lens because light from only one point in space is collected in this configuration. Two dimensional imaging is optimized when a lens is placed at a distance S_1 where S_1>f as shown in Fig. 6.2. The rays from all points at distance S_1 will be focused to corresponding points at distance S_2 by a lens constructed of material with refractive index n_l and with spherical surfaces described by radii of curvature R_1 and R_2.

$$\frac{1}{f} = \frac{1}{S_1} + \frac{1}{S_2} = \left(n_l - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Unlike situation in Fig. 6.1, which only applies to spots, this equation (the thin lens approximation to the lensmaker's equation) applies to all of the points in the plane at distance S_1. Each point in a plane (e.g., sample) is optically mapped to its corresponding position in another plane (e.g., detector). The distance relations in the lens law are illustrated in Fig. 6.2, where the rays from an object to the left of the lens at distance S_1 are focused to an image to the right of the biconvex (R_1>0, R_2>0) lens at distance S_2. The resulting image is magnified by the ratio S_2/S_1, which is explained geometrically using a tool called ray-tracing which also can be used to locate the position of the image. The principles are

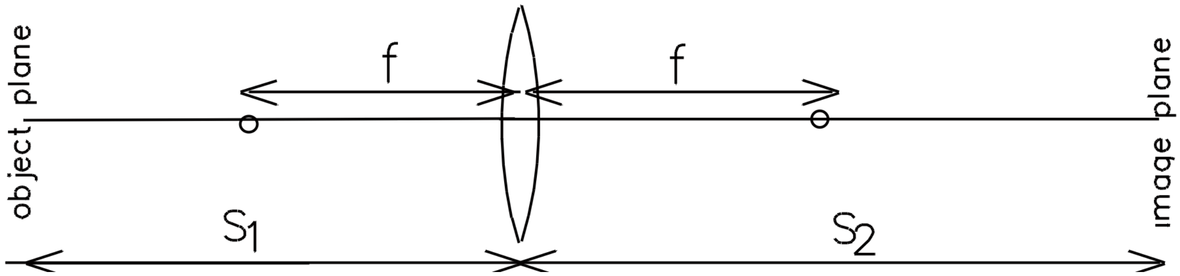

Figure 6.2: Imaging of Object Plane by thin lens

1) all rays parallel to the optical axis (center of lens) must subsequently pass through the focal point (indicated by open circles), and 2) an undeviated (chief) ray passes through the center of the lens, as shown in Fig. 6.3.

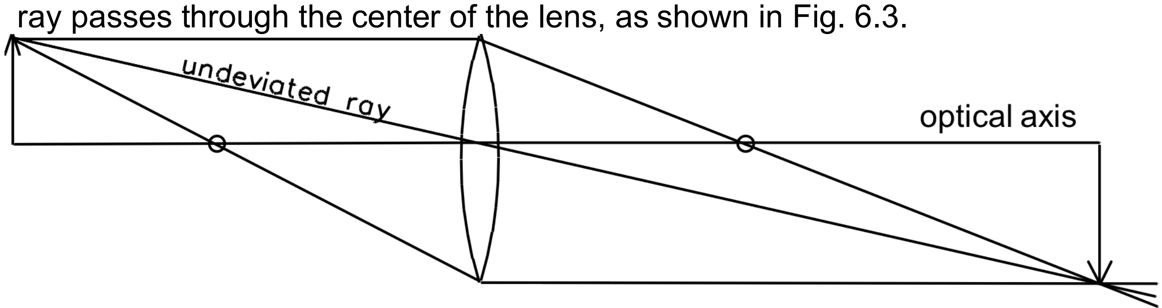

Figure 6.3: Ray Tracing

The image is inverted and increased in size because S_2>S_1. Each point on the object can be mapped to the image by ray-tracing. The simple ray-tracing shown here utilizes the three rays to indicate where the point on the object is focused in the image plane. Physically, all rays from a point in the object plane converge to the same point in the image plane. Consequently, in an ideal lens, the brightness of the image in the focal plan is the total radiant power collected by the optic divided by the source area:

$$\mathcal{E}_{image} = \frac{B_{obj}\Omega_{lens}A_{obj}}{A_{image}} = \frac{B_{obj}}{m^2}\frac{\pi/4}{(F/\#)^2}.$$

Concave lenses are used in conjunction with convex lenses in some optical applications. They diverge collimated beams, having a “virtual” image at -f, as illustrated in Fig. 6.4. It might seem odd that anyone would want to diverge a beam, but remember that they are always used in conjunction with convex lenses to result in eventual convergence. An example of an application of concave lenses is in beam expansion (see Fig. 6.5), where the concave lens saves space relative to two convex lenses. The concave lens also prevents the

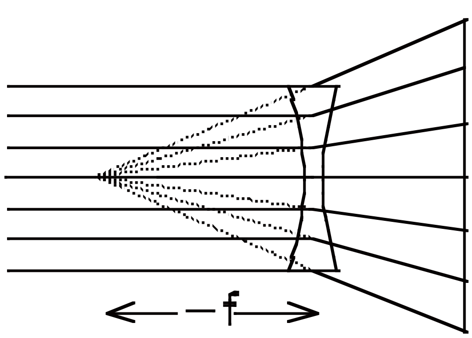

Figure 6.4: Concave Lens

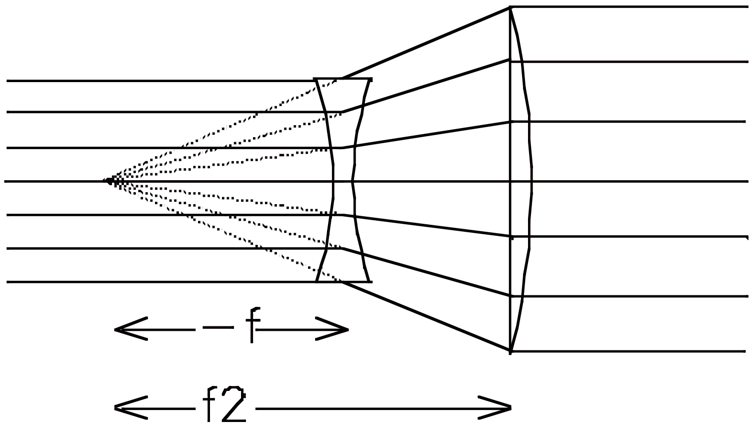

Figure 6.5: Beam Expansion

beam from coming to a real focus, which is essential when using high-power lasers because a focused laser beam can ionize air. Telescopes sometimes use such a concave mirrors to collect and focus the collimated (by the time it reaches us) light emitted by stars. Remember that concave lenses diverge beams as long as the refractive index of the lens is larger than that of the surrounding medium (air). If this were not the case, concave lenses would focus beams.

In imaging, as opposed to beam expansion or collimation, the F-number (used to calculate the collection efficiency) is now larger because S_1>f. Less light is collected. The effective F-number for the calculation of collection efficiency becomes F/#=S_1/D (rather than f/D). The need to image a plane thus reduces the collection efficiency because S_1>f. In microscopy both imaging and high collection efficiency both are achieved. Consequently, spectroscopists are using microscopes as light collection devices more frequently. Microscope objectives are typically made to work at prescribed distances S_1 and S_2, where S_1 is the "working distance" and S_2 is fixed at less than the length of the microscope barrel (160 mm), as shown to the right (Fig. 6.6). Since S_2 is fixed, S_1 depends on the magnification; the sample must be moved to bring the image into focus. What is actually happening when you focus on a specimen is that the sample is being moved to distance S_1. The eyepiece magnifying the image at S_2 is not shown.

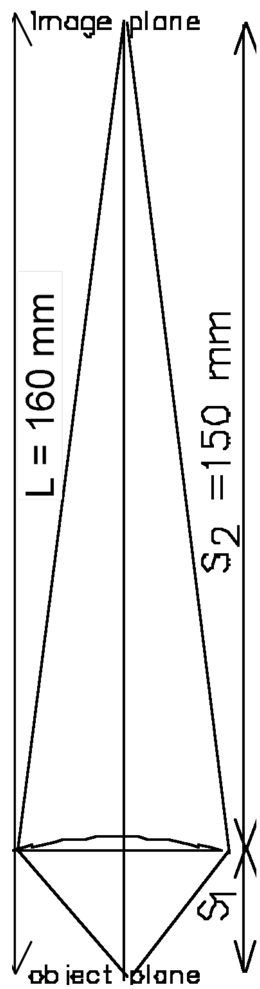

Figure 6.6: Simple Microscope Objective

It is a convention in microscopy to describe the microscope objective by its numerical aperture, NA, rather than is focal length and lens diameter because it is typically a compound lens. The numerical aperture of a lens system is the product of the refractive index of the medium (remember some objectives are immersed in oil) and the sine of the angle that a marginal ray (a ray that exits the lens system at its outer edge) makes with the optical axis (the half-angle of the maximum cone of light that can enter or exit the lens). To calculate the collection efficiency of a microscope objective, its numerical aperture and F/# are inversely related:

$$NA = n\sin\left(\tan^{-1}\left(\frac{1}{2\cdot(F/\#)}\right)\right)$$

(for low NA, NA=$(2F/\#)^{-1}$) , but F/# usually refers to photographic lenses. The maximum lateral resolution provided by a microscope depends on the numerical aperture, though there are many variations in the equation used to express it. One expression is

$$\Delta r = \frac{1.22\lambda}{2NA},$$

where Δr is the smallest distance between 2 distinguishable objects in the image. The longitudinal (z-axis) resolution is related to a property called “depth of focus” which describes the distance along the optical axis through which the image remains focused. The spot size only changes by a factor of √2 over the depth of focus, Δz:

$$\Delta z = \pi r^2/4\lambda$$

The smaller the spot size, the smaller the depth of focus. This is why it is so much harder to focus a 100x microscope objective than it is to focus a 5x or 10x objective.

To this point, this discussion has referred to idealized imaging. In reality there are some complications that must be kept in mind. First, the equations discussed here apply to the thin lens approximation, which is why we did not carefully define whether S_i is measured to the edge or to the middle of the lens. In practice, one calculates approximately where the image is expected and uses adjustable optical mounts so that the focal plane can be adjusted to the optimal position. Second, all lenses have aberrations. The most common aberration is the spherical aberration. Most lenses are sections of spheres, which do not converge rays to a perfect spot. The further the rays are from the optical axis, the worse the spherical aberration. This is illustrated above. Spherical aberration can be minimized by only using the light close to the optical axis, for example by placing an aperture in the beam. When it is not practical to limit the amount of light, e.g., when the off-axis rays are needed for sensitivity, an aspherical lens such as a parabolic or elliptical lens can be used. It is common for elliptical lenses to be used in fluorescence spectrometers to collect a large solid angle from the source with minimal aberration. If a spherical thin lens must

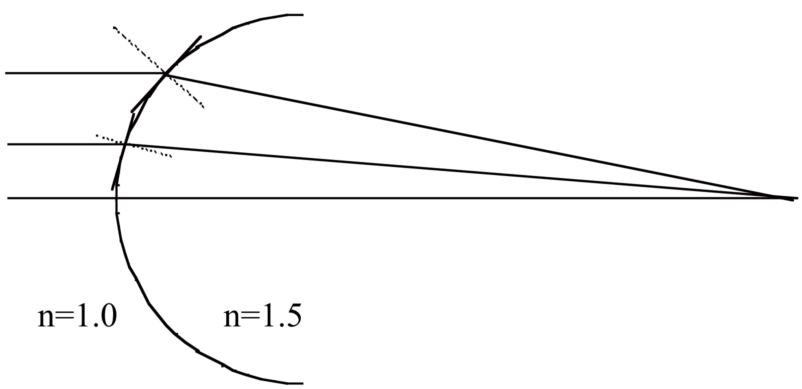

Figure 6.7: Spherical Abberation

be used, its shape determines how much aberration it contributes. A plano-convex lens produces less spherical aberration than a biconvex lens provided that the convex side faces towards the beam source. The next most common aberration is astigmatism, which occurs when a lens is tilted. When tilting cannot be avoided, it is possible to correct for the astigmatism with additional optical elements. Chromatic aberration occurs due to the wavelength dependence of the refractive index, which prevents all wavelengths from focusing to the same spot. Compound lenses are used to correct for chromatic aberration. Finally, it is possible to purchase special lenses that are free of aberrations for a specific wavelength, providing the sharpest images. These are designed by careful ray-tracing to achieve a lens shape that converges all rays to a single diffraction limited spot. The radius of this smallest possible spot size is r,

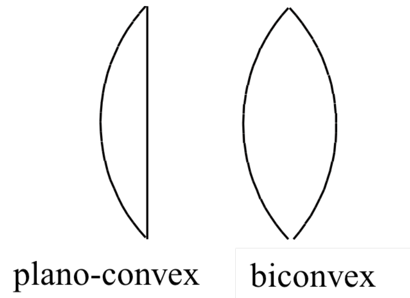

Figure 6.8: Convex Lenses

$$r = 1.22 \cdot \lambda \cdot f / w$$

where w is the 2σ radius of the incoming laser beam. The intensity profile of laser beams is considered to be Gaussian, so r is the 2σ radius of the focused spot. This diffraction-limited spot size is achieved only if the lens has been specially designed to have minimal aberrations at the wavelength and beam diameter specified.

Optical fibers are finding increased application in spectroscopy because of their ability to carry optical signals over irregular paths and into and out of remote or hazardous locations. Optical fibers are fabricated of optical materials (called the core) and coated with a lower refractive index material (called the cladding, often a polymer) to minimize light losses and reinforce the fragile core. Light propagates down the fiber by total internal reflection. Using Snell's law, it can be shown that the numerical aperture of a fiber is

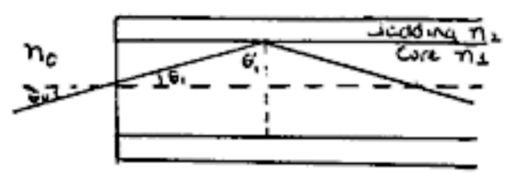

Figure 6.9: Total Internal Reflection inside a fiber optic

$$NA = n_0 \sin\theta_0 = \sqrt{n_{core}^2 - n_{clad}^2}$$

where θ_0 is the entrance angle of the light into the fiber and n_0, n_{core} and n_{clad} are the refractive indices of the external medium, fiber core and cladding, respectively. The throughput of optical fibers depends on many factors including glass (core and cladding) type, glass purity, end polishing, cladding integrity and so on. It is common (and confusing) for transmission losses in fibers to be rated in a gain rather than a loss. The gain is generally reported in decibels, another unitless quantity (remember the steradian?). The gain is given by

$$G = 10 \log \frac{\Phi_{out}}{\Phi_{in}}$$

where Φ_{in} and Φ_{out} are the radiant power accepted and released by a fiber, respectively. On the other hand, just to be sure outsiders don't find it too easy to follow what is going on, when transmission losses are defined per unit length of fiber (or bundle of fibers) this is called the fiber loss or attenuation and is given in dB/m (or dB/km) units! Inscrutable nomenclature aside, the gain and attenuations are both computed from the log transmittance through the fiber.

Curved mirrors are used rather than lenses in some spectrometers. The imaging principles are the same (the lensmaker's equation is reduced to a single surface), but mirrors eliminate reflective losses and chromatic aberrations. Laser cavities utilize curved mirrors because even small losses in laser cavities can result in large decreases in laser power. For applications where the sharpest possible images are required, such as in astronomy, the use of mirrors means that only one surface needs to be described by ray-tracing, and subsequently created by machining. Machining errors as small as microns can cause a disappointing reduction in image quality. The images collected by the Hubbell telescope before it was repaired in 1993 are notable examples. Mirror optics are also important in measurements carried out with sources that are weak or produce frequencies that are often absorbed. IR spectroscopy is a perfect example of this type of application.