Chemistry 620
Analytical Spectroscopy
PROBLEM SET 5: Due 03/20/08

1. At a recent birthday party, a young friend (elementary school) noticed that multicolored rings form across the surface of soap bubbles. Explain the source of the colored rings to your friend. Be complete and precise, but answer in clear terms avoiding jargon. Draw a diagram that illustrates your answer.

   Bubbles are ***thin films of water*** trapped between layers of soap molecules (FYI: fatty side facing air). When a light beam meets the film surface, some of it bounces off the surface, the rest travels across the film. The colored light that makes up ***white light is separated as it travels across the water film*** because the extent to which light changes direction on crossing the film is color dependent. Light beams zigzag as they travel. When two beams zig at the same point, they can combine; when one zigs where the other zags they can cancel each other. When the beams that bounce off the top and bottom of the water film interact, this ***combining & canceling of the colored rays*** reinforces a specific color, determined by film thickness, so we see a ring of that color. The ***colors and ring patterns change as the film thickness and the angle*** we observers make to the film change.

2. For a Fabry-Perot intererometer, when the cavity spacing is set to 0.540 cm, calculate three visible wavelengths close to 600 nm that would be transmitted. How large are the mode orders?

$$m\lambda = 2d\sqrt{\eta^2 - \sin^2\theta}$$
$$\eta \sim 1 \textit{ for gases}$$
$$\theta \sim 0 \textit{ for lasers}$$
$$m\lambda = 2d = 2 \cdot 0.54cm = 1.08cm = 1.08e7nm$$
$$m' = \frac{2d}{600\text{ nm}} = \frac{1.08e7nm}{600nm} = 18000$$
$$m = 17999 : \lambda = \frac{2d}{m} = \frac{1.08e7nm}{17999} = 600.0\overline{3}\,nm$$
$$m = 18000 : \lambda = \frac{2d}{m} = \frac{1.08e7nm}{18000} = 600.00\,nm$$
$$m = 18001 : \lambda = \frac{2d}{m} = \frac{1.08e7nm}{18001} = 599.9\overline{6}\,nm$$

3. Calculate the cavity spacing (d) needed to set a Fabry-Perot interferometer to pass λ = 351 nm in the 10,000th order assuming the dielectric between the reflective plates is air, the mirror reflectances are 0.925 and the beam falls on the device at normal incidence. Calculate the free spectral range, full-width at half-maximum (also the resolution) in wavelength and frequency units.

$$m\lambda = 2d\sqrt{n^2 - \sin^2\theta}$$

$$d = \frac{m\lambda}{2\sqrt{n^2 - \sin^2\theta}} = \frac{10000 \cdot 351nm}{2\sqrt{1-0}} = 1.76e3\mu m$$

$$C_F = \frac{4\rho}{(1-\rho)^2} = \frac{4 \cdot .925}{(1-.925)^2} = 657.8$$

$$F = \frac{\pi\sqrt{C_F}}{2} = 40.29 \qquad FWHM_\delta = \frac{4}{\sqrt{C_F}} = 0.63$$

$$FWHM_\lambda = \frac{FWHM_\delta}{4\pi d\lambda^{-2}} = \frac{2/\sqrt{C_F}}{\pi m\lambda^{-1}} = \frac{2/\sqrt{657.8}}{\pi 10000/351nm} = 0.00087nm$$

$$\Delta\lambda_f = F \cdot FWHM_\lambda = 40.29 \cdot 0.00087nm = 0.035nm$$

$$R_{th} = \pi\sqrt{C_F}\frac{m}{2} = \pi\sqrt{657.8}\frac{10000}{2} = 402,872$$

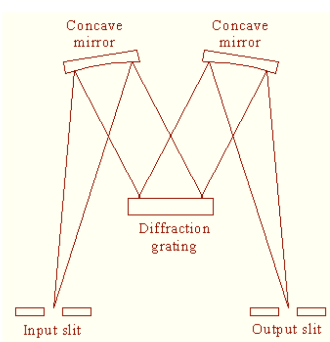
$$FWHM_\lambda = \frac{FWHM_\delta}{\delta\lambda^{-1}} = \frac{4/\sqrt{C_F}}{\delta\lambda^{-1}} = \frac{4/\sqrt{657.8}}{(2\pi 10000)/351nm} = 0.00087nm$$

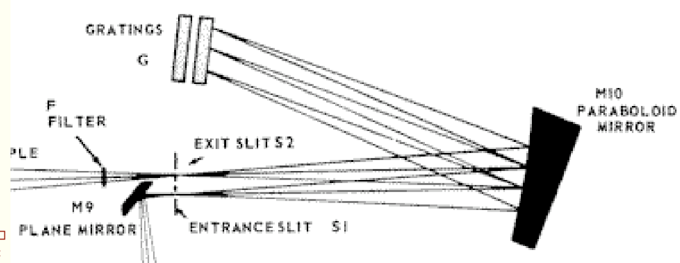
$$\Delta\lambda_f = F \cdot FWHM_\lambda = 40.29 \cdot 0.00087nm = 0.035nm$$

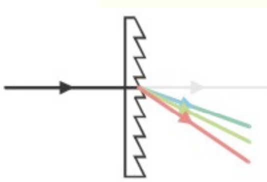
$$FWHM_\nu = \frac{FWHM_\delta}{\delta\nu^{-1}} = \frac{4/\sqrt{C_F}}{\delta\nu^{-1}} = \frac{4/\sqrt{657.8}}{(2\pi 10000)/8.547e14} = 2.12e9Hz$$

$$\Delta\nu_f = F \cdot FWHM_\nu = 40.29 \cdot 2.12e9Hz = 8.55e10Hz$$

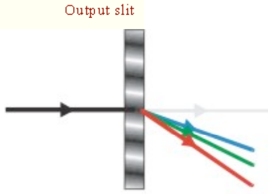
4. One of the most widely used monochrometer (spectrometer) designs is the Czernzy-Turner (CT) design. Do some investigating and make a list of up to five monochromator designs. Your list should include diagrams of each of the designs and a brief explanation of the advantages and disadvantages of the device relative to the CT design.



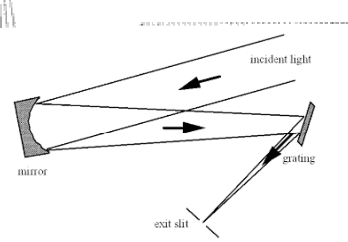


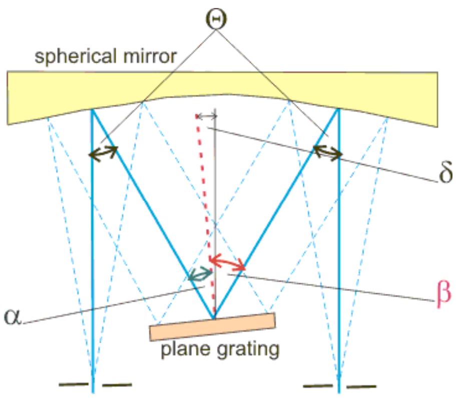


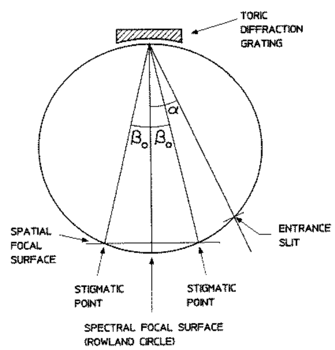
Relief grating

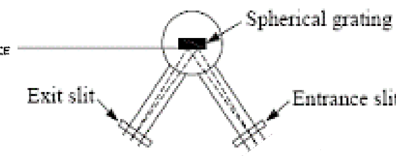


VPH grating









Cz erny-Turner: http://www.cairn-research.co.uk/Support/Technical%20Notes/Monochromator/Principles
Littrow: http://www.geocities.com/antiquesci/PE337/PE337-2.htm
Transmission grating: http://www.ppo.ca/gratings.htm
Monk-Gillieson: http://gratings.newport.com/information/handbook/chapter6.asp#6.2.4
Fastie-Ebert: http://www.thespectroscopynet.com/Index.html?/Monochromators_2.html
Rowland Circle: http://cfa-www.harvard.edu/uvcs/uvcspap/node12.html
Seya Namioka Monochromators: http://mcphersoninc.com/synchrotron/uhvmonochromators/seya.htm

Czerny-Turner: Based on 2 mirrors, 1 grating. In-line or U-turn set up possible.
Littrow: Based on single mirror, 1 or 2 gratings. Small footprint due to folded light path
Transmission grating: shorter optical path between entrance & exit, smaller footprint
Monk-Gillieson: based on single concave mirror & 1 grating; simple design.
Fastie-Ebert: Based on single mirror, so fewer aberrations, simple alignment.
Rowland Circle: Based on single grating, multiple λ may be simultaneously detected.
Seya Namioka Monochromators: Wide separation of entrance & exit slits.

5. Derive an expression for the throughput of a reflective grating monochromator, then compute the radiant power of a beam passed through the monochromator at 532 nm if the spectral radiance of the source is 0.5 W cm^{-2} sr^{-1} nm^{-1} and the features of the grating are given in the list below.

W = 2.00 mm	d = 0.5 µm $groove^{-1}$
H = 5.00 mm	$\theta_{in} = 10^o$
F/# = 3.7	f = 0.25 m

$$\Phi(\lambda) = B(\lambda)\Omega_{mono}A_p$$

$$B(\lambda) = B_0 T_{op} s_g(\lambda) = B_0 T_{op} R_d(\lambda) W$$

$$\Omega_{mono} = \frac{\pi / 4}{(F/\#)^2}$$

$$A_p = WH$$

$$\Upsilon(\lambda) = \frac{\pi/4}{(F/\#)^2} W^2 H T_{op}\left(d\frac{\cos\beta}{|m|f}\right), \text{ assuming } T_{op}=1 \text{ (optimistic)}$$

$$\sin\theta_{out} = \frac{m\lambda}{d} - \sin\theta_{in} = \frac{1\cdot 532nm}{500nm} - \sin 10 = 0.89, \theta_{out} = 62.9$$

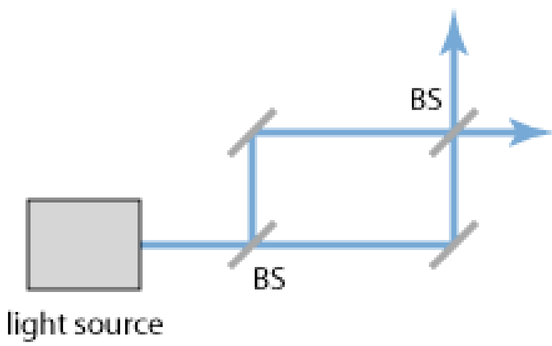
$$\Upsilon(\lambda) =$$

$$= \frac{\pi/4}{(3.7)^2}(2.00mm)^2\, 5.00mm T_{op}\left(500nm\frac{0.455}{250mm}\right)$$

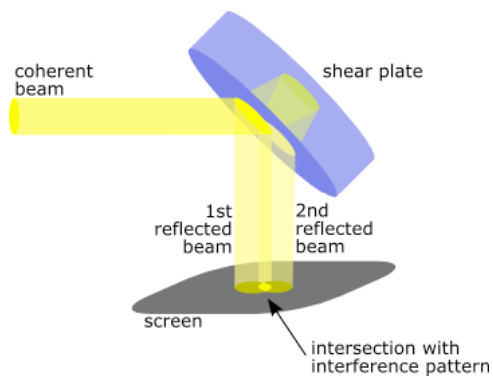
$$= 1.04mm^2 \cdot sr \cdot nm$$

$$\Phi(\lambda) = B(\lambda)\Upsilon(\lambda) = 0.5Wcm^{-2}sr^{-1}nm^{-1}\cdot 1.04e-6mm^2\cdot sr\cdot nm = 5.2e-3W$$

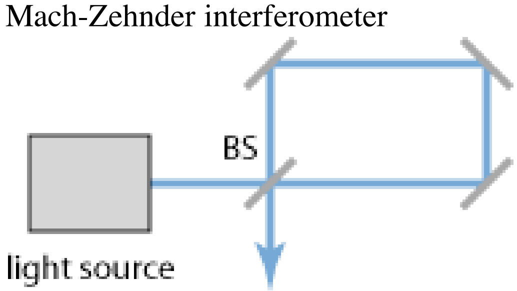
6. The Michelson interferometer is one of the most widely used interferometer designs making spectral measurements. Describe the construction, operation and relative performance of a second interferometer of your choice, excluding the Fabry-Perot.



Mach-Zehnder interferometer



Shearing interferometer



Sagnac interferometer

From http://www.rp-photonics.com/interferometers.html -
MZ uses two separate beamsplitters (BS) to split and recombine the beams, and has two outputs, which can e.g. be sent to photodetectors. The optical path lengths in the two arms will be essentially identical (see figure) when there is no sample. Samples change the distribution of optical intensity at the two outputs, the phase characterizes the sample.

From http://en.wikipedia.org/wiki/Shearing_interferometer
The shearing interferometer is an extremely simple device consisting of a single optical quality glass plate or wedge. It is used to observe interference and to use this phenomenon to test the collimation of light beams, especially from laser sources

http://www.rp-photonics.com/interferometers.html
The Sagnac interferometer uses counterpropagating beams in a ring path, realized e.g. with multiple mirrors or with an optical fiber. If the whole interferometer is rotated e.g. around an axis which is perpendicular to the drawing plane, this introduces a relative phase shift of the counterpropagating beams which is detected as an interference pattern at the ring output related to the rotation speed and changes in the pattern indicate changes in beam properties or system motion.

7. The cavity lifetime described in Section 3 was derived assuming that all losses are reflective. It is clear that scattering, diffraction and absorption losses affect cavity performance. Derive an expression for the cavity lifetime that includes such losses.

In a perfect resonant cavity, radiation persists for an infinitely long time / In real devices, scattering and absorption losses limit residence time in a cavity to a cavity lifetime. In other words the irradiance inside the cavity decays over time:

$$\mathcal{E} = \mathcal{E}_o e^{-t/\tau_c}$$

where τ_c, *the cavity lifetime, is the time required for the irradiance to decay to 37% of its initial value. Alternatively, the radiation loss can be described in terms of specific processes:*

$$\mathcal{E} = \rho_1 \rho_2 \mathcal{E}_o e^{-\alpha t}$$

where ρ_1 *and* ρ_2 *are the reflectivities of the two mirrors at the ends of the cavity and* α *is the rate of losses by absorption, diffraction and scattering. After N roundtrips in the cavity,* $t = 2NL / v = 2NL / (c/n)$, *the irradiance becomes*

$$\mathcal{E} = \mathcal{E}_o e^{-2NL/(c/n)\tau_c} = \left(\rho_1\rho_2\right)^N \mathcal{E}_o e^{-\alpha 2NL/(c/n)}.$$

Solving for the cavity lifetime yields,

$$-\frac{2NL}{(c/n)\tau_c} = \ln\left(\left(\rho_1\rho_2\right)^N e^{-\alpha 2NL/(c/n)}\right) = N\ln\left(\rho_1\rho_2\right) - \frac{2\alpha NL}{c}$$

$$-\frac{(c/n)\tau_c}{2NL} = \frac{1}{N\ln\left(\rho_1\rho_2\right)} - \frac{(c/n)}{2\alpha NL}$$

$$\tau_c = -\frac{2L/(c/n)}{\ln\left(\rho_1\rho_2\right)} + \frac{1}{\alpha} = \frac{2L/(c/n)}{\ln\left(1/\rho_1\rho_2\right)} + \frac{1}{\alpha}$$

See http://hyperphysics.phy-astr.gsu.edu/Hbase/optmod/lascav.html for similar treatment of case without absorption.

8. One of your responsibilities in your new position at Wonderful Instrument Company is to interface with the heads of other departments as the company prepares to release products. Please respond to the following memo.

September 15, 2007

To: Leader, Spectrometer Development Group
From: V.P., Marketing
Re: Resolution

We have been working with our advertising consultants on the layouts for the product brochures and print media ads. The documentation sent by your people uses the terms resolution and resolving power. While it is clear that they are not exactly the same thing, they seem related. What is the distinction between these two terms? Please advise us on when and how to use these two terms correctly. Thank you for your prompt response.

September 17, 2007

To: V.P., Marketing
From: Leader, Spectrometer Development Group
Re: Resolution

Great question. They are distinct. Resolution measures how widely separated adjacent bands are in wavelength (or related) units. It is the separation between minimally distinguishable bands in a spectrum. Therefore it also measures how broad the spectral bands resolved by a spectrometer are. Use this parameter to describe data quality. The resolving power is a unitless score that reports how well a device resolves close bands. Use this parameter to compare devices or configuration settings and changes of a single device.