Chemistry 620
Analytical Spectroscopy
PROBLEM SET 4: Due 03/13/08

1. The human eye is a lens that focuses images on the retina, the screen of the eye. Suppose that the normal focal length of this lens, that is the position in which far away objects are focused on the retina, is 3.5 cm. The eye is able to focus on nearby objects by changing the shape of the lens, and thus its focal length. Calculate the focal length the eye would need to adopt in order for an object 17.5 cm from the eye to be in focus on the retina?

$$\frac{1}{f} = \frac{1}{S_o} + \frac{1}{S_{retina}}$$

$$\frac{1}{3.5cm} = \frac{1}{\infty} + \frac{1}{S_{retina}}, S_{retina} = 3.5cm$$

$$\frac{1}{f} = \frac{1}{17.5cm} + \frac{1}{3.5cm} = \frac{1}{2.9cm}$$

2. A point source of radiant intensity 2.0 Wsr^{-1} is placed at the focal point of a 0.5 cm diameter, f/4.0 lens. What is the focal length of the lens? What is the solid angle collected by the lens? What is the collection efficiency of the lens? What is the irradiance of a beam on a sample placed 0.5 m from the lens?

$$f = (F/\#)_{lens} D_{lens} = 4.0 \cdot 0.5\,cm = 2.0\,cm$$

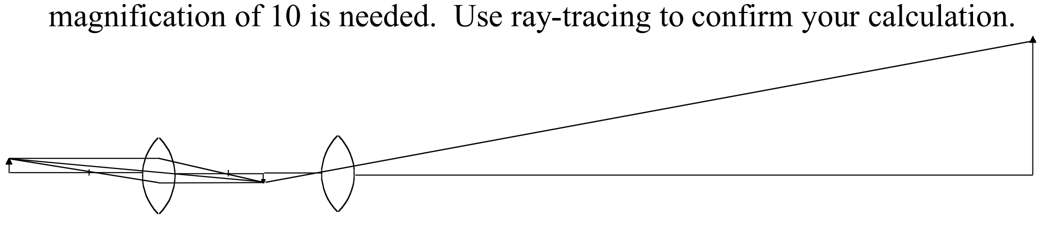
$$\Omega_{lens} = \frac{\pi/4}{(F/\#)^2} = \frac{\pi/4}{16} = 0.049\ sr$$

$$\frac{\Omega_{lens}}{4\pi} = \frac{\pi}{16 \cdot 16 \cdot \pi} = 0.0039\,(0.4\%)$$

$$A_{image} = A_{lens} = \pi\left(\frac{D_{lens}}{2}\right)^2 = \pi\left(\frac{0.5\ cm}{2}\right)^2 = 0.196\,cm^2$$

$$E_i = \frac{\Phi_{image}}{A_{image}} = \frac{I_{source}\Omega_{lens}}{A_{image}} = \frac{2.0Wsr^{-1}0.049\ sr}{0.196\,cm^2} = 0.5Wcm^{-2}$$

3. Using a pair of lenses both having a focal length of 2" and an object position 5" from the first lens, calculate the position needed for the second lens if an overall magnification of 10 is needed. Use ray-tracing to confirm your calculation.



$$\frac{1}{f}=\frac{1}{S_{o1}}+\frac{1}{S_{i1}} \quad \frac{1}{2"}=\frac{1}{5"}+\frac{1}{S_{i1}} \quad 0.3=\frac{1}{S_{i1}} \quad S_{i1}=3.33"$$

$$m_1=-\frac{S_{i1}}{S_{o1}}=\frac{3.\overline{3}"}{5.0"}=0.\overline{6}$$

$$m_{12}=m_1m_2 \quad 10=0.\overline{6}m_2 \quad m_2=15$$

$$\frac{1}{f}=\frac{1}{S_{o2}}+\frac{1}{S_{i2}} \quad \frac{1}{f}=\frac{1}{S_{o2}}+\frac{1}{15S_{o2}} \quad \frac{1}{f}=\frac{16}{15S_{o2}} \quad S_{o2}=2.1\overline{3}"$$

$$d=S_{i1}+S_{o2}=3.33"+2.1\overline{3}"=5.47"$$

4. The objective and the eyepiece of a microscope each have a focal length of 2.0 cm. If an object is placed 2.2 cm from the objective, calculate (a) the distance between the lenses when the microscope is adjusted for minimum eyestrain, (b) the magnification of the microscope, and (c) the collection efficiency of the objective.

$$\frac{1}{f_{obj}}=\frac{1}{S_{oo}}+\frac{1}{S_{oi}},\frac{1}{2}=\frac{1}{2.2}+\frac{1}{S_{oi}}\Rightarrow\frac{1.1}{2.2}-\frac{1}{2.2}=\frac{.1}{2.2}=\frac{1}{S_{oi}},S_{io}=22cm$$

$$\frac{1}{f_{eye}}=\frac{1}{S_{eo}}+\frac{1}{S_{ei}},\frac{1}{2}=\frac{1}{2}+\frac{1}{\infty},S_{eo}=2cm$$

$$a) \quad d=S_{io}+S_{eo}=22cm+2cm=24cm;$$

Since the image produced by the objective is real, the magnification it provides is simply S_o/S_i. In the case of the eyepiece, the image distance is infinitely far away, so the angular magnification, which measures the ratio of the apparent (angular) size of the object viewed through the eyepiece and the apparent size of the object held at a standard distance from the eye, usually 25 cm. Happily, the angular sizes are proportional to the distances for most objects.

$$b) \quad M_{uscope}=M_{obj}M_{eye}=\left(-\frac{S_i}{S_o}\right)\left(\frac{250\,mm}{f_{eye}}\right)=\left(-\frac{220\,mm}{22\,mm}\right)\left(\frac{250\,mm}{20\,mm}\right)=-125$$

Using the magnification as a guide to estimate the NA (http://www.microscopy.fsu.edu/primer/anatomy/numaperture.html). High NA objectives require immersion oil, n>1.

$$c) \quad \frac{\Omega}{4\pi}=\frac{1}{16(F/\#)^2}=\frac{1}{16\left(2\cdot\tan\left(\sin^{-1}\left(\frac{NA}{n}\right)\right)^{-1}\right)^2}=\frac{1}{16\left(\left(2\tan\left(\sin^{-1}\left(\frac{1.2}{1.4}\right)\right)\right)^{-1}\right)^2}=0.10$$

5. If you focus the beam from a 50 mW Ar+ laser with a 3 mm beam waist to a diffraction-limited spot using a lens with a 50 mm focal length, what would be the spot size and the irradiance of the beam on the spot?

$$r_{spot} = 1.22\lambda f / w = 1.22 \cdot 514\,nm \cdot \frac{50mm}{3mm} = 10.5\mu m (10.0\mu m)_{488}$$

$$A_{spot} = \pi r_{spot}^2 = 3.43e2\mu m^2 \frac{10^4\,cm^2}{10^{12}\,\mu m^2} = 3.43e-6cm^2 (3.12e-6cm^2)_{488}$$

$$\mathcal{E} = \frac{\Phi_{beam}}{A_{spot}} = \frac{50mW}{3.43e-6cm^2} = 1.46e4Wcm^{-2} (1.60e4Wcm^{-2})_{488}$$

6. Fiberoptics are replacing lenses in some spectroscopic instrumentation because they can transmit light over long and/or irregular paths. If a 40 mW source beam is launched down a 50 µm diameter fiber that has n_{core} and n_{clad} equal to 1.52 and 1.43 and the attenuation of the fiber is 0.25 dBm^{-1} what is radiant flux exiting a 2.0 m segment of fiber?

$$\theta_0 = \sin^{-1}\left(\frac{\sqrt{\eta_{core}^2 - \eta_{clad}^2}}{\eta_0}\right) = \sin^{-1}\left(\frac{\sqrt{1.52^2 - 1.43^2}}{1}\right) = 20.1$$

$$F/\# = \left(2\tan\theta_0\right)^{-1} = 1.36;$$

$$\Omega = \frac{\pi/4}{\left(F/\#\right)^2} = 0.424sr$$

$$G = KL = 10\log_{10}\frac{\Phi_{out}}{\Phi_{in}} \Rightarrow \Phi_{out} = \Phi_{in}10^{-\frac{K}{10}L}$$

$$\Phi_{out} = \Phi_{in}10^{-\frac{K}{10}L} = I\Omega 10^{-\frac{K}{10}L} = \left(\frac{40mW}{4\pi sr}\right)\cdot 0.424sr \cdot 10^{-\frac{0.25\frac{db}{m}\cdot 2m}{10}} = 1.20mW$$

7. Compute the acceptance cone of the fiber described in question #6 and the radiant power exiting the 2 m fiber segment if the laser beam and lens in question #5 is focused on the end of the fiber.

$$A_{spot}^{laser} = \pi r_{spot}^2 = 3.1416\cdot\left(10.5\mu m\right)^2 \Rightarrow 3.43e-6cm^2$$

$$A_{spot}^{fiber} = \pi r_{spot}^2 = 3.1416\cdot\left(25\mu m\right)^2 \Rightarrow 19.6e-6cm^2$$

$$\theta_0 = \sin^{-1}\left(\frac{\sqrt{\eta_{core}^2 - \eta_{clad}^2}}{\eta_0}\right) = \sin^{-1}\left(\frac{\sqrt{1.52^2 - 1.43^2}}{1}\right) = 20.1$$

$$\Phi_{out} = \Phi_{in}10^{-\frac{K}{10}L} = \mathcal{E}A10^{-\frac{K}{10}L} = 1.46e4Wcm^{-2}\cdot 3.43e-6cm^2 \cdot 10^{-\frac{0.25\frac{db}{m}\cdot 2m}{10}} = 44.6mW$$

8. One of the issues raised in *Chi* is the fact that placing a lens a focal length away from a source will collimate the light but collect only a small spot. On the other hand, placing the lens further away from the source images more of the source but with lower collection efficiency. Estimate the collection efficiency and image brightness of a broadband white light 0.25Wsr^{-1}cm^{-2} extended source collected by a 1.5 cm diameter lens with a focal length equal to 7.5 cm placed 15 cm away from the source. Compare this to the collection efficiency when the lens is moved to the focal length, assuming the source area viewed is the size of a diffraction limited spot. Where is the source image in this case?

$$\Omega_{lens} = \frac{A}{d^2} = \frac{\pi/4}{(F/\#)^2} = \frac{\pi/4}{(S_1/D)^2} = \frac{\pi D^2/4}{S_1^2} \text{ also can be } \frac{\pi/4}{(f/D)^2}$$

$$D = 1.5\ cm \quad f = 7.5\ cm \quad S_1 = 15cm$$

$$\Omega_1(F/\# = 10) = \frac{\pi/4}{(15\ cm/1.5\ cm)^2} = 7.85e-3sr$$

$$\frac{\Omega}{4\pi} = \frac{1}{[4(F/\#)]^2} = \frac{1}{16(10)^2} = 6.25e-4$$

$$\mathcal{E}_{image}(F/\# = 10) = \frac{B_{obj}}{m^2}\frac{\pi/4}{(F/\#)^2} = \frac{0.25Wsr^{-1}cm^{-2}}{1}\frac{\pi/4\,sr}{(10)^2} = 1.96e-3Wcm^{-2}$$

$$D = 1.5\ cm \quad f = 7.5\ cm \quad S_1 = 7.5\ cm$$

$$\Omega_2(F/\# = 5) = \frac{\pi/4}{(7.5\ cm/1.5\ cm)^2} = 3.14e-2sr$$

$$\frac{\Omega}{4\pi} = \frac{1}{[4(F/\#)]^2} = \frac{1}{16(5)^2} = 2.5e-3, \text{ Image at infinity}$$

Since image is infinitely far, how to estimate m? Using the notions that the lens is collecting light from a diffraction limited spot on the source surface and that the reddest beam forms the largest spot :

$$\mathcal{E}_{image}(F/\# = 5) = \frac{B_{obj}}{m^2}\frac{\pi/4}{(F/\#)^2} = \frac{B_{obj}}{\left(\frac{A_{image}}{A_{obj}}\right)^2}\frac{\pi/4}{(F/\#)^2} =$$

$$= \frac{0.25Wsr^{-1}cm^{-2}}{\left(\frac{\pi(1.5\ cm)^2}{\pi(1.22\cdot 800nm\cdot 7.5\ cm/1.5\ cm)^2}\right)}\frac{\pi/4\,sr}{(5)^2} = 8.31e-10\,Wcm^{-2}$$

While the notion that light is passed from a diffraction limited spot is extreme, this calculation shows that improved collection efficiency doesn't necessarily mean increased throughput.