Chemistry 620
Analytical Spectroscopy
PROBLEM SET 3: Due 03/06/08

1. Derive an expression for the transmittance of a monochromatic beam propagating normal to dielectric interface from the expression for the reflectance. Assume neither of the media absorbs the radiation. Use the expression to compute the transmittance of a normal beam across an air/glass (n_{glass} = 1.55) interface.

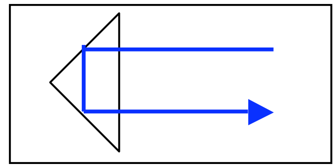
$$\tau = 1-\rho = 1-\frac{(n_i-n_t)^2}{(n_i+n_t)^2} = \frac{(n_i+n_t)^2}{(n_i+n_t)^2} - \frac{(n_i-n_t)^2}{(n_i+n_t)^2}$$
$$= \frac{n_i^2+2n_in_t+n_t^2}{(n_i+n_t)^2} - \frac{n_i^2-2n_in_t+n_t^2}{(n_i+n_t)^2}$$
$$= \frac{4n_in_t}{(n_i+n_t)^2} = \frac{4\cdot1\cdot1.5}{(1+1.5)^2} = \frac{6}{6.25} = 0.96$$

2. By what angles are the two highest gain lines of the Ar+ laser ($\lambda_{bl/gr}$ = 488.0 nm & λ_{gr} = 514.5 nm) refracted upon entering (from air) a dielectric medium that has a number density equal to 7×10^{27} m^{-3} and natural (resonance) frequency equal to 5.4×10^{15} Hz at an incident angle of 20^o? Assume that there is no absorption.

Assuming the dielectric is ideal:

$$n^2 = 1+\frac{Ne^2}{m_e\varepsilon_0(\omega_0^2-\omega^2)}$$
$$\omega_{bl} = 2\pi c/\lambda_{bl} = 2\pi\cdot2.998e8ms^{-1}/4e-7m = 4.709e15s^{-1}$$
$$\omega_{rd} = 2\pi c/\lambda_{rd} = 2\pi\cdot2.998e8ms^{-1}/8e-7m = 2.355e15s^{-1}$$
$$n_{bl}^2 = 1+\frac{7e27\frac{1}{m^3}\cdot2.56e-38C^2}{9.11e-31kg\cdot8.8542e-12\frac{C^2}{Nm^2}\left((5.4e15s^{-1})^2-(4.709e15s^{-1})^2\right)}$$
$$= 1+\frac{17.92e-11\frac{C^2}{m^3}}{8.066e-42kg\frac{C^2}{kgm^3s^{-2}}\left(2.916e31s^{-2}-2.217e31s^{-2}\right)}$$
$$= 1+\frac{17.92e-11}{8.066e-42(0.699e31)} = 1+\frac{17.92e-11}{5.38e-11} = 1+3.33;\quad n_{bl} = 2.08$$
$$n_{rd}^2 = 1+\frac{7e27\frac{1}{m^3}\cdot2.56e-38C^2}{9.11e-31kg\cdot8.8542e-12\frac{C^2}{Nm^2}\left((5.4e15s^{-1})^2-(2.355e15s^{-1})^2\right)}$$
$$= 1+\frac{17.92e-11}{8.066e-42(2.361e31)} = 1+\frac{17.92e-11}{19.04e-11} = 1+0.94;\quad n_{rd} = 1.39$$
$$\theta_t = \sin^{-1}\left(\frac{n_i}{n_t}\sin\theta_i\right);\theta_{bl} = \sin^{-1}\left(\frac{1.003}{2.08}\sin20\right) = 9.5;\ \theta_{rd} = \sin^{-1}\left(\frac{1.003}{1.39}\sin20\right) = 14.3$$

3. A 45-45-90 prism is used to totally reflect light as shown in the figure on the right. What is the minimum index of refraction of the prism needed for this to work (assuming the prism is used in air)? Illustrate (Draw) and compute what would happen if this device were used to point the dual wavelength Ar+ laser beam described in question #2.



$$\theta_c \geq \sin^{-1}\left(\frac{n_2}{n_1}\right) \Rightarrow \theta_c \geq \sin^{-1}\left(\frac{n_{air}}{n_{prism}}\right)$$

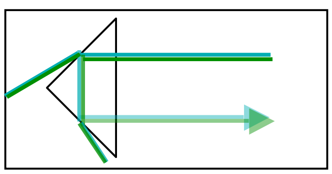
In this prism θ_i=45, assuming the prism sits in air

$$n_{prism} \geq \frac{n_2}{\sin\theta_c} \geq \frac{1.003}{\sin 45} = 1.42$$

If an Ar^+ beam is launched into a $n_{prism} \geq 1.42$ prism, both beams will be reflected.

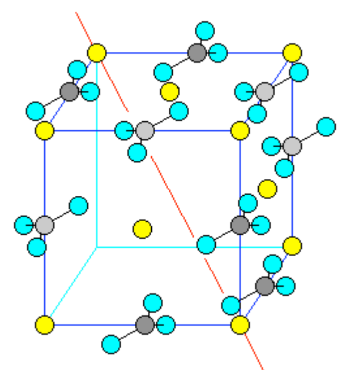
If an Ar^+ beam is launched into a $n_{prism} < 1.42$ prism, both beams will be refracted.

$$\theta_r = \sin^{-1}\left(\frac{n_{prism}}{n_{air}}\sin\theta_i\right) = \sin^{-1}\left(\frac{1.39}{1.003}\sin 45\right) = 78.5$$



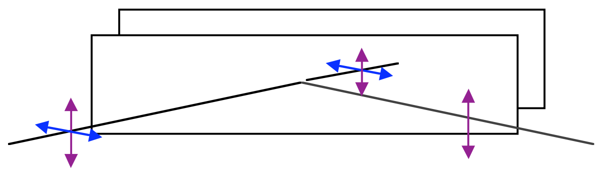
Beam loss at interfaces enhanced for clarity

4. Calcite has a large birefringence because the carbonate groups lie in parallel planes normal to the optic axis. Explain why the polarization of the carbonate group will be less when the electric field is polarized parallel to the optic axis than perpendicular to it. What does this imply about a wave's speed in calcite when its electric field is linearly polarized parallel or perpendicular to the optic axis. Figure source: http://www.uwgb.edu /DutchS/GRAPHIC0/ROCKMIN/ATOM-STRUCT/Calcite-NaCl.gif



The carbonate group is flat, so changes in electron distribution and crystal polarization will occur primarily in the plane parallel to the carbonyl bond(s) which is perpendicular to the optic axis. Perpendicular electric fields overlap and interact substantially with the carbonate groups. Electric fields polarized parallel to the optic axis attempt to push electrons out of the plane of the molecule, so not much happens. The implication is that the speed of waves with the electric field polarized perpendicular to the optic axis (parallel to carbonate plane) is slower (thus the refractive index is larger) than the speed for parallel waves in calcite.

5. One cheap way to make a polarizer is to use a stack of N microscope slides. The problem is that even assuming negligible absorption the transmitted light is dim. Use conservation laws to show that the total transmittance of a stack of N slides is $T=(1-\rho)^{2N}$ for normal incidence. Derive an expression for transmission through the stack inclined at Brewster's angle. Derive an expression for the extinction ratio for the stack at Brewster's angle



$$\rho_{\parallel}=\left(\frac{\tan(\theta_i-\theta_t)}{\tan(\theta_i+\theta_t)}\right)^2 \qquad \rho_{\perp}=\left(\frac{\sin(\theta_i-\theta_t)}{\sin(\theta_i+\theta_t)}\right)^2$$

$$T_{\parallel}=\left(\frac{n_t\cos\theta_t}{n_i\cos\theta_i}\right)\left(\frac{2\sin\theta_t\cos\theta_i}{\sin(\theta_i+\theta_t)\cos(\theta_i-\theta_t)}\right)^2 \qquad T_{\perp}=\left(\frac{n_t\cos\theta_t}{n_i\cos\theta_i}\right)\left(\frac{2\sin\theta_t\cos\theta_i}{\sin(\theta_i+\theta_t)}\right)^2$$

At Brewster's angle, $\theta_B+\theta_t=90$ and $\theta_t=90-\theta_B$. Consequently,

$$\rho_{\parallel}=\left(\frac{\tan(2\theta_B-90)}{\tan(90)}\right)^2=0 \qquad \rho_{\perp}=\frac{\sin^2(2\theta_B-90)}{\sin^2(90)}=\cos^2(2\theta_B)$$

$$T_{\parallel}=1-\rho_{\parallel}=1 \qquad T_{\perp}=1-\rho_{\perp}=\sin^2(2\theta_B)$$

After one interface

$$\mathcal{E}_{\parallel,T}=T_{\parallel}\mathcal{E}_{\parallel,0}=(1-\rho_{\parallel})\mathcal{E}_{\parallel,0} \qquad \mathcal{E}_{\perp,T}=T_{\perp}\mathcal{E}_{\perp,0}=(1-\rho_{\perp})\mathcal{E}_{\perp,0}$$

After two interfaces

$$\mathcal{E}_{\parallel,T}=T_{\parallel}T_{\parallel}\mathcal{E}_{\parallel,0}=(1-\rho_{\parallel})^2\mathcal{E}_{\parallel,0} \qquad \mathcal{E}_{\perp,T}=T_{\perp}T_{\perp}\mathcal{E}_{\perp,0}=(1-\rho_{\perp})^2\mathcal{E}_{\perp,0}$$

After N slides (two interfaces each)

$$\mathcal{E}_{\parallel,T}=(T_{\parallel}T_{\parallel})^N\mathcal{E}_{\parallel,0}=(1-\rho_{\parallel})^{2N}\mathcal{E}_{\parallel,0} \qquad \mathcal{E}_{\perp,T}=(T_{\perp}T_{\perp})^N\mathcal{E}_{\perp,0}=(1-\rho_{\perp})^{2N}\mathcal{E}_{\perp,0}$$

After N plates, the fraction of the beam polarized $\perp$ has been attenuated, the parallel part of the beam has not since $T_{\parallel}=1$. So the transmittances are

$$T_{\parallel}^{2N}=1; \qquad T_{\perp}^{2N}=(1-\rho_{\perp})^{2N}=\left(\sin^2(2\theta_B)\right)^{2N}$$

The light transmission expressions are

$$\mathcal{E}_{\parallel}=T_{\parallel}^{2N}\mathcal{E}_{\parallel,0} \qquad \mathcal{E}_{\perp}=T_{\perp}^{2N}\mathcal{E}_{\perp,0}=(1-\rho_{\perp})^{2N}\mathcal{E}_{\perp,0}=\left(\sin^2(2\theta_B)\right)^{2N}\mathcal{E}_{\perp,0}$$

The extinction coefficient expression, assuming an unpolarized source ($\mathcal{E}_{\parallel,0}=\mathcal{E}_{\perp,0}$) is

$$\frac{\mathcal{E}_{\parallel}}{\mathcal{E}_{\perp}}=\frac{T_{\parallel}^{2N}\mathcal{E}_{\parallel,0}}{T_{\perp}^{2N}\mathcal{E}_{\perp,0}}=\frac{1}{\left(\sin^2(2\theta_B)\right)^{2N}}=\frac{1}{\left(\sin^2\left(2\left(\tan^{-1}\frac{n_{slide}}{n_{air}}\right)\right)\right)^{2N}}$$

6. Suppose n_{fast}=1.47 and n_{slow}=1.51. Calculate the smallest thickness that a crystal could be cut to make a quarter wave plate for 532 nm light.

$$k_x d = 2\pi \qquad k_y d = \frac{5}{2}\pi$$

$$\frac{\omega n_x}{c} d = 2\pi \qquad \frac{\omega n_y}{c} d = \frac{5}{2}\pi$$

$$\frac{2\pi}{\lambda} n_x d = 2\pi \qquad \frac{2\pi}{\lambda} n_y d = \frac{5}{2}\pi$$

$$\frac{2\pi}{\lambda} n_x d - \frac{2\pi}{\lambda} n_y d = 2\pi - \frac{5}{2}\pi$$

$$\frac{2\pi}{\lambda}\left(n_x - n_y\right) d = \frac{\pi}{2}$$

$$\frac{4}{\lambda}\left(n_x - n_y\right) d = 1$$

$$4\left(n_x - n_y\right) d = \lambda$$

$$d = \frac{\lambda}{4\left(n_x - n_y\right)} = \frac{532nm \cdot \frac{10^6\,\mu m}{10^9\,nm}}{4\left|1.47 - 1.51\right|} = 3.325\mu m$$

7. Compute the irradiance of a randomly polarized beam transmitted by a system composed of a perfect linear polarizer with its transmission axis set at 20° to vertical placed between a pair of crossed polarizers (perpendicular transmission axes) if the source power is 50 mW and the beam diameter at the source, 25 mm, is smaller than the diameter of any of the polarizers. (You should assume that the polarizers have no effect on the beam size.) How much light would be observed if the perfect linear polarizer were a quarter-wave plate instead?

Beam irradiance on any surface larger than beam waist is

$$\mathcal{E} = \frac{\Phi}{A} = \frac{4\Phi}{\pi d^2} = \frac{4 \cdot 50mW}{\pi \cdot (2.5cm)^2} = 10.2\frac{mW}{cm^2}.$$

Before the linear polarizer is placed between the crossed polarizers (#1 & #3), no light can pass through the system. After the linear polarizer is in place, Polarizer #1 accepts half the incident light. The linear polarizer (#2) accepts almost 90% of the light passed by the vertical polarizer ($\cos^2 20$=0.88). The transmitted ray vibrating at 20° can be viewed as the combination of vertical and horizontal components. The horizontal polarizer passes the horizontal component, which is just over one tenth the intensity of the beam ($\cos^2 70$=0.12). The output beam is

$$\mathcal{E}_{out} = T\mathcal{E}_{in} = 0.5 \cdot 0.88 \cdot 0.12 \cdot 10.2\frac{mW}{cm^2} = 0.54\frac{mW}{cm^2}$$

The light intensity observed through a quarter wave plate depends on the angle the incident light makes with its optic axis. If the vertically polarized beam transmitted by the first polarizer is oriented parallel or perpendicular to the optic axis, it is still vertically polarized and the output intensity is zero. If it is between these, say at 45^{o}, the linearly polarized light becomes circularly polarized. The transmission of the horizontal component of the circularly polarized light will be transmitted.

$$\mathcal{E}_{out} = T\mathcal{E}_{in} = 0.5 \cdot \cos^2 q \mathcal{E}_{in} = 0.5 \cdot 0.5 \cdot 10.2 \frac{mW}{cm^2} = 2.55 \frac{mW}{cm^2}$$

8. Suppose you were given only a linear polarizer and a half-wave plate. How would you determine which was which, assuming you all you have is a source of natural (unpolarized) light.

   The linear polarizer only passes a fraction of the source light, while the quarter wave plate has no apparent impact on light intensity (only relative phases changed). So the item that passes more light is the half wave plate.