Chemistry 620
Analytical Spectroscopy
PROBLEM SET 1: Due 02/21/08

1. Even if you've never studied lasers before, you've encountered theory that explains why a laser system must be based on an active medium that has more than two energy levels. Think about this and explain this fact. Describe the advantages of a four-level laser system over a three-level system.

   The relative population of two states in equilibrium is described by the Boltzmann distribution
   $$\frac{N_2}{N_1} = \exp\left(\frac{E_1 - E_2}{kT}\right)$$
   In this regime, a persistent population inversion would require a negative absolute temperature and is therefore impossible. Another way to say this is that in a 2-level system, the rates of absorption and stimulated and spontaneous emission must be balanced, preventing population inversion. It is more difficult to maintain a population inversion in a 3-level system than in a 4-level system because lasing repopulates the lower energy level, reducing the population inversion. In a 4-level system, the population inversion is facilitated by the fast non-radiative deactivation process, which keeps the population of the lower state low, returning atoms or molecules to the ground state from which they can be re-excited to the lasing transition.

2. Consider a ruby laser made with a 10 cm long ruby rod and mirrors of reflectivity 0.99 and 0.74. (Ruby is Al_2O_3 doped with Cr.) Assume that the Cr concentration is 1.58 x 10^{19} cm^{-3} and the transition (absorption/stimulated emission) cross section is 1.27 x 10^{-20} cm^2 at the lasing wavelength of 694 nm.

   a) Calculate the transmittance of the ruby rod in the absence of the pump.
   $$T = \exp\left(-\sigma\ell[Cr]\right) = \exp\left(-1.27e-20cm^2 \cdot 10cm \cdot 1.58e19cm^{-3}\right)$$
   $$= \exp\left(-2.0066\right) = 0.134$$

   b) Calculate the threshold population inversion and the number density of the upper energy level of the lasing transition at the threshold.
   $$\left(N_j - N_i\right)_{thr} = \frac{-\ln \rho_1\rho_2}{2\sigma\ell}$$
   $$= \frac{0.311}{2 \cdot 1.27e-20cm^2 \cdot 10cm} = 1.225e18cm^{-3}$$

   $$N_j - N_i = 0.1225e19cm^{-3}$$
   $$N_j + N_i = 1.58e19cm^{-3}$$
   $$2N_j = 1.7025e19cm^{-3}$$
   $$N_j = 8.51e18cm^{-3}$$

3. In a typical He-Ne laser operating at 633 nm the length of the gain medium (cavity) is 0.5 m. The reflectivities of the cavity mirrors are 0.998 and 0.980 for the high reflector and output coupler, respectively. The beam suffers single pass losses totaling of 1.6%. Calculate the *gain threshold*, *threshold population inversion* and *longitudinal mode spacing* in nm for this system assuming the cross-section for stimulated emission is 65e-22 m^2.

$$G_{thr} = \sqrt{\frac{1}{(1-\ell)\rho_1\rho_2}} = e^{\sigma\left(N_j - N_i\right)_{thr} l}$$

$$G_{thr} = \sqrt{\frac{1}{(1-\ell)\rho_1\rho_2}} = \sqrt{\frac{1}{(1-2\cdot 016)0.998\cdot 0.98}} = 1.028 \text{ over the whole cavity}$$

$$\gamma(\nu) \geq \frac{1}{2L}\ln\left(\frac{1}{(1-\ell)\rho_1\rho_2}\right) = \frac{1}{2L}\ln\left(\frac{1}{(1-2\cdot 016)0.998\cdot 0.98}\right) = 0.055m^{-1}$$

$$\Delta\mathrm{N}_{\mathrm{thr}} = \frac{\ln\left(1/(1-\ell)\rho_1\rho_2\right)}{2\sigma_{SE}l} = \frac{\ln\left(1/(1-2\cdot 016)0.998\cdot 0.98\right)}{2\cdot 65e-22m^2\cdot 0.5\,m} = 8.42e18m^{-3}$$

$$\Delta\nu = \frac{c/n}{2L} = \frac{3e8m/s}{2\cdot 0.5\,m} = 300\,MHz$$

$$\Delta\lambda = \frac{c}{\nu^2}\Delta\nu = \frac{3e8m/s}{\left(4.74e14Hz\right)^2}300\,MHz = 4.005e-4nm$$

4. Calculate the average power produced by a Q-switched Nd:YAG laser pumped to five times its threshold population inversion. You can look for an equation for the pulse energy stored in the cavity, but you can write it yourself if you think about what the population inversion is measuring (this time the units help). The Nd:YAG rod fills the 0.1 m cavity and has a 40 mm^2 cross-sectional area. The cavity mirror reflectivities are 99.99 and 97.5 percent. The beam suffers 2.5% round trip absorption and scattering losses. The repetition rate is 10 Hz. Optical properties of the Nd:YAG active medium are not given, you must find them.

σ_{SE}values vary ($7e-19\,cm^2$, $28e\text{-}10\,cm^2$). Using the threshold population inversion is

$$\Delta N_{thr} = \frac{\ln\left(1/(1-\ell)\rho_1\rho_2\right)}{2\sigma_{SE}l} = \frac{\log(1/(1-.025).9999\cdot .975)}{2\cdot 7e-23m^2\cdot 0.1m} = 1.574e21m^{-3}$$

The amount of energy stored in the cavity is

$E_{pulse} = m\Delta N_{thr}\cdot V_{medium}\cdot h\nu/2$ (Pop. inversion consumed when half the atoms relax.)

$$= 5\cdot 1.574e21m^{-3}\cdot\left(0.1m\times 40\,mm^2\cdot\left(\frac{1m}{10^3\,mm}\right)^2\right)\cdot\left(6.626e-34Js\cdot 2.82e14\right)/2$$

$$= 5\cdot 1.574e21m^{-3}\cdot\left(4e-6\,m^3\right)\cdot 1.87e-19\,J/2$$

$$= 0.0029\,J$$

the average power is

$$\bar{\Phi} = E_{pulse}\mathrm{PRR}{=}0.0029\,J\cdot 10\,Hz = 0.029W$$

5. One of the reasons that photomultiplier tubes (PMTs) have been such important devices for so long is that they are capable of measuring very low light levels. Consider the PMT response to a single photon. If the anode signal is monitored in 2.5 ns time bins, compute the current that would be observed if the PMT exhibited a gain equal to 10^7 and all the secondary photoelectrons arrived within a single time bin. Compute the voltage this pulse would produce if it were measured across a 50 ohm resistor. Explain the difference between photon counting and analog signal acquisition for measuring low light signals using PMTs including advice about when each acquisition mode should and should not be used.

$$i = er = me\frac{N}{\Delta t} = \frac{10^7 \cdot 1.6e-19C \cdot 1}{2.5e-9s} = 6.4e-4A$$

$$V = iR = 6.4e-4A \cdot 50\Omega = 0.032V$$

Analog signal acquisition uses RC circuitry to amplify and process the current produced by arriving photons (and noise) in the PMT. The RC components have time constants so the current generated by the PMT is "integrated" by the electronics into a time-dependent electrical (I or V) signal. Photon counting acquisition is used to capitalize on the fact that each arriving photon generates a pulse of electrons that can be distinguished from noise by magnitude. PC uses a discriminator to set a threshold for the pulse height and a counter to record the detection of pulses above the threshold. Analog detection when radiant power is low is likely to produce imprecise measurements. PC should not be used when radiant power is high because of pulses will arrive faster than the discriminator/counter can detect them (bunching).

6. A photomultiplier tube has a radiant cathodic responsivity of 8.0 mA W^{-1} at 365 nm. It is operated under conditions that produce a gain, m, of 1.5e5 and a collection efficiency, η, of 0.88. If 5.8e-11 A of anodic current can just be detected, what is the minimum detectable flux density in watts and photons per second?

$$R = 8.0e-3\ AW^{-1}$$

$$m = 1.5e5$$

$$\eta = 0.88$$

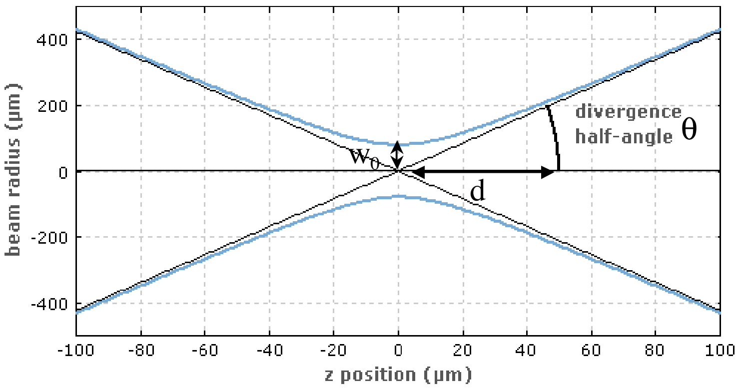
$$i_{\min} = 5.8e-11A$$

$$\begin{aligned}\Phi_{\min} &= i_{\min}/m\eta R \\ &= 5.8e-11A/1.5e5 \cdot 0.88 \cdot 8.0e-3\ AW^{-1} \\ &= 5.49e-8\ W\end{aligned}$$

$$\begin{aligned}(\Phi_{\min})_{hv} &= \Phi_{\min} \cdot 1\ \text{photon}\Big/\frac{\text{hc}}{\lambda} \\ &= 5.49e-8\ W \cdot 1\ \text{photon}\Big/\frac{\text{6.626e-34Js} \cdot \text{3e8m/s}}{365e-9\ m} \\ &= 1.0e5\ \text{photon/s}\end{aligned}$$

7. The Encyclopedia of Laser Physics and Technology defines the beam divergence as a measure for how fast the beam expands far from the beam waist. This source also points out that the $1/e^2$ beam divergence half-angle is $\lambda/(\pi w_0)$, where λ is the wavelength (in the medium) and w_0 the beam radius at the beam waist.

a. Use this result to find the rough size of the smallest patch of light one can get on the moon when a beam of diameter ~ 10 mm from a Nd:YAG laser is pointed at it in lunar ranging experiments.



http://www.rp-photonics.com/beam_divergence.html

The divergence half angle for the Nd:YAG is

$$\theta = \frac{\lambda}{\pi w_0} = \frac{1064nm}{3.1416 \cdot 5e6nm} = 6.67e-5rad$$

The distance between the earth and moon surfaces is 3.762e8m.

The radius of the spot drawn by the beam is

$$r_{spot} = d \tan\theta = 3.762e8m \cdot 1.1e-6 = 4.38e2m$$

So, the area of the spot on the moon's surface is

$$A_{spot} = \pi r_{spot}^2 = 3.1416(4.38e2m)^2 = 6.03e5m^2$$

b. Compute the irradiance of the beam on the surface of a 2.5 cm diameter photodiode on the moon's surface.

Using a 100W laser that experiences no intensity losses

$$\mathcal{E} = \frac{\Phi}{A} = \frac{100W}{1.62e5m^2} = 6.2e-4W/m^2$$

c. Compute the current that would be generated in a photodiode that has a responsivity of 0.10 AW^{-1} and efficiency of 0.93.

Using a one square inch (25.4mm) photodiode:

$$i = \eta R E A_{PD} = 0.93 \cdot 0.10AW^{-1} \cdot \left(6.2e-4Wm^{-2} \cdot 6.25e-4m^2\right) = 36nA$$