A new method for analyzing joint symmetry and normalcy, with an application to analyzing gait

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Abstract

A new method for analyzing symmetry and normalcy of gait patterns is explained. This method utilizes eigenvectors to compare waveforms. The right and left limbs of a single subject are compared to determine symmetry. To determine normalcy, a single limb from a subject is compared to a normative file created from the average of healthy control subjects. The analysis method provides four measures of symmetry and normalcy: trend phase, trend symmetry/normalcy, range amplitude ratio, and range offset. The trend symmetry/normalcy is a comparison of the shapes of the waveforms for each limb. The range amplitude ratio is a comparison of the range of motion of each limb. The range offset is a comparison of the range in which each limb operates. The exact methods and clinical applications are provided.

Keywords: Symmetry; Normalcy; Gait

1. Introduction

A common method of analyzing gait symmetry involves the symmetry index, $\text{SI}(\%) = ((X_R - X_L)/0.5(X_R + X_L)) \times 100$ first proposed by Robinson et al. [1]. $X_R$, the value for the right leg, and $X_L$, the value for the left leg, refer to distinct points in a selected waveform. When the index value is zero, the gait is symmetrical. This index has been used to analyze symmetry in long distance runners, healthy subjects, subjects with chronic stroke and leg length discrepancy, and amputees [2].

Another method to analyze symmetry is the ratio index $R = X_R/X_L$, which has been used to compare peak velocity of below-knee amputees [3], lower limb prosthetic alignment in amputees [4], temporal gait asymmetries in runners [5], and subjects with knee osteoarthritis [6]. Vagenas and Hoshizaki [7] also developed a ratio $I_a = ((L - R)/\max(L, R)) \times 100$, where $L$ and $R$ are the values for the left and right legs, similar to $X_L$ and $X_R$ of the previous indices. Limitations of these ratios include relatively small asymmetry, unknown location of asymmetry, and low sensitivity [2]. Statistical means of determining asymmetry include correlation coefficients, principal component analysis [8], and analysis of variance [2]. Indices and statistical means typically use a single point or a limited set of points from waveforms to determine symmetry and do not analyze the entire curve or gait cycle.

2. Technique description

The purpose of this paper is to present a method of assessing joint symmetry that utilizes the entire selected waveform. The proposed method involves calculating four distinct measures: trend symmetry, phase shift, range amplitude ratio, and range offset. A fifth measure consists of a recalculated trend symmetry value that accounts for phasing differences between waveform pairs.

The measure of trend symmetry utilizes eigenvectors to compare time-normalized right leg and left leg gait cycles in the following manner. Each waveform is translated by
subtracting its mean value from every value in the waveform (Eq. (1)): 

\[
\begin{align*}
\{ X_T \} &= \{ X_i \} - \{ X_m \} \\
\{ Y_T \} &= \{ Y_i \} - \{ Y_m \}
\end{align*}
\]  

(1)

for every \( i \)th pair of \( n \) rows of waveform data

where \( X_T \) and \( Y_T \) are the translated elements of one data point from the right and left waveforms, respectively, \( X_i \) and \( Y_i \) the original elements of one data point from the right and left waveforms respectively, and \( X_m \) and \( Y_m \) are the mean values of each waveform.

Translated data points from the right and left waveforms are entered into a matrix \( (M) \), where each pair of points is a row. The rectangular matrix \( M \) is premultiplied by its transpose \( (M^T M) \) to form a square matrix \( S \), and the eigenvectors are derived from the square matrix \( S \). To simplify the calculation process, we applied a singular value decomposition (SVD) to the translated matrix \( M \) to determine the eigenvectors, since SVD performs the operations of multiplying \( M \) by its transpose and extracting the eigenvectors.

Each row of \( M \) is then rotated by the angle formed between the eigenvector and the \( X \)-axis (\( \theta \)) so that the points lie around the \( X \)-axis (Eq. (2)): 

\[
\begin{align*}
\{ X_R \} &= \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} X_T \end{bmatrix} \\
\{ Y_R \} &= \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} Y_T \end{bmatrix}
\end{align*}
\]  

(2)

for every \( i \)th pair of \( n \) rows of waveform data

where \( X_T \) and \( Y_T \) are the translated elements of one data point from the right and left waveforms respectively, \( X_R \) and \( Y_R \) the rotated elements of one data point from the right and left waveforms respectively, and \( \theta \) is the angle formed by the eigenvector and the \( X \)-axis.

The variability of the points is then calculated along the \( X \) and \( Y \)-axes, where the \( Y \)-axis variability is the variability \emph{about} the eigenvector, and the \( X \)-axis variability is the variability \emph{along} the eigenvector. The trend symmetry value is calculated by taking the ratio of the variability about the eigenvector to the variability along the eigenvector, and is expressed as a percent. A value of 0% indicates perfect symmetry.

The principal eigenvector extracted from the matrix describes the orientation of the distribution of points in matrix \( M \) such that the variability of \( M \) is maximized along the eigenvector. This vector can be thought of in much the same manner as a regression line with one important distinction: the regression line is oriented such that the variability of \( M \) \emph{about} the line of fit is minimized, whereas the eigenvector is oriented such that the variability of \( M \) \emph{along} the line of fit (eigenvector) is maximized. In cases where the waveforms are close in shape, the regression line and the eigenvector would be closely aligned, but in cases where the shapes or magnitudes of the waveforms are different, the two lines would diverge (Fig. 1). Using the principal eigenvector to analyze the variance of the distribution of points formed by the paired waveform data provides a measure of trend symmetry that is unaffected by differences in magnitude between the two waveforms.

Fig. 1. Hip and ankle data from a right below-knee amputee. (A) Hip flexion right (solid) and left (dashed) flexion waveform data. (B) Scatterplot of right hip vs. left hip with eigenvector (solid line) and regression line (dashed line) superimposed. Note that the regression line and the eigenvector lie nearly coincident with each other. (C) Ankle flexion right (solid) and left (dashed) flexion waveform data. (D) Scatterplot of right ankle vs. left ankle with eigenvector (solid line) and regression line (dashed line) superimposed. Note that the regression line and the eigenvector diverge as a result of differences in range of motion between the two waveforms.
A second measure of symmetry examines the phase relationship between waveforms. To do this, we calculated the trend symmetry between the time-normalized right and left limb waveforms for the sagittal plane joint angles. One waveform is phase-shifted in 1% increments (e.g. sample 100 becomes sample 1, sample 1 becomes sample 2, etc.) and the trend symmetry is recalculated for each shift. The phase shift is then determined by identifying the index at which the smallest value for trend symmetry occurs. The phase corrected trend symmetry value that identifies the phase shift is also used as an indicator of symmetry. The phase shift is limited to ±15%. Phase shifts are calculated using sagittal plane joint data because waveforms in the frontal and transverse planes have considerably less magnitude and considerably more variability in terms of their range and their maximum and minimum values, which can result in phase shifts that are clearly inappropriate. We postulated that any phase shift exhibited by the joint would affect all three axes, and that the magnitude of the shift would be most effectively derived from the axis with the largest range magnitude.

We also calculated two additional measures of symmetry between waveforms. Range offset, a measure of the differences in operating range of each limb, is calculated by subtracting the average of the right-side waveform from the average of the left-side waveform. Range amplitude ratio quantifies the difference in range of motion of each limb, and is expressed as a ratio of the range of motion of the left limb to that of the right limb.

![Graphs showing different phase shifts and symmetry measures](image-url)
3. Discussion

Fig. 2 demonstrates examples of using this method to analyze symmetry. Each pair of figures contains time series data of knee flexion angles normalized to 100 points for right and left limbs and a scatterplot of the right versus left curves. In the scatterplot, the right-side waveform is plotted on the X-axis, and the left-side waveform is plotted on the Y-axis.

In Fig. 2A, the left leg curve was created by adding 10° to each point of the right-side curve. In this case, the range offset is 10. Since the range offset is positive, the left side is operating in a greater range of flexion than the right side, specifically ten degrees greater. If the range offset were negative, the left side would be more extended than the right side. A range offset value of zero would indicate both sides operate within the same range of motion.

The left leg curve in Fig. 2C was created by doubling the right-side curve. The range amplitude ratio value is 2, indicating that the left side has a greater range of motion than the right side; in this case, twice as much range of motion. If the range amplitude ratio were less than one, it would mean that the left side had a smaller range of motion than the right.

A value of one would indicate that each curve had the same range of motion.

In Fig. 2E, the left leg curve was created by adding a sine wave with amplitude of 10, frequency of 0.01, and phase of 0 to the right-side curve. The trend symmetry for this curve is 5.17%, an indication that the right and left curves are different. If the curves were the same shape, the trend symmetry would be zero, as in Fig. 2B and D. The larger the trend symmetry, the greater the difference in the shape of the curves. Trend symmetry can be compared between joints. For example, if the trend symmetry at the knee is 3.2 and the trend symmetry at the hip is 1.5, one can state that the hips are more symmetrical than the knees.

In Fig. 2G, the left limb curve was created by shifting the left limb waveform in Fig. 2A by 10%. The comparison of the right limb waveform to the shifted left limb waveform produced a trend symmetry of 35.01%. When the phase shift operation was applied to this waveform pair, it correctly identified the magnitude of the shift as 10%, and the minimum trend symmetry of the realigned waveform was calculated to be 0%. Because of the effect a phase shift can have on the trend symmetry measure, we calculate trend similarities from raw waveform pairs as well as from phase aligned waveform pairs. Only sagittal plane waveforms are used to make the phase adjustments. Phase shifts can be calculated for each individual joint or for a single joint and then applied to the other joints within the limb. If the latter is performed, it is recommended to use the most proximal joint (e.g. the hip for the leg) because it is the least complex of the curves generated. In our experience, the most complex of the curves (the ankle) generally has a greater phase shift value than the knee and hip.

3.1. Symmetry norm values

The symmetry measures were calculated using gait data [9] from 96 healthy subjects (49 females, 47 males, age = 9.1 ± 4.2 years, height = 131.9 ± 24.6 cm, weight = 33.8 ± 18.7 kg) to determine normative values. These data were collected with approval from the institutional review board. Trend symmetry, range offset, and range amplitude ratio values were calculated for the hip, knee, and ankle joints in each of the three planes of motion. Phase shifts were determined for the sagittal plane joint angles and then used to determine the minimum trend symmetry for all

<table>
<thead>
<tr>
<th>Symmetry</th>
<th>Trend symmetry</th>
<th>Phase shift (% Gait cycle)</th>
<th>Minimum trend symmetry</th>
<th>Range amplitude ratio</th>
<th>Range offset (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hip</td>
<td></td>
<td></td>
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<tr>
<td>95% CI</td>
<td>0.83</td>
<td>−2.3, 2.7</td>
<td>0.30</td>
<td>0.86, 1.11</td>
<td>−5.8, 6.0</td>
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<td>0.05</td>
<td>1.04</td>
<td>−3.8</td>
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<td>0.98</td>
<td>−5.6</td>
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<td>0.24</td>
<td>1.02</td>
<td>2.0</td>
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<td>Knee</td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>0.85, 1.12</td>
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<td>3.5</td>
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<td></td>
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<td>−3</td>
<td><strong>24.76</strong></td>
<td><strong>1.72</strong></td>
<td>−5.6</td>
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<td><strong>17.56</strong></td>
<td><strong>1.30</strong></td>
<td>−3.7</td>
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</table>

Bold numbers indicate values outside the 95% confidence interval. Normative 95% confidence interval symmetry values for the trend symmetry, phase shift and minimum trend symmetry for 96 healthy subjects walking at a self-preferred pace are also shown.
three planes of motion. Table 1 contains the 95% confidence interval values for the hip, knee, and ankle sagittal plane variables.

3.2. Application to clinical data

Fig. 3 illustrates three examples of the application of this symmetry method to clinical data. For purposes of brevity, only sagittal plane data are reported in this paper. Subject A is a healthy female subject (age = 29 years, height = 155 cm, weight = 56 kg) who walked in an unbraced condition and in a braced condition with limited plantarflexion of the right ankle. Subject B is a male right-side below-knee amputee (age = 52, height = 180 cm, mass = 100 kg). Each waveform in Fig. 3 is an ensemble average of 10 trials. The waveforms were used to calculate trend symmetry, phase shift, minimum trend symmetry, range amplitude ratio and range offset (Table 1). These measures quantify the changes in symmetry as a function of treatment intervention.

3.2.1. Subject A

Symmetry values for Subject A in the unbraced condition fell within the 95% confidence interval (CI) of normal subjects for all measures (Table 1). With the exception of the minimum trend symmetry at the hip, the addition of the brace to the right ankle affected the symmetry values at the ankle only. The addition of the ankle brace caused the trend symmetry values at the ankle to exceed the 95% CI, and the phase shift increased to 3% of the gait cycle (Table 1). The trend symmetry increased from 0.94% in the unbraced condition to 29.01% in the braced condition, indicating the right ankle waveform followed a different trend compared to the left ankle. Correcting for the phase shift of 3% decreased the trend symmetry to 24.76%, but this value still exceeds the 95% CI, indicating that the right and left ankle were not operating in similar manners. The range amplitude ratio of 1.72 with the brace was also above the 95% CI for the ankle. This is due to the fact that the plantarflexion was limited in the right ankle, causing the right ankle to have a smaller range of motion than the freely mobile left ankle. Range offset values were within the 95% CI for all joints. In this healthy subject, the addition of the ankle brace produced substantial changes in ankle symmetry, and a slight change in symmetry at the hip.

3.2.2. Subject B

Subject B, the below-knee amputee subject, had symmetry values exceeding the 95% CI at the knee and ankle. At the knee, both the trend symmetry and the phase...
shift were above the upper limits of the 95% CIs. Correcting for the phase shift of 4% did not appreciably change the trend symmetry, indicating that the knee waveforms exhibited different trends at their natural phrase orientation. As expected, the ankles also showed dissimilar trends, producing a trend symmetry value well above the upper limit of the 95% CI. The range amplitude ratio value was above the 95% CI indicating that the left side operated with a larger range of motion (1.3 times greater) than the right ankle (the prosthetic side). No substantial difference in symmetry was noted for the hip joints.

4. Normalcy value

The eigenvector approach to determining joint symmetry can also be used to provide normalcy values if one side is referenced to a normal curve instead of to the contralateral limb. This approach provides insights into the manner in which adjustments in symmetry occur. Specifically, the approach will reveal whether changes in symmetry are due to one leg only or due to changes in both legs. This approach will also allow clinicians/researchers to track gait changes over time or following therapeutic intervention.

Data from Subjects A and B were also used to illustrate the use of normalcy values. To calculate the 95% CI for normalcy, waveforms from each of the healthy subjects were compared to the normative waveforms calculated from the entire group of 96 subjects, and resulting values from right and left limbs were combined.

4.1. Subject A

Subject A walked at a self-selected pace in two conditions, unbraced and with a right ankle brace (Table 2). With the addition of a right ankle brace, the right ankle became much less normal, while the left ankle remained in the normal range. The phase shift of the right ankle exceeded the upper limit of the 95% CI with the addition of the brace. Applying the phase shift correction did not decrease the trend normalcy value (corresponds to trend

<table>
<thead>
<tr>
<th>Normalcy</th>
<th>Trend normalcy</th>
<th>Phase shift (% gait cycle)</th>
<th>Minimum trend normalcy</th>
<th>Range amplitude ratio</th>
<th>Range offset (°)</th>
</tr>
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<tbody>
<tr>
<td>Hip</td>
<td></td>
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<td>0, 7.02</td>
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<td><strong>52.99</strong></td>
<td>10</td>
<td>2.69</td>
<td>1.40</td>
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</table>

Bold numbers indicate values outside the 95% confidence interval. Normative 95% confidence interval normalcy values for the trend normalcy, phase shift and minimum trend normalcy for 96 healthy subjects walking at a self-preferred pace are also included.
Symmetry value) to within the normal range, indicating that
the ankle motion did not follow a normal trend. The range
offset exceeded the normal range, as well. The range offset
increased from 1.2° of dorsiflexion in the unbraced
condition, to 8.7° of dorsiflexion in the braced condition.

Normalcy values for the knee showed no substantial changes
with the addition of the ankle brace. Trend normalcy values
for the hip indicate that the trend symmetry value fell outside
the 95% CI because of changes in the right hip. The results of
this comparison help to explain differences in symmetry
between the subject’s unbraced and braced conditions. The
change in trend normalcy for the right ankle (braced) was
much greater than the left ankle, and the loss of range of
motion increased the range offset. The normalcy values of
the left ankle changed only slightly to compensate for the
lack of motion of the right ankle due to the addition of the
ankle brace.

4.2. Subject B

While the hip waveforms were found to be symmetrical,
raw trend normalcy values indicated an abnormal trend in
both right and left hip waveforms. Phase-corrected trend
normalcy values at the hip indicated that both hips were
within the 95% CI and would be considered normal.
Conversely, the knee joint exhibited abnormal patterns of
motion on the healthy side as evidenced by both raw and
phase-corrected trend normalcy values. Both right and left
ankles produced trend normalcy values and phase shift
values above the 95% CI. Applying the phase shift
correction to the left ankle normalized the trend normalcy,
while the right side’s phase-corrected trend normalcy
remained above the 95% CI. Results of the analysis indicate
that this subject exhibits abnormal phase characteristics of
the kinematics of all three joints. In addition, raw trend
normalcy values are near or above the upper limits of the
95% CI for all three joints. However, phase-corrected trend
normalcy values indicate that the more significant kinematic
changes are occurring in the healthy knee and the prosthetic
side hip.

5. Precautions

The approach for measuring joint symmetry and
normalcy presented here is intended for waveform data
only, and would not be appropriate for measuring symmetry
for discrete data points such as temporal-spatial measures. In
addition, the approach should only be applied to waveforms
of like kind. For example, sagittal plane knee data should
only be compared to sagittal plane knee data. It would not be
advisable to use this approach to symmetry analysis to
compare a sagittal plane knee curve to a sagittal plane ankle
curve.

6. Conclusion

The eigenvector approach to determining joint symmetry
and normalcy provides useful information in quantifying
and tracking changes in joint motion.

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