Seminar in Business Processes and Operations Management

BUEC 865
Dr. C. Kydd

Exponential Smoothing Model

Uses a combination of most recent data point and most recent forecast
Weight (smoothing constant) is assigned to data and forecast points
Weights can be varied between 0 and 1

Exponential Smoothing Model - formulas

$$F_{t+1} = \alpha D_t + (1-\alpha) F_t$$

or

$$F_{t+1} = F_t + \alpha (D_t - F_t)$$

F_{t+1} = Forecast for the next period (Period t+1)

α = Smoothing constant ($0 <= \alpha <=1$)

D_t = Actual data for current period (Period t)

F_t = Forecast for current period made in last period

Smoothing constant

α = smoothing constant

$0 <= \alpha <= 1$
$.1 <= \alpha <= .4$ usually

$$\text{Forecast for next pd.} = \alpha \begin{bmatrix}\text{current}\\ \text{demand}\end{bmatrix} + (1-\alpha)\begin{bmatrix}\text{previous}\\ \text{forecast}\end{bmatrix}$$

Exponential Smoothing Model - formulas

$$F_{t+1} = \alpha D_t + (1-\alpha)\, F_t$$

or

$$F_{t+1} = F_t + \alpha\,(D_t - F_t)$$

F_{t+1} = Forecast for the next period (Period t+1)

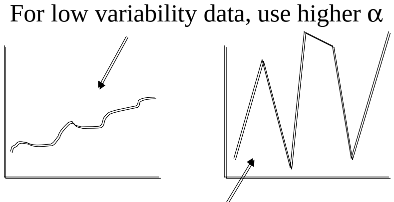
α = Smoothing constant ($0 <= \alpha <= 1$)

D_t = Actual data for current period (Period t)

F_t = Forecast for current period made in last period

Smoothing Constant

For low variability data, use higher α



For high variability data, use lower α

Exponential Smoothing Formulas

$$F_{t+1} = \alpha D_t + (1-\alpha)\, F_t$$

$$F_t = \boxed{\alpha D_{t-1} + (1-\alpha)\, F_{t-1}}$$

$$F_{t+1} = \alpha\, (D_t) + (1-\alpha)\, [\, \alpha D_{t-1} + (1-\alpha)\, F_{t-1}\,]$$

$$F_{t-1} = \boxed{\alpha D_{t-2} + (1-\alpha)\, (F_{t-2})}$$

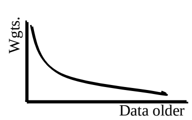
Exponential Smoothing Formulas *(cont.)*

$$F_{t+1} =$$

$$\alpha D_t + (1-\alpha)\, [\, \alpha D_{t-1} + (1-\alpha)\, (\alpha D_{t-2} + (1-\alpha) F_{t-2})]$$

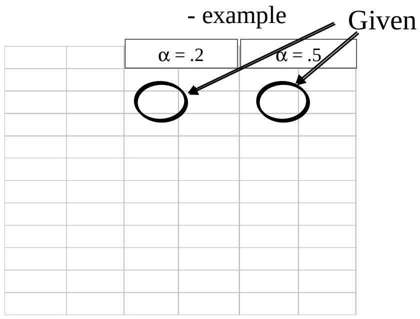
Exponential Smoothing Formulas *(cont.)*

Data	Weight	*example*
D_t	α	.3
D_{t-1}	$\alpha\,(1-\alpha)$	.21
D_{t-2}	$\alpha\,(1-\alpha)^2$	.147



Exponential Smoothing Model

- example



Comparison of Exponential Smoothing Models

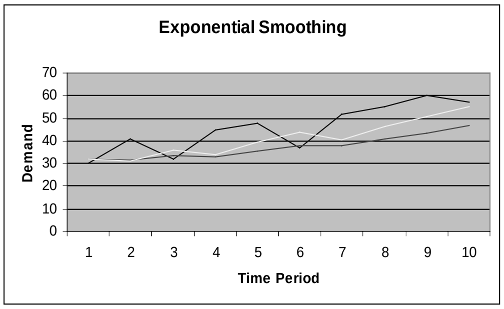
E.S. α = .2	E.S. α = .5
MAD = 9.31	MAD = 7.37
Bias = 8.39	Bias = 4.83

E.S. with α = .5 yields better MAD and Bias, so provides better model

Exponential Smoothing model - tracking signals with α = .2

T.S.$_1$ = -1	T.S.$_6$ = 4.5
T.S.$_2$ = 1.3	T.S.$_7$ = 5.7
T.S.$_3$ = 1.4	T.S.$_8$ = 6.9
T.S.$_4$ = 2.9	T.S.$_9$ = 8.0
T.S.$_5$ = 4.1	T.S.$_{10}$ = 9.0

Exponential Smoothing Forecasts



Double Smoothing

givens

Pd.	D_t	F_t'	F_t''	$F_t'-F_t''$	$b_t=F_t''-F_{t-1}''$
1	30	32	32	0	
2	41	31.6	32	-0.4	0
3	32	33.5	31.9	1.6	-0.1
4	45	33.2	32.2	1	0.3
5	48	35.6	32.4	3.2	0.2
6	37	38.1	33	5.1	0.6
7	52	37.9	34	3.9	1
8	55	40.7	34.8	5.9	0.8
9	60	43.6	36	7.6	1.2
10	57	46.9	37.5	9.4	1.5

$F_{t+1} = \alpha D_t + (1-\alpha)\, F_t$ (single)

$F''_{t+1} = \alpha F_t' + (1-\alpha)\, F_t''$ (double)

Double Smoothing

Pd.	D_t	F_t'	F_t''	$F_t'-F_t''$	$b_t=F_t''-F_{t-1}''$
1	30	32	32	0	
2	41	31.6	32	-0.4	0
3	32	33.5	31.9	1.6	-0.1
4	45	33.2	32.2	1	0.3
5	48	35.6	32.4	3.2	0.2
6	37	38.1	33	5.1	0.6
7	52	37.9	34	3.9	1
8	55	40.7	34.8	5.9	0.8
9	60	43.6	36	7.6	1.2
10	57	46.9	37.5	9.4	1.5

Final Forecast = $F_t' + (F_t' - F_t'') + b_t$

Double Smoothing (cont.)

Final Forecast	e_t
31.2	9.8
35	-3
34.5	10.5
39	9
43.8	-6.8
42.8	9.2
47.4	7.6
52.4	7.6
57.8	-0.8

$$\text{MAD} = \frac{64.3}{9} = 7.14$$

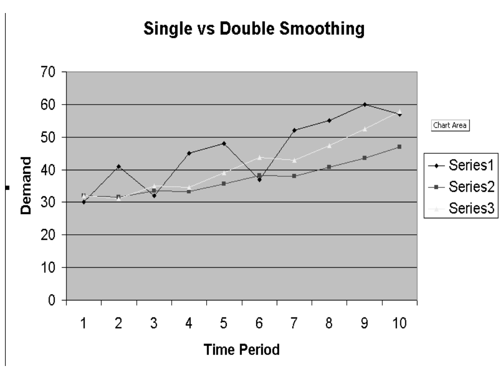
$$\text{Bias} = \frac{43.10}{9} = 4.79$$

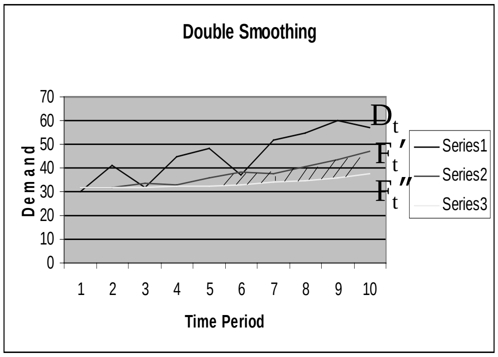
$$\text{Final Forecast} = F_t' + (F_t' - F_t'') + b_t$$

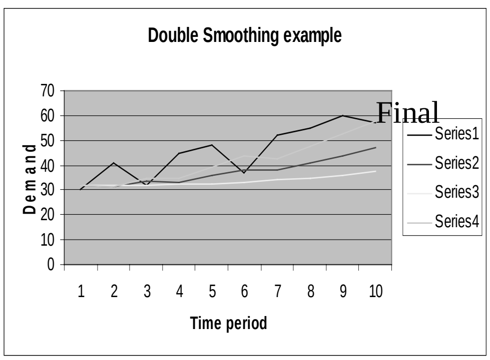
Comparison of Exponential Smoothing Models

E.S. α = .2	E.S. α = .2 (double)
MAD = 9.31	MAD = 7.14
Bias = 8.39	Bias = 4.79

Double E.S. yields better MAD and Bias, so provides better model







Problem 1: Forecasting

You are given sales data for 10 periods.

1. Apply the four time series models to forecast sales for periods 6-11.

2. Compare results and suggest which is best and why.

For these, use sales and time period only.

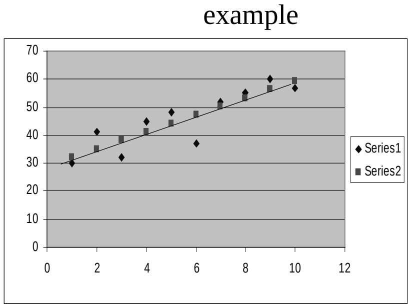
Problem 1 Data

Period	Sales	Mkt price	GNP	CPI
1	35725	57.63	115	161.2
2	47180	78.50	125	170.5
3	54965	62.88	132	181.5
4	63220	53.75	140	195.4
5	66315	50.00	150	217.4
6	57730	45.00	163	246.8
7	62700	38.50	178	272.4
8	60025	62.38	195	289.1
9	74590	74.38	206	298.4
10	83900	78.38	213	311.1

Simple Linear Regression

Attempts to fit a straight line to set of data points
Uses least squares estimate to determine line (minimizes sum of squared deviations between points and line)
Uses equation Y = a + bX where **Y** is dependent variable, **X** is independent variable, **a** is Y-intercept, **b** is slope

Simple Linear Regression - example



Simple Linear Regression

Mass Transit Quarterly Ridership Data

Qtr., X	Volume, Y
1	3.5
2	8
3	5.5
4	10
5	9.5
6	14
7	11.5
8	16
9	15.5
10	20

$Y=a + bX$ or

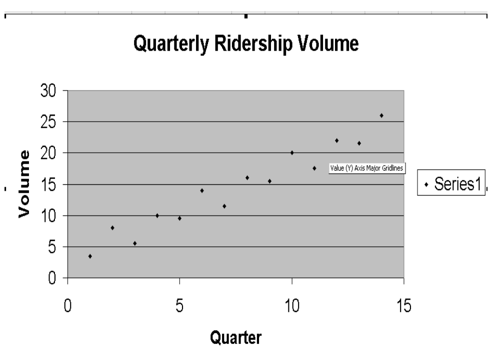
$T_x=a + bX$

Dependent var.

Independent var.

a= Y-intercept

b = slope



Regression slope and intercept

	Coefficients	Standard Error	t Stat
Intercept	2.7252747	0.956410186	2.84948
X Variable 1	1.5461538	0.11232468	13.765

Slope

Simple Linear Regression

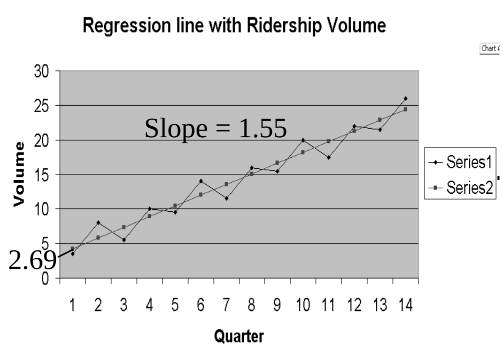
Quarterly Ridership Problem (cont'd.)

$T_x = a + bX = 2.69 + 1.55X$

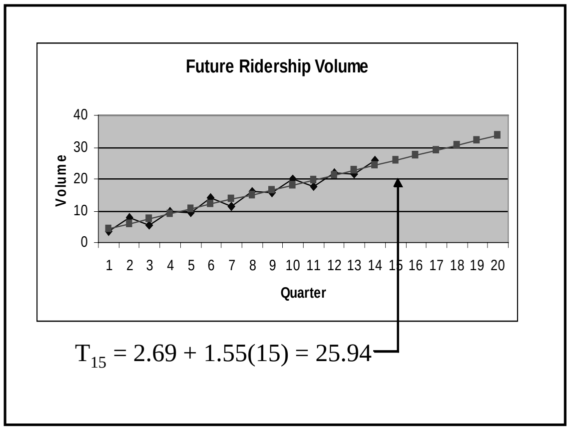
Y-intercept slope

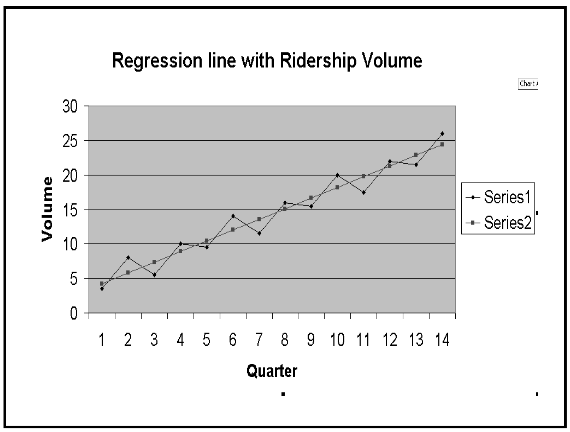
To predict future values of T_x (points on the regression line):

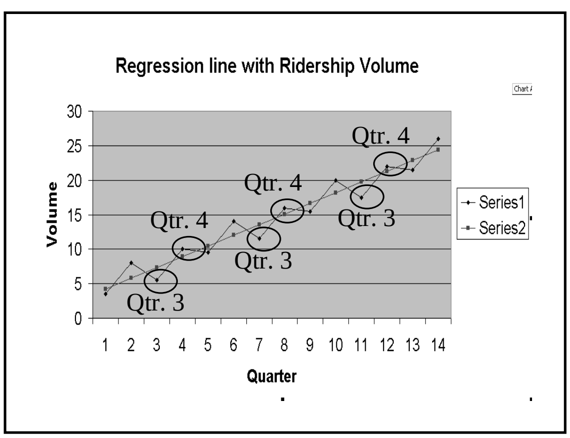
$T_{15} = 2.69 + 1.55(15) = 25.94$
$T_{16} = 2.69 + 1.55(16) = 27.49$
$T_{17} = 2.69 + 1.55(17) = 29.04$



SUMMARY OUTPUT	
Regression Statistics	
Multiple R	0.96976268
R Square	0.94043966
Adjusted R Square	0.9354763
Standard Error	1.69420473
Observations	14


Future Ridership Volume
Volume
Quarter
T_{15} = 2.69 + 1.55(15) = 25.94


Regression line with Ridership Volume
Volume
Quarter
Series1
Series2


Regression line with Ridership Volume
Volume
Quarter
Qtr. 4
Qtr. 4
Qtr. 4
Qtr. 3
Qtr. 3
Qtr. 3
Series1
Series2

Incorporating Seasonality

1. Compute measure of seasonality

$$S = \frac{Y}{T} = \frac{\text{Actual volume}}{\text{Computed trend value}}$$

2. Take average ($\bar{S}$) of all relevant S's

3. Apply $\bar{S}$ to appropriate values of T to forecast into the future

Finding Seasonal Factor for Quarter 4

1. Compute measure of seasonality

$$\left.\begin{array}{l} T_4 = 2.69 + 1.55\ (4) = 8.89 \\ T_8 = 2.69 + 1.55\ (8) = 15.09 \\ T_{12} = 2.69 + 1.55\ (12) = 21.29 \end{array}\right\} \text{Computed T values}$$

$$\left.\begin{array}{l} S_4 = 10/8.89 = 1.12 \\ S_8 = 16/15.09 = 1.06 \\ S_{12} = 22/21.2 = 1.03 \end{array}\right\}$$

2. Average S's

$\bar{S}_4 = 1.07$

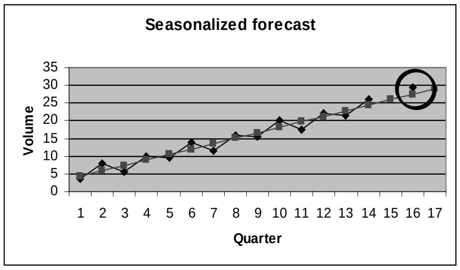
Finding Seasonal Factor for Quarter 4

3. Apply $\bar{S}$ to appropriate values of T

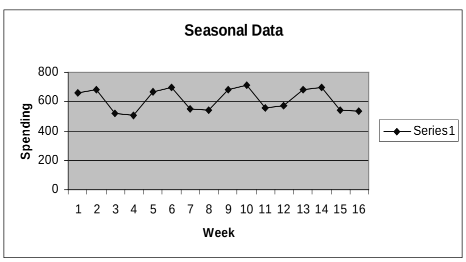
$$T_{16} = 2.69 + 1.55(16) = 27.49$$

$$\text{Adjusted } T_{16} = 27.49\ (1.07) = 29.41$$

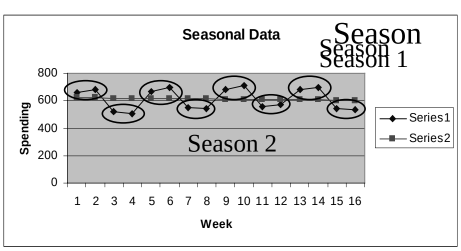
$\bar{S}_4$



Seasonality - Identifying seasons



Identifying seasons



Problem 1 (cont'd.)

Use simple regression to forecast values for sales, market price, CPI and GNP for periods 11-13.

Comment on how well each forecast fits the data.

In each case, time will be the independent variable.

Use POM for Windows/Forecasting/Time series module.

Forecasting Quiz

Find forecast for April and May using

1) 2-month moving average

2) Exp. Smooth. with $\alpha = .4$

	Actual	Forecast
Jan.	30 units	
Feb.	45 units	
Mar.	55 units	50
Apr.	65 units	
May	70 units	

Forecasting Quiz

Find forecast for April and May using

3) Double smoothing model with $\alpha = .4$

(again assume $F''_{Mar} = 50$)

Which model works best here?

Forecasting Quiz - Solution

Find forecast for April and May using

1) 2-month moving average

F(April) = (45 + 55)/2 = 50 e = 15

F(May) = (55 + 65)/2 = 60 e = 10

MAD = 25/2 = 12.5

Bias = 25/2 = 12.5

Forecasting Quiz

Find forecast for April and May using

2) E.S. model with α=.4

$$F(April) = .4(\underbrace{55}_{D_{Mar}})+.6(\underbrace{50}_{F_{Mar}}) = 52 \quad e = 13$$

$$F(May) = .4(\underbrace{65}_{D_{Apr}})+.6(\underbrace{52}_{F_{Apr}}) = 57.2 \quad e = 12.8$$

MAD = 25.8/2 = 12.9

Bias = 25.8/2 = 12.9

Forecasting Quiz

Find forecast for April and May using

3) Double smoothing model with α = .4

(again assume F''_{Mar} = 50)

$$F''(April) = .4(\underbrace{50}_{F'_{Mar}})+.6(\underbrace{50}_{F''_{Mar}}) = 50$$

$$F''(May) = .4(\underbrace{52}_{F'_{Apr}})+.6(\underbrace{50}_{F''_{Apr}}) = 50.8$$

Forecasting Quiz

	D_t	F_t'	F_t"	F_t'-F_t"	b_t	Final F
Jan.	30 units					
Feb.	45 units					
Mar	55 units	50	50			
Apr.	65 units	52	50	2	0	54
May	70 units	57.2	50.8	6.4	.8	64.4

$e_{Apr} = 11$ MAD = 16.6/2=8.3

$e_{May} = 5.6$ Bias = 16.6/2=8.3