ANOVA Design Summary

[Calculating the F-Ratio for a 1-Way ANOVA with Independent Groups 2](#_Toc287802193)

[Design 2](#_Toc287802194)

[Calculations for: 1-way ANOVA, Independent Groups 2](#_Toc287802195)

[Example: 1-way ANOVA with independent groups and 1 factor (A) with 3 levels. 2](#_Toc287802196)

[SPSS Syntax 3](#_Toc287802197)

[Calculating the F-Ratio for a 2-Way ANOVA with Independent Groups 4](#_Toc287802198)

[Design (2x3) 4](#_Toc287802199)

[Calculations for: 2-way ANOVA, Independent Groups 4](#_Toc287802200)

[Example: 2-way ANOVA with independent groups: factor (A) has 2 levels, factor B has 3 levels. 4](#_Toc287802201)

[SPSS Syntax 6](#_Toc287802202)

[Calculating the F-Ratio for a 1-Way ANOVA with Repeated Measures 7](#_Toc287802203)

[Design 7](#_Toc287802204)

[Calculations for: 1-way ANOVA, Repeated Measures 7](#_Toc287802205)

[Example: 1-way ANOVA with repeated measures 7](#_Toc287802206)

[SPSS Syntax 9](#_Toc287802207)

[Calculating the F-Ratio for a 2-Way ANOVA with Repeated Measures on Both Factors 10](#_Toc287802208)

[Design 10](#_Toc287802209)

[Calculations for: 2-way ANOVA, Repeated Measures 11](#_Toc287802210)

[Example: 2-way ANOVA with repeated measures 11](#_Toc287802211)

[SPSS Syntax 14](#_Toc287802212)

[Calculating the F-Ratio for a 2-Way ANOVA with Repeated Measures on One Factor (B) 15](#_Toc287802213)

[Design 15](#_Toc287802214)

[Calculations for: 2-way ANOVA with Repeated Measures on One Factor (B) 15](#_Toc287802215)

[Example: 2-way ANOVA with repeated measures on one factor (B) 16](#_Toc287802216)

[SPSS Syntax 18](#_Toc287802217)

# Calculating the F-Ratio for a 1-Way ANOVA with Independent Groups

## Design

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | a1 | a2 | a3 |  |
|  | s1 | s6 | s11 |
|  | s2 | s7 | s12 |
|  | s3 | s8 | s13 |
|  | s4 | s9 | s14 |
|  | s5 | s10 | s15 | **T** | **T2** |
| **ΣAi** | **Σa1** | **Σa2** | **Σa3** | **Σ(Σa1+ Σa2+ Σa3)** | **T2** |
| **Σ(Ai)2** | **Σ(a1)2** | **Σ(a2)2** | **Σ(a3)2** |  |
| **Σ(AS)2** | **Σ(a1si)2** | **Σ(a2si)2** | **Σ(a3si)2** |

## Calculations for: 1-way ANOVA, Independent Groups

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Source | df | Expanded df | Formula | Letter Code | Coded Formula |
| A | a-1 | a-1 | $$\frac{ΣA^{2}}{s}-\frac{T^{2}}{as}$$ | [A] | [A] – [T] |
| S/A | a(s-1) | as-a | $$Σ(AS)^{2}-\frac{ΣA^{2}}{s}$$ | [AS] | [AS] – [A] |
| Total | as-1 | as-1 | $$Σ(AS)^{2}-\frac{T^{2}}{as}$$ | [T] | [AS] - [T] |
|  |
| Where: | A = sum of scores in each level of factor A |
|  | a = number of levels of factor A |
|  | T = total sum of scores |
|  | s = number of subjects in each level of factor A |

## Example: 1-way ANOVA with independent groups and 1 factor (A) with 3 levels.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | a1 | a2 | a3 |  |
|  | 16 | 4 | 2 |
|  | 18 | 6 | 10 |
|  | 10 | 8 | 9 |
|  | 12 | 10 | 13 |
|  | 19 | 2 | 11 | T | T2 |
| ΣAi | 75 | 30 | 45 | 150 | 22500 |
| Σ(Ai)2 | 5625 | 900 | 2025 |  |
| **Σ(AS)2** | **1185** | **220** | **475** |

**Calculation of Sums of Squares:**

SSA = [A] = $\left[\frac{5625+900+2025}{5} - \frac{22500}{3\*5}\right]$ = 1710 – 1500 = 210

SSS/A = [AS] = $\left[\left(1185+220+475\right)- \frac{5625+900+2025}{5}\right]$ = 1880 – 1710 = 170

SST = [T] = $\left[\left(1185+220+475\right)- \frac{22500}{3\*5}\right]$ = 1880 – 1500 = 380

**Equations for Calculating F-Ratio:**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Source | SS | df | Mean Square (MS) | F-Ratio |
| A | SSA | a-1 | $$\frac{SS\_{A}}{df\_{A}}$$ | $$\frac{MS\_{A}}{MS\_{S/A}}$$ |
| S/A | SSS/A | a(s-1) | $$\frac{SS\_{S/A}}{df\_{S/A}}$$ |  |
| Total | SST | as-1 |  |  |

**Calculation of F-Ratio:**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Source | SS | df | Mean Square (MS) | F-Ratio |
| A | 210 | 2 | 105 | 7.41\* |
| S/A | 170 | 12 | 14.17 |  |
| Total | 380 | 14 |  |  |

**Post-Hoc Tests Between Levels of Factor A:**

Fischer’s LSD test: Independent t-tests with Bonferroni correction to alpha (α/(# of comparisons).

Alternatively, Tukey HSD and Scheffe post-hoc tests can also be used. Both of these tests hold experimentwise error constant and do not require further correction.

## SPSS Syntax

ONEWAY Value BY Group

 /STATISTICS DESCRIPTIVES HOMOGENEITY

 /POSTHOC=TUKEY LSD BONFERRONI ALPHA(0.05).

# Calculating the F-Ratio for a 2-Way ANOVA with Independent Groups

## Design (2x3)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| a1b1 | a1b2 | a2b1 | a2b2 | a3b1 | a3b2 |
| s1 | S5 | S9 | s13 | S17 | S21 |
| s2 | S6 | s10 | S14 | S18 | S22 |
| s3 | S7 | S11 | s15 | S19 | S23 |
| s4 | S8 | s12 | s16 | S20 | S24 |

## Calculations for: 2-way ANOVA, Independent Groups

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Source | df | Expanded df | Formula | Letter Code | Coded Formula |
| A | a-1 | a-1 | $$\frac{ΣA^{2}}{bs}-\frac{T^{2}}{abs}$$ | [A] | [A] – [T] |
| B | b-1 | b-1 | $$\frac{ΣB^{2}}{as}-\frac{T^{2}}{abs}$$ | [B] | [B] – [T] |
| AxB | (a-1)(b-1) | ab-a-b+1 | $$\frac{Σ(AB)^{2}}{s}$$ | [AB] | [AB] – [A] – [B] + [T] |
| S/A | ab(s-1) | abs-a | $$Σ(ABS)^{2}-\frac{Σ(AB)^{2}}{s}$$ | [ABS] | [ABS] – [AB] |
| Total | abs-1 | abs-1 | $$Σ(ABS)^{2}-\frac{T^{2}}{abs}$$ | [T] | [ABS] - [T] |
|  |
| Where: | A = sum of scores in each level of factor A |
|  | a = number of levels of factor A |
|  | B = sum of scores in each level of factor B |
|  | B = number of levels of factor B |
|  | T = total sum of scores |
|  | s = number of subjects in each level of factor A |

## Example: 2-way ANOVA with independent groups: factor (A) has 2 levels, factor B has 3 levels.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| a1b1 | a1b2 | a2b1 | a2b2 | a3b1 | a3b2 |
| 1 | 15 | 13 | 6 | 9 | 14 |
| 4 | 6 | 5 | 18 | 16 | 7 |
| 0 | 10 | 7 | 9 | 18 | 6 |
| 7 | 13 | 15 | 15 | 13 | 13 |

**Cell and Margin Means**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **a1** | **a2** | **a3** | **Sum** |
| **b1** | 12 | 40 | 56 | **108** |
| **b2** | 44 | 48 | 40 | **132** |
| **Sum** | **56** | **88** | **96** | ***240*** |

**Calculation of Sums of Squares:**

SSA = [A] = $\left[\frac{56^{2}+88^{2}+96^{2}}{2\*4} - \frac{240^{2}}{3\*2\*4}\right]$ = 2512 – 2400 = 112

SSB = [B] = $\left[\frac{108^{2}+132^{2}}{3\*4} - \frac{240^{2}}{3\*2\*4}\right]$ = 2424 – 2400 = 24

SSAxB = [AB] = $\left[\frac{12^{2}+44^{2}+40^{2}+48^{2}+56^{2}+40^{2}}{4} - \frac{56^{2}+88^{2}+96^{2}}{2\*4}-\frac{108^{2}+132^{2}}{3\*4}+\frac{240^{2}}{3\*2\*4}\right]$

= 2680 – 2512 – 2424 +2400 = 144

SSS/AB = [ABS] = [(122+442+…562+402) - $\frac{12^{2}+44^{2}+40^{2}+48^{2}+56^{2}+40^{2}}{4}$] = 3010 – 2680 = 330

SST = [T] = $\left[(12^{2}+44^{2}+…56^{2}+40^{2}) - \frac{240^{2}}{3\*2\*4}\right]$ = 3010 – 2400 = 610

**Equations for Calculating F-Ratio:**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Source | SS | df | Mean Square (MS) | F-Ratio |
| A | SSA | a-1 | $$\frac{SS\_{A}}{df\_{A}}$$ | $$\frac{MS\_{A}}{MS\_{S/AB}}$$ |
| B | SSB | b-1 | $$\frac{SS\_{B}}{df\_{B}}$$ | $$\frac{MS\_{B}}{MS\_{S/AB}}$$ |
| AxB | SSAxB | (a-1)(b-1) | $$\frac{SS\_{AXB}}{df\_{AxB}}$$ | $$\frac{MS\_{AxB}}{MS\_{S/AB}}$$ |
| S/AB | SSS/A | ab(s-1) | $$\frac{SS\_{S/AB}}{df\_{S/AB}}$$ |  |
| Total | SST | abs-1 |  |  |

**Calculation of F-Ratio:**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Source | SS | df | Mean Square (MS) | F-Ratio |
| A | 112 | 2 | 56 | 3.06 |
| B | 24 | 1 | 24 | 1.31 |
| AxB | 144 | 2 | 72 | 3.93\* |
| S/AB | 330 | 18 | 18.33 |  |
| Total | 610 | 23 |  |  |

**Post-Hoc Tests Between Levels of Factor A within a Level of Factor B or Between Levels of Factor B within a level of Factor A:**

Fischer’s LSD test: Independent t-tests with Bonferroni correction to alpha (α/(# of comparisons).

Alternatively, Tukey HSD and Scheffepost-hoc tests can also be used. Both of these tests hold experimentwise error constant and do not require further correction. SPSS will not calculate these tests for simple main effects.

## SPSS Syntax

UNIANOVA Score BY A B

 /METHOD=SSTYPE(3)

 /INTERCEPT=INCLUDE

\*Compare marginal means for factor A\*

 /EMMEANS=TABLES(A) COMPARE ADJ(BONFERRONI)

\*Compare marginal means for factor B\*

 /EMMEANS=TABLES(B) COMPARE ADJ(BONFERRONI)

\*Compare simple main effects for factor A within levels of factor B\*

 /EMMEANS=TABLES(A\*B) COMPARE (A) ADJ(BONFERRONI)

\*Conduct Levene’s test for homogeneity of variance\*

 /PRINT=HOMOGENEITY DESCRIPTIVE

 /CRITERIA=ALPHA(.05)

 /DESIGN=A B A\*B.

# Calculating the F-Ratio for a 1-Way ANOVA with Repeated Measures

## Design

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | a1 | a2 | a3 | SUM |
|  | s1 | s1 | s1 | **Σs1** |
|  | s2 | s2 | s2 | **Σs2** |
|  | s3 | s3 | s3 | **Σs3** |
|  | s4 | s4 | s4 | **Σs4** |
|  | s5 | s5 | s5 | **Σs5** |
| **SUM** | **Σa1** | **Σa2** | **Σa3** | **Σas** |

## Calculations for: 1-way ANOVA, Repeated Measures

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Source | df | Expanded df | Formula | Letter Code | Coded Formula |
| A | a-1 | a-1 | $$\frac{ΣA^{2}}{s}-\frac{T^{2}}{as}$$ | [A] | [A] – [T] |
| S | s-1 | s-1 | $$\frac{ΣS^{2}}{a}-\frac{T^{2}}{as}$$ | [S] | [S] – [T] |
| AxS | (a-1)(s-1) | as-a-s+1 | $$Σ(AS)^{2}-\frac{ΣA^{2}}{s}-\frac{ΣS^{2}}{a}+\frac{T^{2}}{as}$$ | [AS] | [AS] – [A] – [S] + [T] |
| Total | as-1 | as-1 | $$Σ(AS)^{2}-\frac{T^{2}}{as}$$ | [T] | [AS] - [T] |
|  |
| Where: | A = sum of scores in each level of factor A |
|  | a = number of levels of factor A |
|  | S = sum of scores for each subject  |
|  | s = number of subjects |
|  | T = total sum of scores |

## Example: 1-way ANOVA with repeated measures

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | a1 | a2 | a3 | a4 | a5 | a6 | SUM |
| s1 | 7 | 3 | 2 | 2 | 1 | 1 | 16 |
| s2 | 4 | 8 | 3 | 8 | 1 | 2 | 26 |
| s3 | 7 | 6 | 3 | 1 | 5 | 4 | 26 |
| s4 | 8 | 6 | 1 | 0 | 2 | 0 | 17 |
| s5 | 7 | 2 | 3 | 0 | 1 | 3 | 16 |
| s6 | 6 | 3 | 3 | 1 | 1 | 1 | **15** |
| s7 | 4 | 2 | 0 | 0 | 0 | 0 | **6** |
| s8 | 6 | 7 | 5 | 1 | 3 | 2 | **24** |
| **SUM** | **49** | **37** | **20** | **13** | **14** | **13** | **146** |

**Calculation of Sums of Squares:**

SSA = [A] = $\left[\frac{49^{2}+37^{2}+20^{2}+13^{2}+14^{2}+13^{2}}{8} - \frac{146^{2}}{6\*8}\right]$ = 588 – 444.08 = 143.92

SSs = [S] = $\left[\frac{16^{2}+26^{2}+26^{2}+17^{2}+16^{2}+15^{2}+6^{2}+24^{2}}{6} - \frac{146^{2}}{6\*8}\right]$ = 498.33 – 444.08 = 54.25

SSAxB = [AB] = $\left[ (7^{2}+4^{2}+7^{2}….1^{2}+0^{2}+2^{2})-\left[A\right]-[S]+\frac{146^{2}}{6\*8}\right]$ = 740 – 588 - 498.33 + 444.08 = 97.75

SST = [T] = $\left[(7^{2}+4^{2}+7^{2}….1^{2}+0^{2}+2^{2}) - \frac{146^{2}}{6\*8}\right]$ = 740 – 444.08 = 295.92

**Equations for Calculating F-Ratio:**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Source | SS | df | Mean Square (MS) | F-Ratio |
| A | SSA | a-1 | $$\frac{SS\_{A}}{df\_{A}}$$ | $$\frac{MS\_{A}}{MS\_{AxS}}$$ |
| S | SSs | s-1 | $$\frac{SS\_{s}}{df\_{s}}$$ |  |
| AxS | SSAxs | (a-1)(s-1) | $$\frac{SS\_{AxS}}{df\_{AxS}}$$ |  |
| Total | SST | as-1 | $$\frac{SS\_{T}}{df\_{T}}$$ |  |

**Calculation of F-Ratio:**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Source | SS | df | Mean Square (MS) | F-Ratio |
| A | 143.92 | 5 | 28.78 | 10.32\* |
| S | 54.25 | 7 | 7.75 |  |
| AxS | 97.75 | 35 | 2.79 |  |
| Total | 295.92 | 47 |  |  |

**Post-Hoc Tests Between Levels of Factor A:**

Fischer’s LSD test: Dependent t-tests with Bonferroni correction to alpha (α/(# of comparisons).

Alternatively, Tukey HSD and Scheffe post-hoc tests can also be used. Both of these tests hold experimentwise error constant and do not require further correction. SPSS will not calculate these tests for this design.

## SPSS Syntax

GLM a1 a2 a3 a4 a5 a6

 /WSFACTOR=A 6 Simple

 /METHOD=SSTYPE(3)

 /EMMEANS=TABLES(A) COMPARE ADJ(BONFERRONI)

 /PRINT=DESCRIPTIVE

 /CRITERIA=ALPHA(.05)

 /WSDESIGN=A.

# Calculating the F-Ratio for a 2-Way ANOVA with Repeated Measures on Both Factors

## Design

**ABS Matrix**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| a1 b1 | a1 b2 | a1 b3 | a1 b4 | a2 b1 | a2 b2 | a2 b3 | a2 b4 |
| s1 | s1 | s1 | s1 | s1 | s1 | s1 | s1 |
| s2 | s2 | s2 | s2 | s2 | s2 | s2 | s2 |
| s3 | s3 | s3 | s3 | s3 | s3 | s3 | s3 |
| s4 | s4 | s4 | s4 | s4 | s4 | s4 | s4 |
| s5 | s5 | s5 | s5 | s5 | s5 | s5 | s5 |

**AS Matrix**

(each inner cell represents the sum across all levels of B, so:

A1 S1 = A1B1S1 + A1B2S1 + A1B3S1 + A1B4S1)

|  |  |  |  |
| --- | --- | --- | --- |
|  | a1 | a2 | **SUM** |
| **s1** | A1 S1 | A2 S1 | **S1** |
| **s2** | A1 S2 | A2 S2 | **S2** |
| **s3** | A1 S3 | A2 S3 | **S3** |
| **s4** | A1 S4 | A2 S4 | **S4** |
| **s5** | A1 S5 | A2 S5 | **S5** |
| **SUM** | **A1** | **A2** |  |

**BS Matrix**

(each inner cell represents the sum across all levels of A, so:

B1 S1 = A1B1S1 + A2B1S1)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | b1 | b2 | b3 | b4 | **SUM** |
| **s1** | B1 S1 | B2 S1 | B3 S1 | B4 S1 | **S1** |
| **s2** | B1 S2 | B2 S2 | B3 S2 | B4 S2 | **S2** |
| **s3** | B1 S3 | B2 S3 | B3 S3 | B4 S3 | **S3** |
| **s4** | B1 S4 | B2 S4 | B3 S4 | B4 S4 | **S4** |
| **s5** | B1 S5 | B2 S5 | B3 S5 | B4 S5 | **S5** |
| **SUM** | **B1** | **B2** | **B3** | **B4** |  |

**AB Matrix**

(each inner cell represents the sum across all subjects, so:

A1 B1 = A1B1S1 + A1B1S2+ A1B1S3 + A1B1S4+ A1B1S5)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | b1 | b2 | b3 | b4 | **SUM** |
| **a1** | A1 B1 | A1 B2 | A1 B3 | A1 B4 | **A1** |
| **a2** | A2 B1 | A2 B2 | A2 B3 | A2 B4 | **A2** |
| **SUM** | **B1** | **B2** | **B3** | **B4** |  |

## Calculations for: 2-way ANOVA, Repeated Measures

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Source | df | Expanded df | Formula | Letter Code | Coded Formula |
| A | a-1 | a-1 | $$\frac{ΣA^{2}}{bs}-\frac{T^{2}}{abs}$$ | [A] | [A] – [T] |
| S | s-1 | s-1 | $$\frac{ΣS^{2}}{ab}-\frac{T^{2}}{abs}$$ | [S] | [S] – [T] |
| AxS | (a-1)(s-1) | as-a-s+1 | $$\frac{Σ(AS)^{2}}{b}-\frac{ΣA^{2}}{bs}-\frac{ΣS^{2}}{ab}+\frac{T^{2}}{abs}$$ | [AS] | [AS] – [A] – [S] + [T] |
| B | b-1 | b-1 | $$\frac{ΣB^{2}}{as}-\frac{T^{2}}{abs}$$ | [B] | [B] – [T] |
| BxS | (b-1)(s-1) | bs-b-s+1 | $$\frac{Σ(BS)^{2}}{a}-\frac{ΣB^{2}}{as}-\frac{ΣS^{2}}{ab}+\frac{T^{2}}{abs}$$ | [BS] | [BS] – [B] – [S] + [T] |
| AxB | (a-1)(b-1) | ab-a-b+1 | $$\frac{Σ(AB)^{2}}{s}-\frac{ΣA^{2}}{bs}-\frac{ΣB^{2}}{as}+\frac{T^{2}}{abs}$$ | [AS] | [AB] – [A] – [B] + [T] |
| AxBxS | (a-1)(b-1)(s-1) | Abs-ab-as-bs+a+b+s-1 | $Σ\left(ABS\right)^{2}-\frac{Σ\left(AB\right)^{2}}{s}-\frac{Σ\left(AS\right)^{2}}{b}-\frac{Σ\left(BS\right)^{2}}{a}+\frac{ΣA^{2}}{bs}+\frac{ΣB^{2}}{as}+\frac{ΣS^{2}}{ab}-\frac{T^{2}}{abs}$  | [ABS] | [ABS] – [AB]-[AS] – [BS]+[A]+[B] + [S] - [T] |
| Total | abs-1 | abs-1 | $$Σ(ABS)^{2}-\frac{T^{2}}{abs}$$ | [T] | [ABS] - [T] |
|  |
| Where: | A = sum of scores in each level of factor A |
|  | a = number of levels of factor A |
|  | B = sum of scores in each level of factor B |
|  | b = number of levels of factor B |
|  | S = sum of scores for each subject  |
|  | s = number of subjects |
|  | T = total sum of scores |

## Example: 2-way ANOVA with repeated measures

**ABS Matrix**

|  |  |  |
| --- | --- | --- |
|  | a1 | a2 |
|  | b1 | b2 | b3 | b4 | b1 | b2 | b3 | b4 |
| s1 | 3 | 5 | 9 | 6 | 5 | 6 | 11 | 7 |
| s2 | 7 | 11 | 12 | 11 | 10 | 12 | 18 | 15 |
| s3 | 9 | 13 | 14 | 12 | 10 | 15 | 15 | 14 |
| s4 | 4 | 8 | 11 | 7 | 6 | 9 | 13 | 9 |
| s5 | 1 | 3 | 5 | 4 | 3 | 5 | 9 | 7 |

**AS Matrix**

|  |  |  |  |
| --- | --- | --- | --- |
|  | a1 | a2 | **SUM** |
| **s1** | 23 | 29 | **52** |
| **s2** | 41 | 55 | **96** |
| **s3** | 48 | 54 | **102** |
| **s4** | 30 | 37 | **67** |
| **s5** | 13 | 24 | **37** |
| **SUM** | **155** | **199** | 354 |

**BS Matrix**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | b1 | b2 | b3 | b4 | **SUM** |
| **s1** | 8 | 11 | **20** | **13** | **52** |
| **s2** | 17 | 23 | **30** | **26** | **96** |
| **s3** | 19 | 28 | **29** | **26** | **102** |
| **s4** | 10 | 17 | **24** | **16** | **67** |
| **s5** | 4 | 8 | **14** | **11** | **37** |
| **SUM** | **58** | **87** | 117 | 92 | 354 |

**AB Matrix**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | b1 | b2 | b3 | b4 | **SUM** |
| **a1** | 24 | 40 | **51** | **40** | **155** |
| **a2** | 34 | 47 | **66** | **52** | **199** |
| **SUM** | **58** | **87** | 117 | 92 | 354 |

**Calculation of Sums of Squares:**

SSA = [A] = $\left[\frac{155^{2}+199^{2}}{4\*5} - \frac{354^{2}}{2\*4\*5}\right]$ = 3181.30 – 3132.90 = 48.40

SSs = [S] = $\left[\frac{52^{2}+96^{2}+102^{2}+67^{2}+37^{2}}{2\*4} - \frac{354^{2}}{2\*4\*5}\right]$ = 3522.75 – 3132.90 = 389.85

SSAxS = [AS] = $\left[ \frac{(23^{2}+41^{2}+48^{2}….54^{2}+37^{2}+24^{2})}{4}-\left[A\right]-[S]+\frac{354^{2}}{2\*4\*5}\right]$ = 3577.50 – 3181.30 - 3522.75 + 3132.90 = 6.35

SSB = [B] = $\left[\frac{58^{2}+87^{2}+117^{2}+92^{2}}{2\*5} - \frac{354^{2}}{2\*4\*5}\right]$ = 3308.60 – 3132.90 = 175.70

SSBxS = [BS] = $\left[ \frac{(8^{2}+17+19^{2}….26^{2}+16^{2}+11^{2})}{2}-\left[B\right]-[S]+\frac{354^{2}}{2\*4\*5}\right]$ = 3714.00 – 3308.60 - 3522.75 + 3132.90 = 15.55

SSAxB = [AB] = $\left[ \frac{(23^{2}+34^{2}+40^{2}….66^{2}+40^{2}+52^{2})}{5}-\left[A\right]-[B]+\frac{354^{2}}{2\*4\*5}\right]$ = 3360.40 – 3181.30 - 3308.60 + 3132.90 = 3.40

SSAxBxS = [ABS] = $\left[ (3^{2}+7^{2}+9^{2}….14^{2}+9^{2}+7^{2})-\left[AB\right]-[AS]-[BS]+[A]+[B]+[S]-\frac{354^{2}}{2\*4\*5}\right]$

= 3778.00 – 3360.40 - 3577.50 -3360.40 + 3181.30 +3308.60 +3522.75 -3132.90 = 5.85

SST = [T] = $\left[(3^{2}+7^{2}+9^{2}….14^{2}+9^{2}+7^{2}) - \frac{354^{2}}{2\*4\*5}\right]$ = 3778.00 – 3132.90 = 645.10

**Equations for Calculating F-Ratio:**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Source | SS | df | Mean Square (MS) | F-Ratio |
| A | SSA | a-1 | $$\frac{SS\_{A}}{df\_{A}}$$ | $$\frac{MS\_{A}}{MS\_{AxS}}$$ |
| S | SSs | s-1 | $$\frac{SS\_{s}}{df\_{s}}$$ |  |
| AxS | SSAxS | (a-1)(s-1) | $$\frac{SS\_{AxS}}{df\_{AxS}}$$ |  |
| B | SSB | b-1 | $$\frac{SS\_{B}}{df\_{B}}$$ | $$\frac{MS\_{B}}{MS\_{BxS}}$$ |
| BxS | SSBxS | (b-1)(s-1) | $$\frac{SS\_{BxS}}{df\_{BxS}}$$ |  |
| AxB | SSAxB | (a-1)(b-1) | $$\frac{SS\_{AxB}}{df\_{AxB}}$$ | $$\frac{MS\_{AxB}}{MS\_{AxBxS}}$$ |
| AxBxS | SSAxBxS | (a-1)(b-1)(s-1) | $$\frac{SS\_{AxBxS}}{df\_{AxBxS}}$$ |  |
| Total | SST | abs-1 | $$\frac{SS\_{T}}{df\_{T}}$$ |  |

**Calculation of F-Ratio:**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Source | SS | df | Mean Square (MS) | F-Ratio |
| A | 143.92 | 1 | 48.40 | 30.44\* |
| S | 54.25 | 4 | 97.46 |  |
| AxS | 97.75 | 4 | 1.59 |  |
| B | 175.70 | 3 | 58.57 | 45.05\* |
| BxS | 15.55 | 12 | 1.30 |  |
| AxB | 3.40 | 3 | 1.13 | 2.31 |
| AxBxS | 5.85 | 12 | 0.49 |  |
| Total | 645.10 | 39 |  |  |

**Post-Hoc Tests Between Levels of Factor A within a Level of Factor B or Between Levels of Factor B within a level of Factor A:**

Fischer’s LSD test: Dependent t-tests with Bonferroni correction to alpha (α/(# of comparisons).

Alternatively, Tukey HSD and Scheffepost-hoc tests can also be used. Both of these tests hold experimentwise error constant and do not require further correction. SPSS will not calculate these tests for this design.

## SPSS Syntax

GLM a1b1 a1b2 a1b3 a1b4 a2b1 a2b2 a2b3 a2b4

 /WSFACTOR=A 2 Simple B 4 Simple

 /METHOD=SSTYPE(3)

\*Compare marginal means for factor A\*

 /EMMEANS=TABLES(A) COMPARE ADJ(BONFERRONI)

\*Compare marginal means for factor B\*

 /EMMEANS=TABLES(B) COMPARE ADJ(BONFERRONI)

\*Compare simple main effects for factor B within levels of factor A\*

\*Factor A has 2 levels, so it is not necessary to test simple main effects\*

\*of factor A within levels of factor B\*

 /EMMEANS=TABLES(A\*B) COMPARE (B) ADJ(BONFERRONI)

 /PRINT=DESCRIPTIVE

 /CRITERIA=ALPHA(.05)

 /WSDESIGN=A B A\*B.

# Calculating the F-Ratio for a 2-Way ANOVA with Repeated Measures on One Factor (B)

## Design

**ABS Matrix**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| a1 b1 | a1 b2 | a1 b3 | a1 b4 | a2 b1 | a2 b2 | a2 b3 | a2 b4 |
| s1 | s1 | s1 | s1 | s6 | s6 | s6 | s6 |
| s2 | s2 | s2 | s2 | s7 | s7 | s7 | s7 |
| s3 | s3 | s3 | s3 | s8 | s8 | s8 | s8 |
| s4 | s4 | s4 | s4 | s9 | s9 | s9 | s9 |
| s5 | s5 | s5 | s5 | s10 | s10 | s10 | s10 |

**AS Matrix**

(each inner cell represents the sum across all levels of B, so:

A1 S1 = A1B1S1 + A1B2S1 + A1B3S1 + A1B4S1)

|  |  |  |  |
| --- | --- | --- | --- |
|  | a1 |  | a2 |
| **s1** | A1 S1 | s6 | A2 S6 |
| **s2** | A1 S2 | s7 | A2 S7 |
| **s3** | A1 S3 | s8 | A2 S8 |
| **s4** | A1 S4 | s9 | A2 S9 |
| **s5** | A1 S5 | s10 | A2 S10 |
| **SUM** | **A1** |  | **A2** |

**AB Matrix**

(each inner cell represents the sum across all subjects, so:

A1 B1 = A1B1S1 + A1B1S2+ A1B1S3 + A1B1S4+ A1B1S5)

|  |  |  |  |
| --- | --- | --- | --- |
|  | a1 | a2 | **SUM** |
| **b1** | A1 B1 | A2 B1 | **B1** |
| **b2** | A1 B2 | A2 B2 | **B2** |
| **b3** | A1 B3 | A2 B3 | **B3** |
| **b4** | A1 B4 | A2 B4 | **B4** |
| **SUM** | **A1** | **A2** | T |

## Calculations for: 2-way ANOVA with Repeated Measures on One Factor (B)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Source | df | Expanded df | Formula | Letter Code | Coded Formula |
| A | a-1 | a-1 | $$\frac{ΣA^{2}}{bs}-\frac{T^{2}}{abs}$$ | [A] | [A] – [T] |
| S/A | a(s-1) | as-a | $$\frac{Σ(AS)^{2}}{b}-\frac{ΣA^{2}}{bs}$$ | [AS] | [AS] – [A] |
| B | b-1 | b-1 | $$\frac{ΣB^{2}}{as}-\frac{T^{2}}{abs}$$ | [B] | [B] – [T] |
| AxB | (a-1)(b-1) | ab-a-b+1 | $$\frac{Σ(AB)^{2}}{s}-\frac{ΣA^{2}}{bs}-\frac{ΣB^{2}}{as}+\frac{T^{2}}{abs}$$ | [AB] | [AB] – [A] – [B] + [T] |
| BxS/A | a(b-1)(s-1) | abs-ab-as+a | $Σ\left(ABS\right)^{2}-\frac{Σ\left(AB\right)^{2}}{s}-\frac{Σ\left(AS\right)^{2}}{b}+\frac{ΣA^{2}}{bs}$  | [ABS] | [ABS] – [AB]-[AS] – [BS]+[A]+[B] + [S] - [T] |
| Total | abs-1 | abs-1 | $$Σ(ABS)^{2}-\frac{T^{2}}{abs}$$ | [T] | [ABS] - [T] |
|  |
| Where: | A = sum of scores in each level of factor A |
|  | a = number of levels of factor A |
|  | B = sum of scores in each level of factor B |
|  | b = number of levels of factor B |
|  | S = sum of scores for each subject  |
|  | s = number of subjects |
|  | T = total sum of scores |

## Example: 2-way ANOVA with repeated measures on one factor (B)

**ABS Matrix**

|  |  |
| --- | --- |
| a1 | a2 |
|  | b1 | b2 | b3 | b4 |  | b1 | b2 | b3 | b4 |
| s1 | 3 | 5 | 9 | 6 | s6 | 5 | 6 | 11 | 7 |
| s2 | 7 | 11 | 12 | 11 | s7 | 10 | 12 | 18 | 15 |
| s3 | 9 | 13 | 14 | 12 | s8 | 10 | 15 | 15 | 14 |
| s4 | 4 | 8 | 11 | 7 | s9 | 6 | 9 | 13 | 9 |
| s5 | 1 | 3 | 5 | 4 | s10 | 3 | 5 | 9 | 7 |

**AS Matrix**

|  |  |  |  |
| --- | --- | --- | --- |
|  | a1 |  | a2 |
| **s1** | 23 | s6 | 29 |
| **s2** | 41 | s7 | 55 |
| **s3** | 48 | s8 | 54 |
| **s4** | 30 | s9 | 37 |
| **s5** | 13 | s10 | 24 |
| **SUM** | **155** |  | **199** |

**AB Matrix**

|  |  |  |  |
| --- | --- | --- | --- |
|  | a1 | a2 | **SUM** |
| **b1** | 24 | 34 | **58** |
| **b2** | 40 | 47 | **87** |
| **b3** | 51 | 66 | **117** |
| **b4** | 40 | 52 | **92** |
| **SUM** | **155** | **199** | 354 |

**Calculation of Sums of Squares:**

SSA = [A] = $\left[\frac{155^{2}+199^{2}}{4\*5} - \frac{354^{2}}{2\*4\*5}\right]$ = 3181.30 – 3132.90 = 48.40

SSS/A = [AS] = $\left[ \frac{(23^{2}+41^{2}+48^{2}….54^{2}+37^{2}+24^{2})}{4}-\frac{155^{2}+199^{2}}{4\*5}\right]$ = 3577.50 – 3181.30 = 396.20

SSB = [B] = $\left[\frac{58^{2}+87^{2}+117^{2}+92^{2}}{2\*5} - \frac{354^{2}}{2\*4\*5}\right]$ = 3308.60 – 3132.90 = 175.70

SSAxB = [AB] = $\left[ \frac{(23^{2}+34^{2}+40^{2}….66^{2}+40^{2}+52^{2})}{5}-\left[A\right]-[B]+\frac{354^{2}}{2\*4\*5}\right]$ = 3360.40 – 3181.30 - 3308.60 + 3132.90 = 3.40

SSBxS/A = [ABS] = $\left[ (3^{2}+7^{2}+9^{2}….14^{2}+9^{2}+7^{2})-\left[AB\right]-[AS]+[A]\right]$

= 3778.00 – 3360.40 - 3577.50 -3360.40 + 3181.30 = 21.40

SST = [T] = $\left[(3^{2}+7^{2}+9^{2}….14^{2}+9^{2}+7^{2}) - \frac{354^{2}}{2\*4\*5}\right]$ = 3778.00 – 3132.90 = 645.10

**Equations for Calculating F-Ratio:**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Source | SS | df | Mean Square (MS) | F-Ratio |
| A | SSA | a-1 | $$\frac{SS\_{A}}{df\_{A}}$$ | $$\frac{MS\_{A}}{MS\_{S/A}}$$ |
| S/A | SSS/A | a(s-1) | $$\frac{SS\_{S/A}}{df\_{S/A}}$$ |  |
| B | SSB | b-1 | $$\frac{SS\_{B}}{df\_{B}}$$ | $$\frac{MS\_{B}}{MS\_{BxS/A}}$$ |
| AxB | SSAxB | (a-1)(b-1) | $$\frac{SS\_{AxB}}{df\_{AxB}}$$ | $$\frac{MS\_{AxB}}{MS\_{BxS/A}}$$ |
| BxS/A | SSAxBxS | a(b-1)(s-1) | $$\frac{SS\_{BxS/A}}{df\_{BxS/A}}$$ |  |
| Total | SST | abs-1 | $$\frac{SS\_{T}}{df\_{T}}$$ |  |

**Calculation of F-Ratio:**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Source | SS | df | Mean Square (MS) | F-Ratio |
| A | 48.40 | 1 | 48.40 | 0.97 |
| S/A | 396.20 | 8 | 49.52 |  |
| B | 175.70 | 3 | 58.57 | 65.81\* |
| AxB | 3.40 | 3 | 1.13 | 1.27 |
| BxS/A | 21.40 | 24 | 0.89 |  |
| Total | 645.10 | 39 |  |  |

**Post-Hoc Tests Between Levels of Factor A (independent groups) within a Level of Factor B (repeated measure)**

Fischer’s LSD test: Independent t-tests with Bonferroni correction to alpha (α/(# of comparisons).

Alternatively, Tukey HSD and Scheffepost-hoc tests can also be used. Both of these tests hold experimentwise error constant and do not require further correction.

**Post-Hoc Tests Between Levels of Factor B (repeated measures) within a level of Factor A independent group):**

Fischer’s LSD test: Dependent t-tests with Bonferroni correction to alpha (α/(# of comparisons).

Alternatively, Tukey HSD and Scheffepost-hoc tests can also be used. Both of these tests hold experimentwise error constant and do not require further correction. SPSS will not calculate these tests for this design.

## SPSS Syntax

GLM b1 b2 b3 b4 BY A

 /WSFACTOR=B 4 Simple

 /METHOD=SSTYPE(3)

\*Compare marginal means for factor A\*

 /EMMEANS=TABLES(A) COMPARE ADJ(BONFERRONI)

\*Compare marginal means for factor B\*

 /EMMEANS=TABLES(B) COMPARE ADJ(BONFERRONI)

\*Compare simple main effects for factor B within levels of factor A\*

\*Factor A has 2 levels, so it is not necessary to test simple main effects\*

\*of factor A within levels of factor B\*

 /EMMEANS=TABLES(A\*B) COMPARE (B) ADJ(BONFERRONI)

 /PRINT=DESCRIPTIVE

 /CRITERIA=ALPHA(.05)

 /WSDESIGN=B

 /DESIGN=A.