Discrete Random Variable
Problem 4.27

- A rock concert producer has scheduled an outdoor concert on a Saturday.
- If it does not rain, he expects to make $20,000 profit from the concert.
- If it does rain, the producer will be forced to cancel the concert and lose $12,000 (from fees, advertising, stadium rental and so forth).
- The probability of rain on Saturday is .4.

What is the expected profit from the concert?
Answer: Write out the probability distribution

<table>
<thead>
<tr>
<th>$x$</th>
<th>$20,000$</th>
<th>$-12,000$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(x)$</td>
<td>.6</td>
<td>.4</td>
</tr>
</tbody>
</table>

$E(x) = 20,000 \times .6 + (-12,000) \times .4 = $7,200

For a fee of $1,000 an insurance company will insure against all losses from a rained out concert. If the producer buys the insurance, what is her expected profit from the concert?
**Answer**

<table>
<thead>
<tr>
<th>$x$</th>
<th>$20,000$</th>
<th>$0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(x)$</td>
<td>$.6$</td>
<td>$.4$</td>
</tr>
</tbody>
</table>

$E(x) = (20,000 \times .6 + 0 \times .4) - 1,000 = 11,000$

**Problem 4.27**

- Assuming the forecast is accurate, do you believe the insurance company has charged too much or too little?
**Answer**

- Let .6 be the probability they will not pay out
- Let .4 be the probability they will pay out

<table>
<thead>
<tr>
<th>$x$</th>
<th>$$0$</th>
<th>$$12,000$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(x)$</td>
<td>.6</td>
<td>.4</td>
</tr>
</tbody>
</table>

$E(\text{payment}) = (0 \cdot .6 + 12,000 \cdot .4) = \$4,800$

**Problem 4.51, page 192**

- Suppose you are a purchaser officer for a company. You purchase 5 million electrical switches and your supplier guarantees no more than .1% defectives.
- To check the shipment you randomly sample 500 switches, test them, and find 4 defects.
- **Was the supplier’s claim accurate?**
Answer - Binomial Problem

- Expected defects in the sample of 500 is
  - \( E(x) = 500 \times 0.001 = 0.5 \)
  - Variance = \( 500 \times 0.001 \times 0.999 = 0.4995 \)
  - Std dev = \( 0.7068 \)
  - \( 0.5 \pm (3 \times 0.7068) = -1.62 \text{ to } 2.62 \)

Find the probability of 3 or less defects, using formula or Excel

<table>
<thead>
<tr>
<th>X</th>
<th>p(X)</th>
<th>Cum p(X)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.60638</td>
<td>0.60638</td>
</tr>
<tr>
<td>1</td>
<td>0.30349</td>
<td>0.90987</td>
</tr>
<tr>
<td>2</td>
<td>0.07580</td>
<td>0.98567</td>
</tr>
<tr>
<td>3</td>
<td>0.01259</td>
<td>0.99826</td>
</tr>
</tbody>
</table>

\[ P(x \geq 4) = 1 - 0.99826 = 0.00174 \]
Problem 3.61, page 137

- The probability that a microchip fails in its first use is .1
- If it does not fail immediately, the probability that it lasts one year is .99

The experiment is observing whether a microchip fails or not in its first year.

Think of the following events:
- Event A{microchip lasts through first use}
- Event B{microchip lasts from 1st use through end of year}
Define Events A and B with probabilities

- Event A\{microchip lasts through first use\}
  - \( P(A) = .9 \)
- Event B\{microchip lasts from 1\textsuperscript{st} use through end of year\}
  - \( P(B|A) = .99 \)

Answer

- List the sample points of this experiment—observing whether a microchip fails or not in its first year.
  - There are three!
    - Fails in the first use
    - Fails from first use to one year
    - Does not fail in first year

- The problem asks, what is the probability that it does not fail in the first year?
Assign probabilities
- Fails first use \( p = 0.100 \)
- Fails from first use to 1 year \( p = ? \)
- Does not fail in first year \( p = ? \)

How do we get these?

How to solve this
- Event B is a conditional probability
  - Because \( p(\text{lasts 1}\text{st year}|A) = 0.99 \)
  - And A is the probability that is passes the first test (the complement of failing on first test)
- I should be able to use the formula for conditional probability to solve for the other two sample points

\[
P(A|B) = \frac{P(A \cap B)}{P(B)}\]
Write out conditional probability formula

\[ P(\text{lasts 1 year} \mid A) = \frac{P(\text{last 1 year} \cap A)}{P(A)} = .99 \]
\[ P(\text{lasts 1 year} \mid A) = \frac{P(\text{last 1 year} \cap A)}{.9} = .99 \]
\[ (.9)(.99) = P(\text{last 1 year} \cap A) = .891 \]

Answer

- **Assign probabilities**
  - Fails first use \( p = .100 \)
  - Fails from first use to 1 year \( p = .009 \)
  - Does not fail in first year \( p = .891 \)
  - Since the probabilities must sum to one
Confusion of the Inverse

- You are a doctor and one of your patients has a lump in her breast
- You know that research shows that even with a lump, there is only a 1% chance that it is malignant
  - In other words, 1,000 out of 100,000 such lumps will actually be malignant

- Still you urge a mamogram for your patient
- The literature says that mammograms are
  - 80% accurate for malignant lumps
  - 90% accurate for benign lumps

Suppose the mammogram comes back positive - what is the probability of it truly being malignant?
How to solve it

- Set up a hypothetical table of data
  - Based on 100,000 cases of finding a lump
  - What percent are malignant (a table margin
  - What percent are benign
  - Use the probabilities associated with an accurate malignant or benign test result to complete the table
  - Rows and column margins should sum to 100,000

### Hint: set up a hypothetical data table based on 100,000

<table>
<thead>
<tr>
<th>Lump is</th>
<th>Test +</th>
<th>Test -</th>
</tr>
</thead>
<tbody>
<tr>
<td>Malignant</td>
<td>800</td>
<td>200</td>
</tr>
<tr>
<td></td>
<td>.8</td>
<td>.2</td>
</tr>
<tr>
<td>Benign</td>
<td>9,900</td>
<td>89,100</td>
</tr>
<tr>
<td></td>
<td>.1</td>
<td>.9</td>
</tr>
<tr>
<td>Rows</td>
<td>10,700</td>
<td>89,300</td>
</tr>
</tbody>
</table>
Suppose the mamogram comes back positive – what is the probability of it truly being malignant?

- You might be tempted to say .8, since the test is 80% accurate
- But this is row percentage, saying given a malignant lump, what is the probability of getting a positive test
- But the question is really a conditional probability stated as:
  - **Given the mamogram is positive, what is the probability of it being malignant**
  - This is $\frac{800}{10,700} = .0747$

**Odds and Odds Ratios**

- What is the odds of having a positive test versus a negative test for those who have malignant lump?
  - $\frac{800}{200} = 4$
- What is the odds of having a positive test versus a negative test for those with a benign lump?
  - $\frac{9,900}{89,900} = .1101$
Odds Ratio and its interpretation

- What is the odds ratio of those with malignant to benign lumps
  - 4/.1101 = 36.3

- Those with a malignant lump are 36 times more likely to have a positive test than those with a benign lump

Odds and Odds Ratios

- I could have set up my odds to reflect
  - Odds of a Positive test for malignant versus benign 800/9,900 = .0808
  - Odds of a Negative test for malignant versus benign 200/89,100 = .0022
- And the odds ratio would be nearly the same
  - .0808/.0022 = 36