### FACTORS INFLUENCING CONFIDENCE INTERVALS

#### Confidence Interval for the Mean

\[
\bar{x} \pm z_{\alpha/2} \frac{s}{\sqrt{n}}
\]

#### Confidence Interval for a Proportion

\[
\hat{p} \pm z_{\alpha/2} \sqrt{\frac{pq}{n}}
\]

<table>
<thead>
<tr>
<th>FACTOR</th>
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<th>WHEN FACTOR GETS SMALLER</th>
<th>REASON</th>
<th>EXAMPLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>(n)</td>
<td>sample size</td>
<td>C.I. gets larger</td>
<td>Smaller sample size means larger standard error and less certainty</td>
<td>(25/\sqrt{50} = 3.54) (25/\sqrt{500} = 1.12)</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>Type I error</td>
<td>C.I. gets larger</td>
<td>If you accept smaller Type I error, i.e., the probability that a C.I. will not contain the mean, you have to allow for a larger C.I. to be that sure</td>
<td>(Z_{0.05/2} = 1.96) (Z_{0.01/2} = 2.575)</td>
</tr>
<tr>
<td>(1-\alpha)</td>
<td>% C.I.</td>
<td>C.I. get smaller</td>
<td>A smaller % C.I. means you are willing to accept more Type I error, so we can have a tighter C.I.</td>
<td>95% C.I. uses (Z_{0.05/2} = 1.96) (99%) C.I. uses (Z_{0.01/2} = 2.575)</td>
</tr>
<tr>
<td>(\sigma) or (s)</td>
<td>Std Dev of Population; Std Dev of sample</td>
<td>C.I. gets smaller</td>
<td>If the population has less variability the sampling distribution will also have less variability</td>
<td>(25/\sqrt{50} = 3.54) (5/\sqrt{50} = .71)</td>
</tr>
</tbody>
</table>

### FACTORS INFLUENCING A HYPOTHESIS TEST

#### Test Statistic for a Mean

\[
z^* = \frac{(\bar{x} - \mu_0)}{(s / \sqrt{n})} \quad \text{or} \quad t^* = \frac{(\bar{x} - \mu_0)}{(s / \sqrt{n})}
\]

#### Test Statistic for a Proportion

\[
z^* = \frac{(\hat{p} - p_o)}{\sqrt{\frac{p_o q_o}{n}}}
\]

Note: \(P_o\) comes from the Null Hypothesis

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<tr>
<td>(n)</td>
<td>Sample size</td>
<td>Test Statistic gets smaller</td>
<td>Smaller sample size means larger standard error. The denominator of the test statistic will be larger, and (z^<em>) or (t^</em>) will be smaller</td>
<td>(25/\sqrt{50} = 3.54) (25/\sqrt{500} = 1.12)</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>Type I error</td>
<td>Test Statistic unchanged</td>
<td>The rejection region gets larger making it more difficult to reject (H_0)</td>
<td>(Z_{0.05/2} = 1.96) (Z_{0.01/2} = 2.575)</td>
</tr>
<tr>
<td>One or 2-Tailed Test</td>
<td>Nature of Alternative Hypothesis</td>
<td>Test Statistic unchanged</td>
<td>With a one-tailed test, all of (\alpha) is put into the one tail</td>
<td>Two-Tailed (Z_{0.05/2} = 1.96) One Tailed (Z_{0.05} = 1.645)</td>
</tr>
<tr>
<td>(\sigma) or (s)</td>
<td>Std Dev of Population; Std Dev of sample</td>
<td>Test Statistic gets larger</td>
<td>If the population has less variability, the sampling distribution has less variability, and denominator of the T.S. is smaller</td>
<td>(25/\sqrt{50} = 3.54) (5/\sqrt{50} = .71)</td>
</tr>
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</table>
## DIFFERENCE OF MEANS FORMULAS

**Standard Error estimate for Large Sample Difference of Means**

\[ \hat{\sigma}_{(\bar{x}_1 - \bar{x}_2)} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \]

**Confidence Interval for Large Sample Difference of Means**

\[ (\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \]

**Test Statistic for Large Sample Difference of Means**

\[ z^* = \frac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \]

Note: \( D_0 \) comes from the Null Hypothesis

**Standard Error estimate for Small Sample Difference of Means**

\[ s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} \]

Assumes:
- Independent random samples
- approximate normal distribution
- \( \sigma_1 = \sigma_2 \) so we pool our estimate

\[ \hat{\sigma}_{(\bar{x}_1 - \bar{x}_2)} = \sqrt{s_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2}\right)} \]

**Confidence Interval for Small Sample Difference of Means**

\[ (\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2, n_1+n_2-2.d.f} \sqrt{s_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2}\right)} \]

**Test Statistic for Small Sample Difference of Means**

\[ t^* = \frac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{s_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \]

Note: \( D_0 \) comes from the Null Hypothesis

## DIFFERENCE OF PROPORTIONS FORMULAS

**Standard Error estimate for Large Sample Difference of Proportions for Hypothesis Test**

\[ p = \frac{x_1 + x_2}{n_1 + n_2} \]

\[ \hat{\sigma}_{p_1 - p_2} = \sqrt{pq_p \left(\frac{1}{n_1} + \frac{1}{n_2}\right)} \]

**Test Statistic for Large Sample Difference of Proportions**

\[ z^* = \frac{(p_1 - p_2) - D_0}{\sqrt{pq_p \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \]

Note: \( D_0 \) comes from the Null Hypothesis

**Confidence Interval for Large Sample Difference of Proportions**

\[ (p_1 - p_2) \pm z_{\alpha/2} \sqrt{\frac{pq_1}{n_1} + \frac{pq_2}{n_2}} \]