

CISC303

Homework 1: 45 points

Due Sunday, Feb 14

The proofs for the following problems *must* be in the following form:

- Give the statement that you are going to prove.
- Specify that the statement will be proven by induction on (name the induction parameter).
- Specify the base value of the induction parameter and prove the statement for the base value.
- State the induction hypothesis.
- Perform the induction step using the assumption in the induction hypothesis. Say explicitly why the IH is applicable, that you are applying it, and what that means. E.g. “Because the snake is k feet long, the IH applies, and therefore the snake contains fewer than three mice.”
- Draw the conclusion by induction.

1. Read Kozen pp 3-18; read the standards document on the class webpage.
2. (15 points) Prove by weak mathematical induction that $\sum_{i=1}^k i^2 = k(k+1)(2k+1)/6$
3. (15 points) Given the alphabet $\{0,1,X,Y\}$, suppose that you have the following rules for constructing words in a language L1:

- (a) X1 is the shortest word in language L1.
- (b) Every word in L1, other than X1, is one of the following:
 - i. a word in L1 ending in the character X, followed by the character 1
 - ii. a word in L1 ending in the character Y, followed by the character 0
 - iii. a word in L1 ending in the character 0, followed by the character X
 - iv. a word in L1 ending in the character 1, followed by the character Y

Prove by weak mathematical induction that every word in language L1 consists of an alternating sequence of letters and digits; for example, the string **X1Y0X** is a word in language L1, but the string **X1Y01Y** is not a word in language L1. (Hint: use induction on the length of a word in language L1.)

4. (15 points) Given the alphabet $\{A,B\}$, suppose that you have the following rules for constructing words in a language L2:
 - (a) Any single character from the alphabet $\{A,B\}$ is a word in language L2.
 - (b) Other than single character words, every word in L2 is one of the following:
 - i. a word in L2 ending in the character A, followed by the character B
 - ii. a word in L2 ending in the character B, followed by the character A or the character B.

Prove by weak mathematical induction that language L2 does not contain any word with two successive A's.